Introduction to Generalized Parton Distributions, DVCS and DVMP

Kornelija Passek-K.

Rudjer Boskovic Institute, Croatia

WE-Heraeus Physics School
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Outline

Resolving nucleon structure

SCATTERING

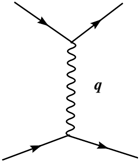
$\rightarrow$ elastic	$(e^- p \rightarrow e^- p)$	}	exclusive
$\rightarrow$ inelastic	$(e^- p \rightarrow e^- \pi p)$	}	exclusive
	$(e^- p \rightarrow e^- X)$	}	inclusive

Resolving nucleon structure

SCATTERING

$\rightarrow$ elastic $(e^- p \rightarrow e^- p)$
$\rightarrow$ inelastic $(e^- p \rightarrow e^- \pi p)$ } exclusive
$(e^- p \rightarrow e^- X)$ } inclusive

ELASTIC SCATTERING on a pointlike particle (s=1/2)

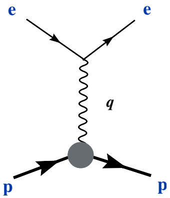


$\gamma^\mu \rightarrow i\mathcal{A}$

$$\rightarrow \frac{d\sigma}{d\Omega_{lab}} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{E'}{E} \left\{ 1 \cos^2 \frac{\theta}{2} - 1 \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\}$$

$$\sim |\mathcal{A}|^2$$

ELASTIC SCATTERING on a composite particle



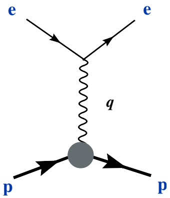
$F_1,\ F_2$ . . . Dirac and Pauli form factors

$$G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2 \qquad G_E^p(0) = 1,\ G_E^n(0) = 0$$

$$G_M = F_1 + \kappa F_2 \qquad G_M^p(0) = 2.79,\ G_M^n(0) = -1.91$$

$$F_1(q^2)\gamma^\mu + \frac{\kappa}{2M} F_2(q^2) i\sigma^{\mu\nu} q_\nu \qquad \to i\mathcal{A}$$

ELASTIC SCATTERING on a composite particle



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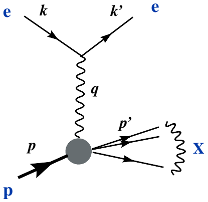
$$\downarrow$$

$$\frac{d\sigma}{d\Omega_{lab}} = \frac{\alpha^2}{4E^2 \sin^4\theta/2} \frac{E'}{E} \left\{ K_2(q^2)\cos^2\frac{\theta}{2} - K_1(q^2)\frac{q^2}{2M^2}\sin^2\frac{\theta}{2} \right\}_{ep\to ep}$$

$$\sim |\mathcal{A}|^2$$

[Rosenbluth formula]

INELASTIC INCLUSIVE SCATTERING



$q = (\nu, \vec{q})$

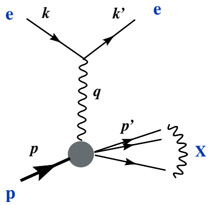
scalars often used:

E', θ (exp.)

q^2, $\nu = \frac{q \cdot p}{M} = E - E'$ (teor.)

q^2, $x = \frac{-q^2}{2 q \cdot p}$ (teor.)

INELASTIC INCLUSIVE SCATTERING



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$$W^{\mu\lambda} = -W_1 g^{\mu\lambda} + \frac{W_2}{M^2} p^\mu p^\lambda + \frac{W_4}{M^2} q^\mu q^\lambda + \frac{W_5}{M^2}(p^\mu q^\lambda + p^\lambda q^\mu)$$

$$q_\mu W^{\mu\lambda} = q_\lambda W^{\mu\lambda} = 0$$

$$d\sigma \sim L^e_{\mu\lambda}\, W^{\mu\lambda}$$

$$\sim \left\{ W_2(q^2,\nu) \cos^2\frac{\theta}{2} + 2 W_1(q^2,\nu) \sin^2\frac{\theta}{2} \right\}_{ep\to eX}$$

W_1, W_2 ...structure functions

DEEP INELASTIC SCATTERING

Bjorken limit:

$$q^2 \to \infty \qquad x = x_B = \frac{-q^2}{2q \cdot p} = cte.$$

$$\nu \to \infty$$

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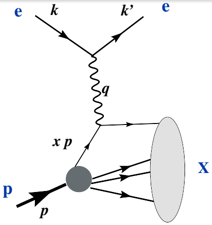

$\to$ sum of elastic e^--parton scatterings

structure functions:

$$M\ W_1(q^2, x) \quad \to \quad F_1(x)$$

$$-\frac{q^2}{2Mx} W_1(q^2, x) \quad \to \quad F_2(x)$$

SCALING VIOLATION IN DEEP INELASTIC SCATTERING

SCALING VIOLATION IN DEEP INELASTIC SCATTERING

structure functions:

$$F_1(x) \rightarrow F_1(x, Q^2)$$
$$F_2(x) \rightarrow F_2(x, Q^2)$$

$$\downarrow$$

ln Q^2 dependence ($Q^2 = -q^2$)

$$\Uparrow$$

parton interactions

PDFs and factorization of DIS

- asymptotic freedom
- factorization

$$\Downarrow$$

PDFs and factorization of DIS

- asymptotic freedom
- factorization

$$\Downarrow$$

structure functions:

$$F_i(x, Q^2) = \sum_a \int dz\ C_i^a(x/z, Q^2/\mu^2)\ f_a(z, \mu^2)$$

μ^2 . . . factorization scale
a . . . parton type

$C_i^a(x/z, Q^2/\mu^2)$ . . . coeficient functions

$f_a(z, \mu^2)$ . . . parton distribution functions (PDFs)

PDFs and factorization of DIS

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- factorization

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PDFs and factorization of DIS

- asymptotic freedom
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$$F_i(x,Q^2) = \sum_a \int dz\, C_i^a(x/z, Q^2/\mu^2)\, f_a(z,\mu^2)$$

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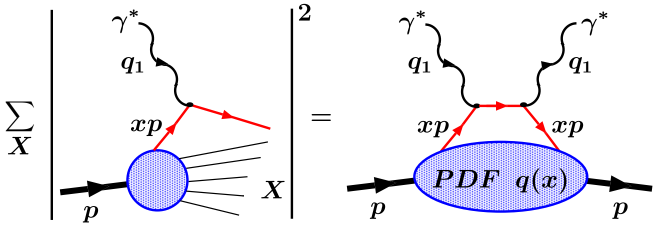
$C_i^a(x/z, Q^2/\mu^2)$ . . . coeficient functions $\to$ pQCD (α_S exp.)

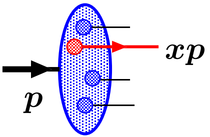
$f_a(z,\mu^2)$ . . . parton distribution functions (PDFs)

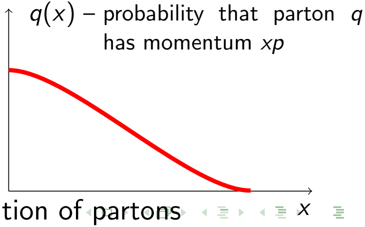
$\to$ nonpert. input + DGLAP evolution equation (pQCD)

Parton distribution functions

- Deeply inelastic scattering

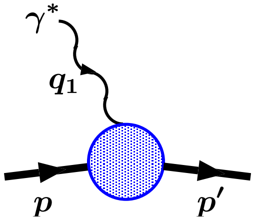


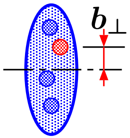




- no information on spatial distribution of partons

Electromagnetic form factors

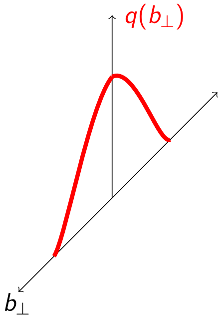




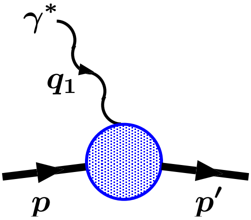
- Form factors → charge distribution

$$\Gamma^\mu(\gamma^* p \to p) = \gamma^\mu F_1(Q^2) + \frac{\kappa_p}{2M_p} i\,\sigma^\mu_\nu q_1^\nu F_2(Q^2)$$

$$q(b_\perp) \sim \int dq_1\, e^{iq_1\cdot b_\perp} F_1(t = q_1^2)$$



Electromagnetic form factors



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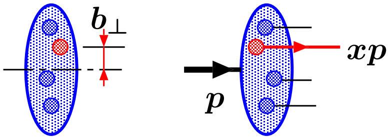

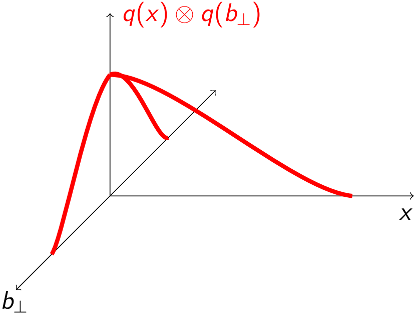

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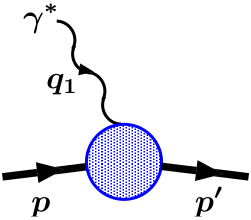

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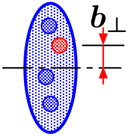
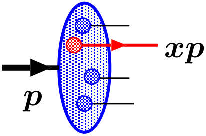
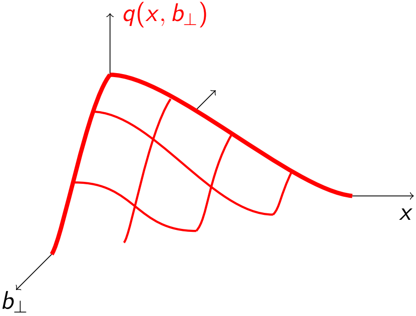
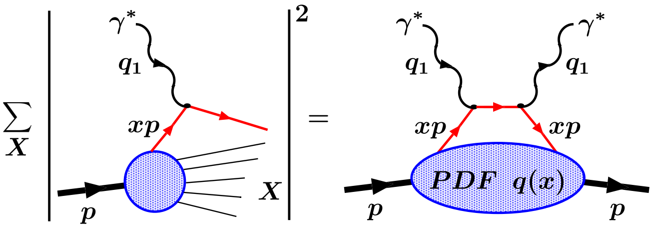






DIS and Compton scattering

- Deeply inelastic scattering $-q_1^2 \equiv Q^2 \to \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p\cdot q_1} \to \mathrm{cte.}$

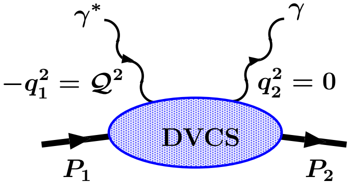


$$\sigma_{tot}(\gamma^* p \to X) \overset{optical\ theorem}{\propto} Im\,\mathcal{A}(\gamma^* p \to \gamma^* p)$$

forward Compton scattering

Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]



$$P = P_1 + P_2$$

$$q = (q_1 + q_2)/2$$

$$\Delta = P_2 - P_1$$

Probing the proton with two photons

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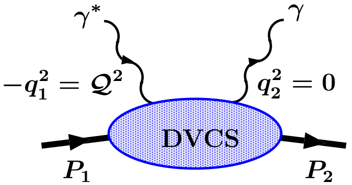


$$P = P_1 + P_2$$
$$q = (q_1 + q_2)/2$$
$$\Delta = P_2 - P_1$$

generalized Bjorken limit:

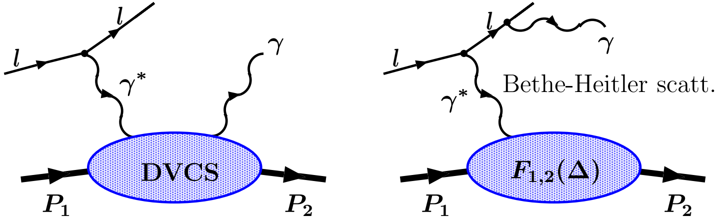
$$-q^2 (\overset{\mathrm{DVCS}}{\simeq} \mathcal{Q}^2/2) \to \infty \qquad \vartheta = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \approx \frac{\eta}{\xi} \overset{\mathrm{DVCS}}{=} 1$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const } (\text{as } x_B) \qquad t = (P_2 - P_1)^2 = \Delta^2$$

$$\sigma \quad \propto \quad |\mathcal{A}(\gamma^* p \to \gamma p)|^2$$

Deeply virtual Compton scattering

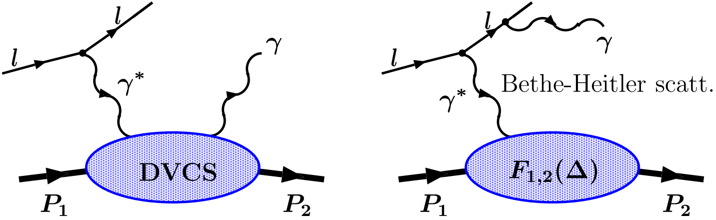
- Measured in $ep \to e\gamma p$



- There is a background process

Deeply virtual Compton scattering

- Measured in $ep \to e\gamma p$



- There is a background process but it can be used to our advantage:

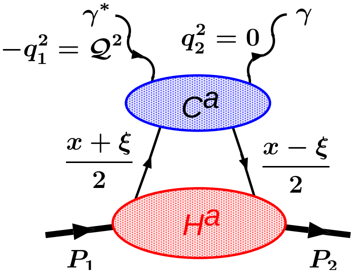
$$\sigma \propto |\mathcal{A}_{\rm DVCS}|^2 + |\mathcal{A}_{\rm BH}|^2 + \mathcal{A}^*_{\rm DVCS}\mathcal{A}_{\rm BH} + \mathcal{A}_{\rm DVCS}\mathcal{A}^*_{\rm BH}$$

- Using $\mathcal{A}_{\rm BH}$ as a referent “source” enables measurement of the phase of $\mathcal{A}_{\rm DVCS}$

Factorization of DVCS $\longrightarrow$ GPDs

$\rightarrow$ cross-section can be expressed in terms of (the squares of)
Compton form factors: $\mathcal{H}(\xi, t, \mathcal{Q}^2)$, $\mathcal{E}(\xi, t, \mathcal{Q}^2)$, $\tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2)$, $\tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2)$, ...

[Collins and Freund '99]



$a = q, G$

- Compton form factor is a convolution:

$$^{a}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2) \; H^a(x, \eta = \xi, t, \mu^2)$$

- $H^a(x, \eta, t, \mu^2)$ — Generalized parton distribution (GPD)

Factorization of DVCS $\longrightarrow$ GPDs

- $C^a(x, \xi, \mathcal{Q}^2/\mu^2)$ . . . hard scattering amplitude

  $\rightarrow$ pQCD

Factorization of DVCS $\longrightarrow$ GPDs

- $C^a(x, \xi, Q^2/\mu^2)$ ... hard scattering amplitude

  $\rightarrow$ pQCD

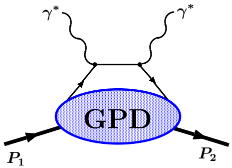
- $H^a(x, \eta = \xi, t, \mu^2)$ ... GPD

  $\rightarrow$ nonperturbative input

  $\rightarrow$ evolution $\Leftarrow$ pQCD (limiting cases DGLAP ($\eta = 0$) and ERBL ($\eta = 1$) evolution equations)

$$\mu^2 \frac{d}{d\mu^2} \mathbf{F}(x, \eta, t, \mu^2) = \int_{-1}^{1} \frac{dy}{2\eta} \mathbf{V}\left(\frac{\eta + x}{2\eta}, \frac{\eta + y}{2\eta}; \eta \middle| \alpha_s(\mu)\right) \cdot \mathbf{F}(y, \eta, t, \mu^2)$$

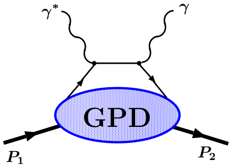
Complementary processes



(double) DVCS

$\boxed{\gamma^* p \to \gamma^* p}$

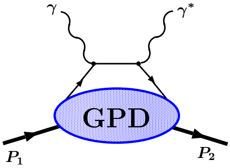
$(ep \to epl^+l^-)$



spacelike DVCS

$\boxed{\gamma^* p \to \gamma p}$

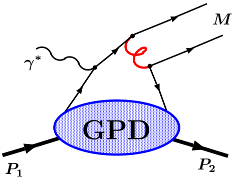
$(ep \to ep\gamma)$



timelike DVCS

$\boxed{\gamma p \to \gamma^* p}$

$(\gamma p \to pl^+l^-)$



Deeply virtual production of mesons (DVMP)

more difficult, but access to flavours

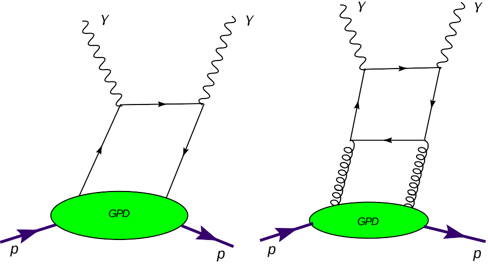
$\boxed{\gamma^* p \to Mp}$

factorization: [Collins, Frankfurt, Strikman '97]

Hard-scattering amplitudes (DV processes vs. meson form factors)

DVCS

$$\gamma^* q \to \gamma q, \; \gamma^* g \to \gamma g$$



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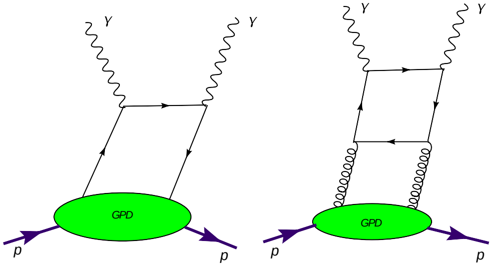
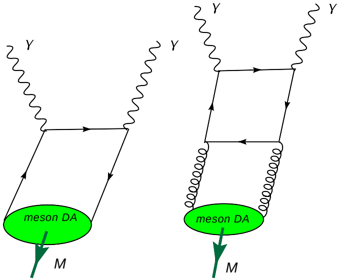


NLO: [Ji et al, Belitsky et al, Mankiewicz et al, '97]

Meson transition form factor

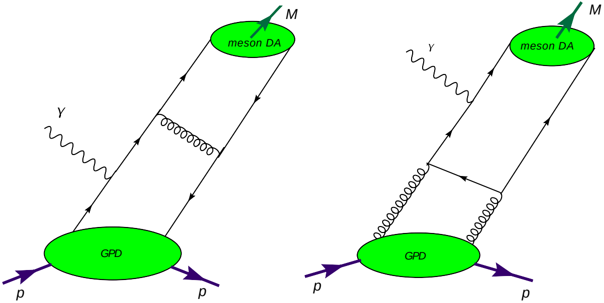
$\gamma^* \gamma \to (q\bar{q}),\ \gamma^* \gamma \to (gg)$



Hard-scattering amplitudes (DV processes vs. meson form factors)

DVMP

$$\gamma^* q \to (q\bar{q})q,\ \gamma^* g \to (q\bar{q})g$$

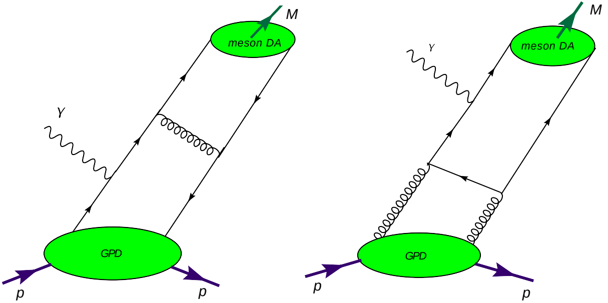


NLO: [Belitsky and Müller '01, Ivanov et al '04]

Hard-scattering amplitudes (DV processes vs. meson form factors)

DVMP

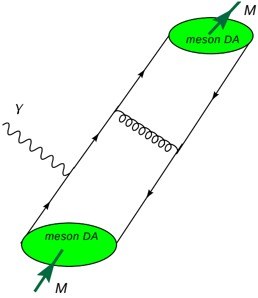
$$\gamma^* q \to (q\bar{q})q, \; \gamma^* g \to (q\bar{q})g$$



NLO: [Belitsky and Müller '01, Ivanov et al '04]

Meson em form factor

$$\gamma^*(q\bar{q}) \to (q\bar{q})$$



Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$\widetilde{F}^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=}$$

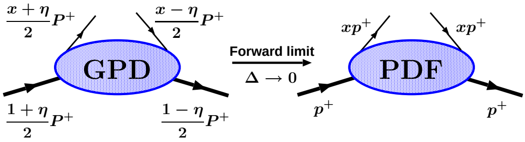
$$\widetilde{F}^g(x,\eta,t=\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)\widetilde{G}_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$

- Decomposing into helicity conserving and non-conserving part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

Properties of GPDs

- Forward limit ($\Delta \to 0$, $\eta \to 0$): $\Rightarrow$ $\widetilde{H}$-GPDs $\to$ PDFs



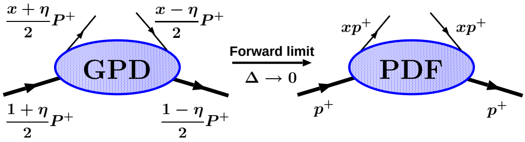
$P = P_1 + P_2$

$\Delta = P_2 - P_1$, $\Delta^2 = t$

$\eta = -\frac{\Delta^+}{P^+}$

Properties of GPDs

- Forward limit ($\Delta \to 0$, $\eta \to 0$): $\Rightarrow$ $\widetilde{H}$-GPDs $\to$ PDFs



$P = P_1 + P_2$

$\Delta = P_2 - P_1,\ \Delta^2 = t$

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- Sum rules: $\Rightarrow$ GPD $\to$ form factors

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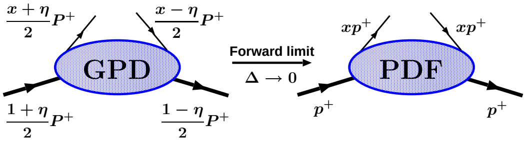

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- Possibility of solution of proton spin problem

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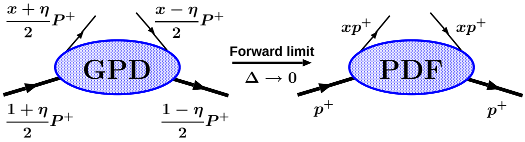

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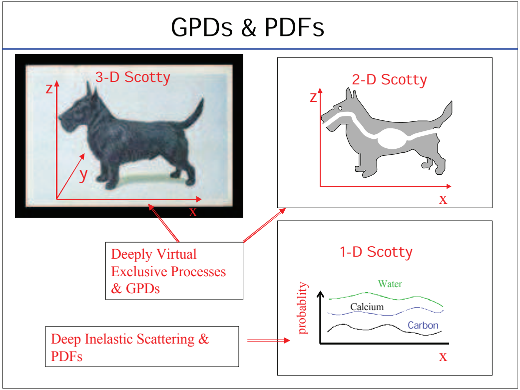
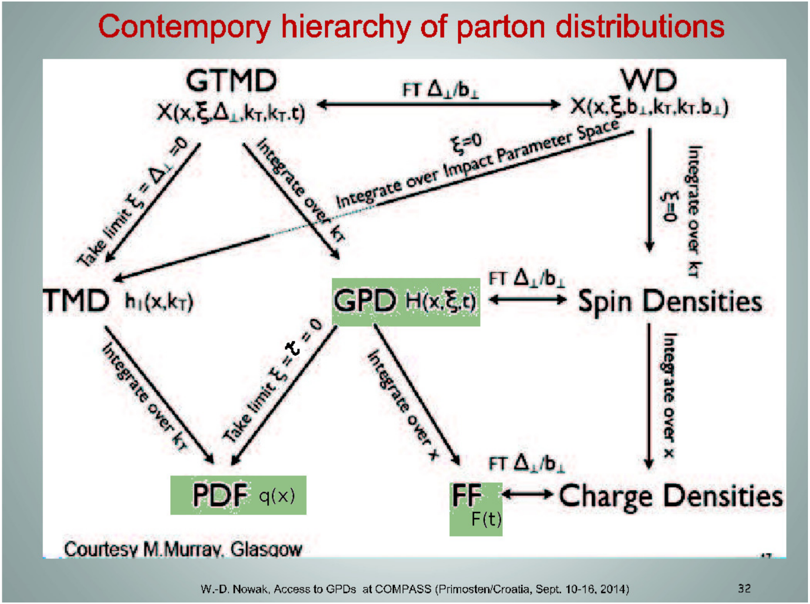
- polinomiality and positivity constraints

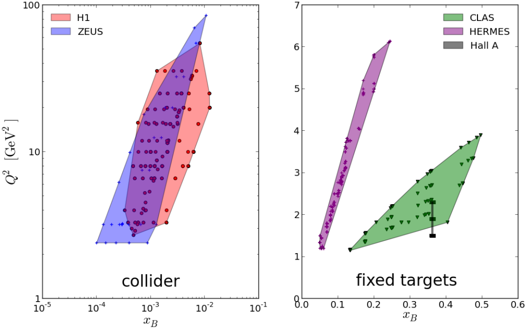


[V. D. Burkert, 2006]


Contempory hierarchy of parton distributions
GTMD
X(x,ξ,Δ⊥,kT,kT,t)
FT Δ⊥/b⊥
WD
X(x,ξ,b⊥,kT,kT,b⊥)
Take limit ξ = Δ⊥ =0
Integrate over kT
ξ=0
Integrate over Impact Parameter Space
ξ=0
Integrate over kT
TMD h1(x,kT)
GPD H(x,ξ,t)
FT Δ⊥/b⊥
Spin Densities
Integrate over kT
Take limit ξ = τ = 0
Integrate over x
Integrate over x
FT Δ⊥/b⊥
PDF q(x)
FF F(t)
Charge Densities
Courtesy M.Murray, Glasgow
W.-D. Nowak, Access to GPDs at COMPASS (Primosten/Croatia, Sept. 10-16, 2014) 32

Experimental status

DVCS



[from Kumericki et al. 2015]

DVMP

- in the last decade: vector meson (ρ, J/Ψ, ϕ) production at H1 and ZEUS, COMPASS, pseudoscalar mesons (π, η) at CLAS ...

$\rightarrow$ new results from COMPASS, JLab12 (EIC)

Towards unravelling GPDs

DVCS: Compton form factors

$$ {}^{a}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2)) \; H^a(x, \xi, t, \mu^2) \qquad a=q,G \text{ or NS,S}(\Sigma, G) $$

DVMP: transition form factors

$$ {}^{a}\mathcal{T}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \int \mathrm{d}y \; T^a(x, \xi, y, \mathcal{Q}^2/\mu^2)) \; H^a(x, \xi, t, \mu^2) \; \phi(y, \mu^2) $$

- Complete deconvolution is impossible and to extract GPDs from the experiment we need to model their functional dependence, or alternatively model form factors for start.

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  When the dimensionality increases, the volume of the space increases so fast that the available data become sparse.

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  When the dimensionality increases, the volume of the space increases so fast that the available data become sparse.
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.

Modeling venues

- double distribution amplitude (DDA) satisfy automatically the polinomiality constraint so many models based on it, or specificaly Radyushkin's DDA (RDDA) (VGG code, [Goeke et al. 01], BMK model [Belitsky, Muller, Kirchner 01], GK model [Goloskokov, Kroll 05]))
- 'aligned jet' model [Freund, McDermott, Strikman 02], polynomials [Belitsky et al. '98], [Liuti et al. '07], [Moutarde '09]
- 'dual model' [Polyakov, Shuvaev 02], [Guzey, Teckentrup 06], [Polyakov 07]
- various models in Mellin-Barnes integral representation [Kumericki, Muller, Passek-K 08, . . .]
- fitting Compton form factors with neural networks [Kumericki, Muller, Schaefer 11]

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DVCS using Mellin-Barnes representation, going to higher-orders and fitting GPDs

K. Kumerički, D. Müller, K. Passek-K.,
Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond, [hep-ph/0703179]

D. Müller, K. Passek-K., T. Lautenschlager, A. Schäfer,
Towards a fitting procedure to deeply virtual meson production - the next-to-leading order case, [arXiv:1310.5394]

K. Kumerički and D. Müller, [arXiv:0904.0458 [hep-ph]]

K. Kumerički, T. Lautenschlager, D. Müller, K. Passek-K., A. Schäfer and M. Meskauskas, [arXiv:1105.0899 [hep-ph]]

K. Kumerički, D. Müller and A. Schäfer, [arXiv:1106.2808 [hep-ph]]

K. Kumerički, D. Müller and M. Murray [arXiv:1301.1230 [hep-ph]]

T. Lautenschlager, D. Müller and A. Schäfer, [arXiv:1312.5493]

- factorization formula for singlet DVCS CFFs:

$$ {}^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; \mathbf{C}(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \; \mathbf{H}(x, \xi, t, \mu^2) $$

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- . . . in terms of conformal moments

(analogous to Mellin moments in DIS: $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$):

$$ = 2 \sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \, \mathbf{H}_j(\xi = \eta, t, \mu^2) $$

$$ H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) $$

. . .

H_j^a even polynomials in η with maximal power η^{j+1}

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$$ \ldots $$

H_j^a even polynomials in η with maximal power η^{j+1}

- series summed using Mellin-Barnes integral over complex j:

$$ = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right)\right] \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi, t, \mu^2) $$

Advantages of conformal moments and Mellin-Barnes representation

- enables simpler inclusion of **evolution** effects
- powerful analytic methods of **complex j** plane are available (similar to complex angular momentum of Regge theory)
- opens the door for interesting **modelling of GPDs**
- possible efficient and stable numerical treatment $\Rightarrow$ stable and fast **computer code** for evolution and fitting
- moments are equal to matrix elements of local operators and are thus directly accessible on the **lattice**

- **NNLO corrections for DVCS** accessible by making use of conformal OPE and known NNLO DIS results

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Modelling conformal moments

$$\mathbf{H}_j(\eta,t) = \underbrace{\begin{pmatrix} N'_\Sigma\, F_\Sigma(t)\, \mathrm{B}\big(1+j-\alpha_\Sigma(0),8\big) \\ N'_G\, F_G(t)\, \mathrm{B}\big(1+j-\alpha_G(0),6\big) \end{pmatrix}}_{\text{Leading partial wave}} + \begin{pmatrix} s_\Sigma \\ s_G \end{pmatrix} \begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}$$

- Leading wave – simplest case:
  (at NLO data can be fitted with leading wave only)
  - Regge-inspired ansatz

$$\alpha_a(t) = \alpha_a(0) + 0.15t \qquad F_a(t) = \frac{j+1-\alpha(0)}{j+1-\alpha(t)} \left(1 - \frac{t}{{M_0^a}^2}\right)^{-p_a}$$

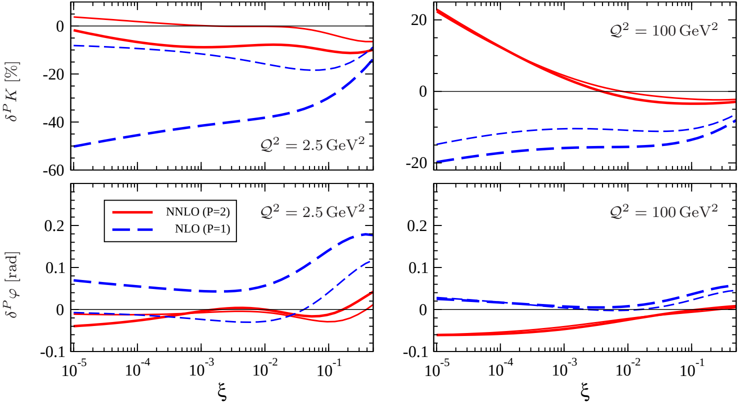
  - for $t = 0$ corresponds to x-space PDFs of the form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)}\, (1-x)^7 \; ; \qquad G(x) = N'_G\, x^{-\alpha_G(0)}\, (1-x)^5$$

  - fit parameters: N_Σ, $\alpha_\Sigma(0)$, $\alpha_G(0)$ (DIS) and M_0^Σ (DVCS)
    ($M_0^G = \sqrt{0.7}$ GeV from J/Ψ prod.)

NLO and NNLO corrections

for generic parameters

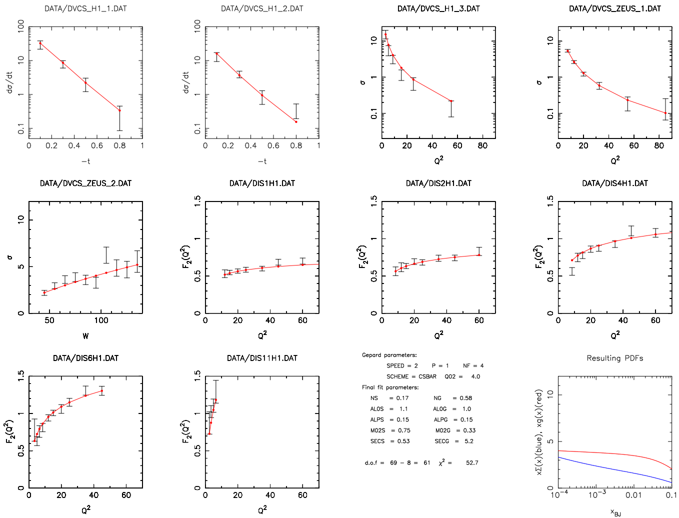


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)=\alpha_\Sigma(0) + 0.05$

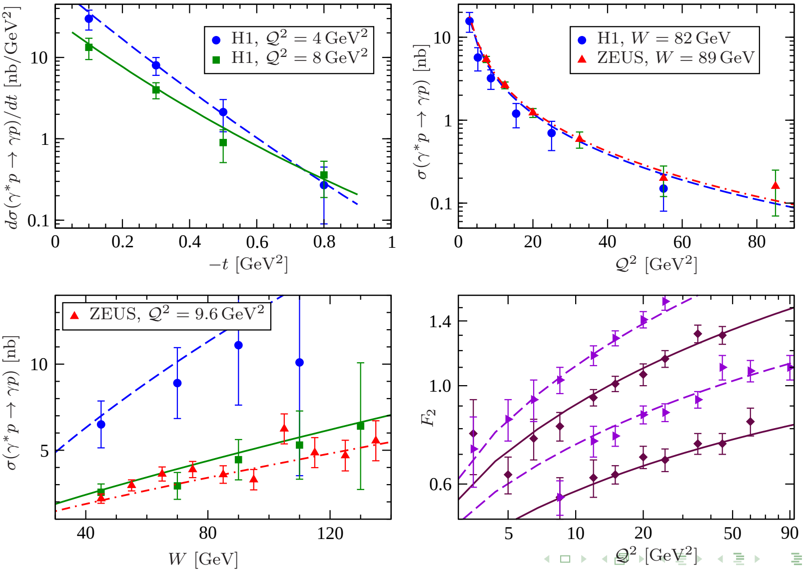
Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0)=\alpha_\Sigma(0) - 0.02$

$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

Fits (GeParD output)

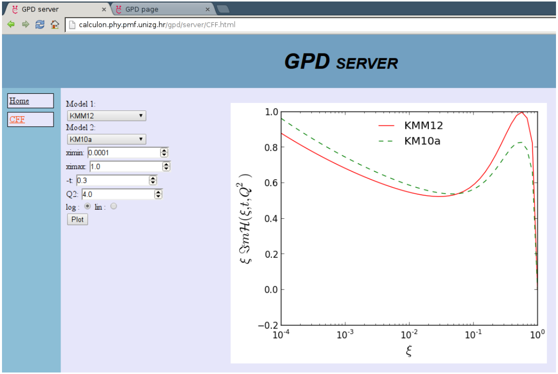


NNLO fit to HERA DVCS+DIS data



GPD page and server

- Durham-like CFF/GPD server page



- binary code for cross sections and KM models available at http://calculon.phy.hr/gpd/

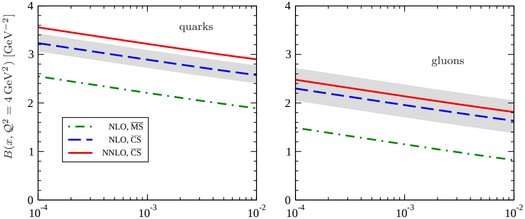
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on x and transversal distance b [Burkardt '00, '02]

$$H(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2)\,,$$

- Average transversal distance :

$$\langle \vec{b}^2 \rangle(x, \mathcal{Q}^2) = \frac{\int d\vec{b}\, \vec{b}^2 H(x, \vec{b}, \mathcal{Q}^2)}{\int d\vec{b}\, H(x, \vec{b}, \mathcal{Q}^2)} = 4B(x, \mathcal{Q}^2)\,,$$

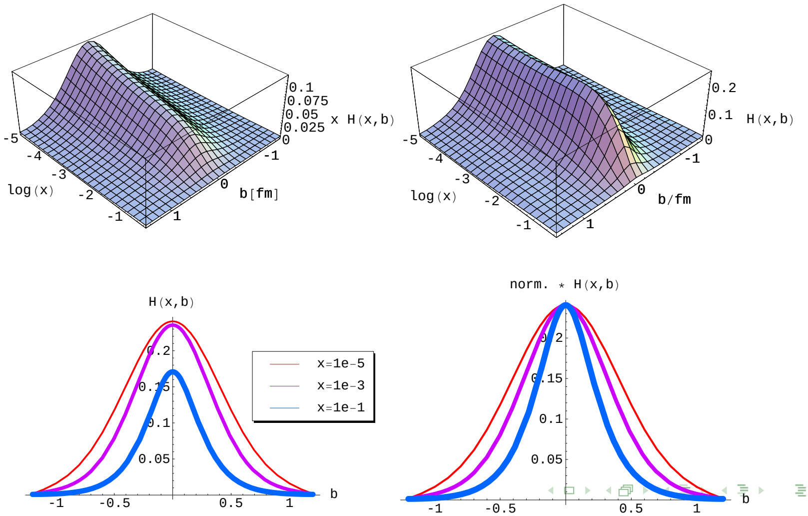


(at $\mathcal{Q}^2 = 4\,\mathrm{GeV}^2$)

$$\langle \vec{b}^2 \rangle_{\mathrm{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04}\,\mathrm{fm}^2$$

Three-dimensional image of a proton

Quarks: Gluons:



Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP . . . different processes offer different insight and should provide more complete picture.

Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP . . . different processes offer different insight and should provide more complete picture.
- Extraction of GPDs is extremely chalenging but efforts for global fits are being made.
- New data are expected from COMPASS and JLab12. DVCS and related processes have a large role in EIC proposal.

The End