

# Black Hole Information Paradox without Hawking Radiation

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**Abstract:** By entangling soft massless particles, one can create an arbitrarily large amount of entanglement entropy that carries an arbitrarily small amount of energy. By dropping this entropy into the black hole (b.h.), one can increase the b.h. entropy by an amount that violates the Bekenstein bound or any other reasonable bound, leading to a version of the b.h. information paradox that does not involve Hawking radiation. Among the many proposed solutions for the standard b.h. information paradox with Hawking radiation, only a few can also resolve this version without Hawking radiation. The assumption that both versions should be resolved in the same way significantly helps to reduce the number of possible resolutions.

**Keywords:** black hole information paradox; entanglement entropy

## 1. Introduction

The prediction of Hawking radiation [1] leads to the black hole (b.h.) information paradox [2]. Namely, the final state of Hawking radiation appears to be a mixed thermal state, and the purity of the full state cannot be encoded into entanglement between radiation and b.h. degrees of freedom, either because the b.h. evaporates completely, or the radiation contains more entropy than can be stored in the b.h. with a limited entropy capacity. Thus, it looks as if the full state of the universe evolves from a pure initial state without Hawking radiation into a mixed state with radiation, which contradicts the general principle of unitarity, according to which pure states of closed quantum systems can evolve only into pure states. There are many proposals for possible resolutions of the paradox, as reviewed in [3–17]; for example, that the principle of unitarity is not valid in quantum gravity, that the black hole has a larger entropy capacity than usually believed, that quantum effects allow information to escape from the black hole despite the classical horizon, that the region behind the horizon does not exist, or that that information is hidden in “parallel worlds”. Additional insight is needed to distinguish the proposals that are promising from those that are not.

The standard mechanism of Hawking radiation based on semiclassical approximation [1] is certainly not completely correct, so one might think that a key to the resolution of the paradox is to better understand the mechanism of Hawking radiation itself. In this paper, however, we find a new version of the b.h. information paradox in which Hawking radiation does not play any important role. This suggests that the mechanism of Hawking radiation as such is not essential for paradox resolution.

Our basic idea is a simple thought experiment consisting of only two steps. In the first step, an entangled pair of low-energy photons is prepared, and in the second step, one member of the pair is dropped into the black hole. In this way, one may significantly increase the b.h. entropy without a significant increase in its mass. This is a paradox, because it seems to violate all reasonable entropy bounds.

This new version of the information paradox is logically independent from the standard one and may be interesting on its own. Nevertheless, our main motivation for studying it is to shed some new light on the standard version of the paradox. It turns out that most of the proposed solutions to the standard b.h. information paradox do not help to solve the new version, suggesting that the new version of the information paradox is an



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even harder problem than the standard one. However, we turn this hardness into a virtue, by proposing that both versions of the paradox should be solved in the same manner. In this way, only a few of the proposed resolutions to the standard b.h. information paradox survive as promising approaches to solve both problems at once.

In the rest of the paper, we elaborate these ideas in more detail.

## 2. Entropy with (Almost) No Energy

Thermal entropy  $S_{th}$  is necessarily associated with energy through the thermodynamic relation  $TdS_{th} = dQ$ , where  $T$  is temperature and  $Q$  is heat. However, thermal entropy is not the only kind of entropy. Namely, thermal entropy is the entropy of systems in (or close to) thermal equilibrium, while many physical systems with entropy are far from thermal equilibrium. With a non-thermal entropy at hand, one can, in principle, increase the entropy of a system without increasing its energy.

The most interesting example is entanglement entropy, which can characterize thermal and non-thermal systems. Let  $|e, 1\rangle$  and  $|e, 2\rangle$  be two orthogonal quantum states with the same energy,  $e$ . Taking them to be the states of some massless field such as the electromagnetic field, the energy  $e$  can, in principle, be arbitrarily small. In the simplest case,  $|e, 1\rangle$  and  $|e, 2\rangle$  are one-particle states, but in principle they can also be many-particle states. From such states one can prepare an entangled state

$$\frac{1}{\sqrt{2}}[|e, 1\rangle_L |e, 2\rangle_R + |e, 2\rangle_L |e, 1\rangle_R] \tag{1}$$

where L and R denote two spatially separated subsystems, one positioned on the “left” side of the laboratory and the other on the “right”. The subsystem on the left is a mixed state described by the density matrix

$$\rho_L = \frac{1}{2}(|e, 1\rangle\langle e, 1|)_L + \frac{1}{2}(|e, 2\rangle\langle e, 2|)_L. \tag{2}$$

This subsystem has energy  $e$  and entanglement entropy  $S = \ln 2$ . In principle, as we said,  $e$  can be arbitrarily small (The energy of  $n$  photons of frequency  $\omega$  is  $e = n\hbar\omega$ . Even though  $n$  is an integer, this energy can be arbitrarily small because  $\omega$  can be arbitrarily small). Hence, it makes sense to consider the limit  $e \rightarrow 0$ , which is a subsystem that carries entropy without carrying energy.

Note that the wave function of a massless particle with energy  $e$  is a wave packet of the spatial width  $W \gtrsim e^{-1}$  (Unless stated otherwise, we work in units  $\hbar = c = k_B = G_N = 1$ ). This means that, in Minkowski spacetime, such a low-energy entropy cannot be packed into a small box. We shall see that it changes drastically when a box in Minkowski spacetime is replaced by a black hole.

## 3. Dropping Entropy into the Black Hole

### 3.1. Theoretical Aspects

Now, suppose that the subsystem on the left is dropped into the black hole of initial mass  $M$ . Initially, the subsystem on the left is far from the black hole and has a large size  $W \gtrsim e^{-1}$ . If  $e^{-1}$  is larger than the b.h. radius  $R = 2M$ , one might naively think that the wave packet on the left cannot be inserted into the black hole. However, as the wave packet approaches the b.h. horizon, it suffers exponential blueshift. From the point of view of the external observer far from the black hole, this means that the wave packet shrinks by an exponential factor and becomes much smaller than the b.h. radius  $R$ . Hence, the black hole can absorb the subsystem on the left, so the b.h. entropy and mass are increased by

$$\delta S = \ln 2, \quad \delta M = e. \tag{3}$$

Now, let us see how it can be used to violate the Bekenstein bound. To simplify the analysis, we shall assume that the black hole does not have angular momentum and

electric charge, and that the states  $|e, 1\rangle$  and  $|e, 2\rangle$  have a zero total angular momentum,  $\mathbf{J} = \mathbf{L} + \mathbf{S} = 0$ . Then, according to the Bekenstein bound [18], the b.h. entropy  $S_{BH}$  cannot be larger than the Bekenstein–Hawking entropy

$$S_{BH} = \frac{A}{4} = 4\pi M^2, \tag{4}$$

where  $A = 4\pi R^2$  is the b.h. area. Taking the differential of (4), one obtains  $dS_{BH} = 8\pi M dM$ , which implies that the Bekenstein bound can be violated if

$$\delta S > 8\pi M \delta M. \tag{5}$$

Using (3), this implies that the Bekenstein bound can be violated if

$$e < \frac{\ln 2}{8\pi} \frac{1}{M} = \frac{\ln 2}{8\pi} \frac{m_{Pl}}{M} m_{Pl}, \tag{6}$$

where in the last equality we restored the units in which the Planck mass  $m_{Pl} = G_N^{-1/2}$  is not equal to one. No known physical law forbids the preparation of quantum states with energy satisfying (6). Indeed, for an  $M$  that is small by astronomical standards (say,  $M \sim 10^9 m_{Pl}$ ), a state of photon(s) with energy  $e$  satisfying (6) can easily be prepared and manipulated with current quantum-optics technologies.

Conceptually, such a violation of the Bekenstein bound is somewhat similar to the Bekenstein-bound violation by the monster states studied in [19]. However, the formation of monster states studied in [19] requires rather unrealistic initial conditions. Our mechanism for Bekenstein-bound violation requires only an ordinary initial black hole and some quite realistic manipulations of photons. For other mechanisms of Bekenstein-bound violation, see also [20] and references therein.

For the sake of nit-picking, it may also be relevant to distinguish two different interpretations of the Bekenstein bound. In one interpretation, a black hole should *always* have entropy equal to (4). So if, indeed, the initial b.h. entropy is equal to (4), the Bekenstein bound will be violated by preparing only one entangled pair (1) followed by the subsequent dropping. In another interpretation, the initial b.h. entropy may be smaller than (4). In this interpretation, one entangled pair (1) may not be sufficient to violate the Bekenstein bound, but it does not make much difference because one can prepare a large number  $N$  of independent entangled pairs of the form (1), and then one can drop into the black hole the left member of each pair. In this way, the entropy and mass increase by  $\delta S = N \ln 2$  and  $\delta M = Ne$ , so the Bekenstein bound can always be violated by a sufficiently large  $N$ . In both interpretations, the only crucial condition for the Bekenstein-bound violation is Equation (6).

If the violation of the Bekenstein bound is not surprising enough, we point out that in a similar way any other reasonable bound can be violated. For instance, if the maximal b.h. entropy scales with “naive” volume  $4\pi R^3/3$ , or with the much larger internal volume [21], one can always find a sufficiently small energy  $e$  that violates that bound. In general, any reasonable entropy bound of the form

$$S_{bh} \leq f(M) \tag{7}$$

with  $f'(M) \equiv df(M)/dM > 0$  can be violated by choosing

$$e < \frac{\ln 2}{f'(M)}. \tag{8}$$

Equation (6) is nothing but a special case of the general condition (8).

### 3.2. Practical Aspects

The blueshift discussed above shrinks a wave packet in the longitudinal direction, but not in the transverse direction. A low-energy wave packet which in the transverse direction is initially much wider than the b.h. radius  $R$  will typically remain so when the wave packet approaches the black hole. Hence, in practice, it is not so easy to achieve wave packet absorption by the black hole. Such a problem does not appear for a theoretical black hole in  $1 + 1$  dimensions, but appears for realistic black holes in  $3 + 1$  dimensions. Here, we discuss various possibilities to resolve this practical problem.

1. *Fine tuning of initial conditions.* Hawking radiation can be described in terms of wave packets [1] that are well-localized near the horizon initially, but widely spread around the black hole in the final state. The Schwarzschild black hole is time-independent, so the wave equation in the Schwarzschild background is time-inversion-invariant. Hence, there exist time-inverted solutions of the wave equation that describe wave packets which are widely spread around the black hole initially but localized near the horizon when the packet approaches the horizon. No known principle of physics forbids the preparation of such wave packets, which makes the wave packet absorption possible in principle. In practice, however, a preparation of such a wave packet would require a fine tuning of the initial conditions, suggesting that this approach might be too difficult in practice.
2. *Engineering tricks.* Instead of dealing with the fine tuning of initial conditions, one can devise various engineering tricks that can help the wave packet to enter the black hole. One possibility is to build a wave guide shaped such that it is wide far from the horizon but narrow near the horizon. Another possibility is to build the quantum-optics laboratory near the horizon, so that the wave packet is localized near the horizon initially. With this second possibility the photons may have a much larger initial energy as measured by observers in the laboratory, because their contribution to the b.h. mass is determined by the small red-shifted energy measured by observers far from the black hole. Of course, keeping the wave guide or the laboratory at a stable position near the horizon may lead to additional practical problems, but in principle such problems are not insurmountable.
3. *Many trials.* An atom can absorb a photon with a wavelength much larger than the size of the atom [22]. The catch, of course, is that such absorption is not a classical deterministic process. The absorption is a quantum “jump”, the probability of which is very small. A black hole can absorb a soft photon in the same sense, the probability of which is very small because the cross section of photon scattering on a black hole is of the same order of magnitude as the b.h. area [23]. The small probability  $p$  of absorption of a single copy of (2) can be overcome by a large number  $N_{\text{tr}}$  of trials, with each trial performed with another copy of (2). In this way, the number  $N$  of successful trials for which the photons are actually absorbed is

$$N \sim pN_{\text{tr}}, \quad (9)$$

which can be made arbitrarily large by taking a sufficiently large  $N_{\text{tr}}$ . For practical purposes, the method with many trials is probably the best.

### 4. Resolution of the Paradox—Useless Approaches

The conclusion that any reasonable entropy bound can be violated by dropping sufficiently soft massless particles into the black hole is a paradox, so it needs to be resolved. Can this paradox be resolved in the same way as the standard b.h. information paradox with Hawking radiation? In the next section, we shall discuss which approaches to the standard information paradox may also be useful for the resolution of our version of the paradox. In this section, we shall first eliminate those approaches that do *not* seem useful for our paradox.

1. *No Hawking radiation.* One logical possibility is that Hawking radiation does not exist [24]. While it obviously avoids the standard b.h. information paradox, it does not help because our paradox does not depend on the existence of Hawking radiation.
2. *New physics for small black holes.* Proposals of this sort include Planck-sized remnants [25], the creation of a baby-universe [3], and the sudden escape of information, perhaps via tunneling into a white hole [26]. Presumably all such events happen when the black hole becomes sufficiently small, which is useless for our purpose because our version of the information paradox exists also for large black holes.
3. *Mild modifications of horizon physics.* One possibility is that quantum fluctuations at the horizon allow a slow leak of information [3]. Another possibility is that pre-Hawking radiation prevents the creation of an apparent horizon [27,28]. Since such scenarios involve rather slow processes (slow leaking or slow pre-Hawking radiation in the examples above), they are essentially useless because their effect may easily be overpowered by dropping many copies of (3) at almost the same time.
4. *Radical modifications of horizon physics.* It has been proposed that quantum gravity effects make the b.h. horizon totally impenetrable, due to a fuzzball [29], an energetic curtain [30], or a firewall [31] at  $R = 2M$ . How could such an impenetrable barrier act in an attempt to drop (3) into the hole? If the dropped particles accumulated in a small region in front of the wall, that would be useless because (8) would again violate any reasonable entropy bound in that small region. Alternatively, if the dropping would result in a fast recoil of the dropped particles such that the particles could not accumulate near  $R = 2M$ , that would resolve our version of the b.h. information paradox. However, such a recoil would be observed in astrophysical black holes such as the one in the center of our galaxy, which is not what we observe. One might argue that we do not observe it yet because the firewall forms only after a very long time (Page time [32]), but then we are back to the problem that the fast dropping of entropy can violate any reasonable entropy bound, long before the firewall forms.
5. *Complementarity.* Even though quantum cloning contradicts unitarity, according to the b.h. complementarity principle [33], it is acceptable as long as no single observer can see both copies. This means that one copy of (2) can be destroyed in the black hole, while the other copy can remain outside of the black hole. The outside copies must either be accumulated near  $R = 2M$  or recoiled, leading to the same problems as with the firewall above.
6. *Decoherence and many worlds.* The radiation of a single Hawking particle is a random quantum event. The particle energy can take any value from a large range of possible values. Unitarity, combined with decoherence induced by the macroscopic environment, implies that the total wave function of the universe contains all the branches corresponding to all possible energies of Hawking particles. While it may help to resolve the standard b.h. information paradox [34–40], here it is useless because the states  $|e, 1\rangle$  and  $|e, 2\rangle$  in (2) have the same energy and can be chosen to be indistinguishable at the macroscopic level. This means that the macroscopic environment cannot distinguish  $|e, 1\rangle$  from  $|e, 2\rangle$  and therefore cannot create different branches.
7. *Soft hair.* It has been argued that a black hole has infinitely many soft supertranslation hairs, such that Hawking radiation can be entangled with that hair [41]. This can help to resolve the standard b.h. information paradox, but here it is useless because it does not influence our mechanism for the violation of entropy bounds before evaporation.

## 5. Resolution of the Paradox—Potentially Useful Approaches

Now, let us discuss those approaches to the standard b.h. information paradox that can also resolve our version of the paradox.

1. *Information destroyed in the singularity.* If, as originally proposed by Hawking [2], any excess of information induced by Hawking radiation is destroyed in the b.h. singularity, then so is any excess of information dropped into the black hole by our mechanism. In this sense, information destruction in the singularity is probably

the simplest resolution of our version of the paradox. It has been argued that such a non-unitary evolution violates energy–momentum conservation or locality [42], but a more careful analysis reveals that this is not the case for systems with a large number of degrees of freedom [43]. Moreover, by treating time as a local quantum observable, such information destruction can be reinterpreted as a unitary process in disguise [44–48].

2. *ER=EPR and islands.* According to the ER = EPR conjecture [49], the left and right subsystems in (1) are connected by a wormhole. Therefore, instead of being destroyed in the singularity at  $r = 0$ , the left subsystem can escape from the black hole through the wormhole. Such an escape that bypasses the horizon resolves our version of the b.h. information paradox. A more precise version of this idea involves a black hole island [50], a region in black hole that due to a wormhole should be thought of as a part of the b.h. exterior, rather than the interior.
3. *Gravitational crystal.* By analogy with condensed-matter physics, it has been proposed that general relativity is merely a macroscopic description of a fluid phase of some unknown fundamental degrees of freedom [51]. These fundamental degrees can also exist in a crystal phase that does not obey the laws of general relativity. Instead of being destroyed in the singularity at  $r = 0$ , any excess of information in the black hole is absorbed by a crystal core formed around the center at  $r = 0$ . The entropy of the core scales with its volume  $V_{core}$ , so, instead of (7), the relevant entropy bound is  $S_{core} \leq \alpha V_{core}$  with  $\alpha \sim 1$ . Consequently, the core continuously grows as new information arrives [51]. In this way, since general relativity is not valid in the crystal phase, the core can penetrate the horizon from the inside and become even larger than  $R = 2M$ .

## 6. Discussion and Conclusions

By dropping the entanglement entropy of low-energy massless particles into the black hole, one can violate any reasonable entropy bound of the form (7), which constitutes a new version of the b.h. information paradox. Unlike the standard version of the paradox [2], the new version does not depend on the existence of Hawking radiation. We have argued that most of the known proposals for the resolution of the standard version of the paradox are not useful for the resolution of this new version. The only proposals for the standard version of the paradox that we found useful for the new version are information destruction [2], ER=EPR/islands [49,50], and the gravitational crystal [51]. However, this does not necessarily imply that all other proposals should be rejected. Perhaps some of the eliminated resolutions can still be useful in some refined form, or perhaps the new version of the paradox should be resolved by ideas that do not depend on the resolution of the standard version. In any case, we believe that our new version of the paradox offers a new insight that may stimulate further fundamental research on the b.h. information paradox.

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