



# Parity from gauge symmetry

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**Abstract** We argue that Left-Right parity symmetry  $\mathcal{P}$  can arise as a discrete remnant of a unified gauge symmetry. The high-energy unification necessarily includes the gauging of the Lorentz symmetry, bringing into the game gravitational interactions, and leading to a gravi-GUT scheme. Parity emerges unbroken below the Planck scale, and can be broken spontaneously at lower energies making contact with the Standard Model. This framework motivates the spontaneous origin of parity violation as in Left-Right symmetric theories with  $\mathcal{P}$ . The possible unifying gauge groups are identified as  $SO(1, 7)$  for gravitational and weak interactions, or  $SO(7, 7)$  for a complete unification.

## 1 Introduction

The chiral asymmetry of weak interactions has been discussed since the seminal work of Lee and Yang on parity violation [1], where the possibility of its restoration was advocated in terms of mirror particles. Rather than duplicating the matter spectrum, a different restoration of parity is achieved in the popular Left-Right symmetric models (LRSM) by extending the weak gauge group, as  $SU_L(2) \times SU_R(2) \times U_{B-L}(1)$  [2–6], see [7] for a review.

Parity restoration demands a discrete symmetry exchanging left with right, which can be realized either as a left-right parity  $\mathcal{P}$  or as a left-right charge conjugation  $\mathcal{C}$  [8]. The latter has a natural UV protection in  $SO(10)$  Grand Unified (GUT) models, as  $\mathcal{C}$  can be found among the gauge generators [9, 10]. The former, on the other hand, is the original and preferred choice if one aims for a true parity-conserving theory at high energy but lacks a UV completion. In this Letter, we propose a possible solution to this long-standing problem.

In analogy with  $\mathcal{C}$  within  $SO(10)$ , one would arrange  $\mathcal{P}$  as a generator of a unified gauge group, such that the discrete

symmetry at low energy can be interpreted as a remnant of a continuous one. However, this approach for  $\mathcal{P}$  is hampered by the fact that a continuous rotation mixing chiralities does not commute with the Lorentz symmetry. Probably, this is the main obstacle to formulating a theory of  $\mathcal{P}$ . A possible approach, rooted in the idea of Kaluza–Klein theories [11], considers parity as part of the 5D Lorentz and coordinate transformations so that it can be obtained as a discrete remnant symmetry in 4D, an idea explicitly considered in e.g. Ref. [12]. The price to pay in this approach is the dependence on the unknown dynamics of dimensional reduction, in addition to issues in anomaly matching [13, 14].

However, since the crucial point is the non-commutation of parity with the Lorentz symmetry, extra dimensions are not strictly necessary: an effective and more economical approach is to promote both as part of unified *internal* gauge symmetry. At high energy, this internal symmetry is completely disentangled from space-time diffeomorphisms, while they are soldered below a breaking scale, where a standard Lorentzian physics emerges. This framework necessarily brings into the game also gravity, in Cartan formulation.

In addition, since in the real world left and right fermions have different weak charges, one shall mix parity not just with Lorentz, but also with the Standard Model (SM) gauge symmetries, leading to a unified group whose gauge fields include the gravitational connection along with standard gauge fields. This is the approach put forward in [15, 16] and implemented as gravi-weak or gravi-GUT setups [17–19]. The unifying group is spontaneously broken by a vacuum expectation value (VEV) of an extended vierbein field at the Planck scale, leading to an unbroken gauge subgroup plus the residual *global* Lorentz symmetry, as we shall review below.<sup>1</sup>

It is thus interesting to uncover the role of parity in this framework. In the present Letter, we investigate the viability

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<sup>1</sup> For unifications involving extra dimensions, see [20–24].

ity of this approach from the point of view of symmetries and show that realizing parity will lead us to select a gravi-weak scenario based on the  $SO(1, 7)$  gauge symmetry, and a complete unification for  $SO(7, 7)$ .

We will conceptually decompose Left-Right parity  $\mathcal{P}$  in two operations: the inversion of space  $I_s$  and the internal action on fields, which we name  $P$ . These two operations can be disentangled at high energy, allowing for the gauging of the internal part  $P$ , i.e. its protection. At low scale, they are soldered and give rise to  $\mathcal{P}$ . Thus,  $\mathcal{P}$  is automatic and protected by the gauging if the theory is assumed to respect basic spatial inversion  $I_s$ . If, on the other hand, we allow for inversion-violating terms at high energy, still internal parity is gauged and just a few  $\mathcal{P}$ -breaking terms are allowed to emerge, notably the topological QCD  $F\tilde{F}$  term, unified with its gravitational analogue  $R\tilde{R}$ .

Finally, we will argue that unlike the usual breaking of gauge to discrete symmetries, the proposed framework does not lead to cosmic strings.

We will discuss first the internal symmetry part and later the breaking which connects with space-time, and finally the implications of this idea.

**Making parity action continuous**

Looking at the action of parity on fields, and ignoring for the moment the weak isospin, we denote, using a Weyl basis in 1+3 dimensions,

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad P = \gamma_0 \equiv \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix} \equiv \mathbf{1} \otimes \sigma_1. \tag{1}$$

As usual, parity swaps fermions as

$$\psi_L \leftrightarrow \psi_R \quad \text{or} \quad \Psi \rightarrow P \Psi. \tag{2}$$

Now, the discrete  $P$  can be enlarged to a continuous symmetry  $U(1)_P$  in the  $(\psi_L, \psi_R)$  space, as follows:

$$U(\alpha) = e^{i\frac{\alpha}{2}X(P-1)} = e^{-i\frac{\alpha}{2}X} \left( \cos \frac{\alpha}{2} + iXP \sin \frac{\alpha}{2} \right), \tag{3}$$

where  $X$  is any matrix which commutes with  $P$  and has  $X^2 = 1$ . As readily checked, parity is a rotation by  $\pi$ :

$$U(\pi) = P, \quad U(0) = U(2\pi) = 1. \tag{4}$$

While  $X = 1$  is a possibility, the other and more interesting choice, to appear in the following, is  $X = n_i \sigma_i \otimes \mathbf{1}$  with  $n^2 = 1$ , for instance  $X = \sigma_3 \otimes \mathbf{1}$ .

**Unifying with Lorentz group**

To promote the above  $U(1)_P$  to a gauge symmetry, one is clearly faced with the fact that parity does not commute with the action of the Lorentz group, in particular, it commutes with angular momentum but not with boosts.

Thus Lorentz has to be included. The simplest and illustrative example is provided by enlarging the Lorentz group to  $SO(1, 4)$ , which contains 3D parity and has precisely  $\Psi$  as

its non-chiral 4-dimensional spinor representation. Labeling the internal directions as  $0, \dots, 4$ , the new internal spacelike direction 4 requires four new generators: three rotations  $R_i$  in the  $i$ -4 planes, and one boost  $K_4$ . One can write

$$\begin{aligned} L_i &= \frac{\sigma_i \otimes \mathbf{1}}{2}, & K_i &= \frac{i\sigma_i \otimes \sigma_3}{2}, \\ R_i &= \frac{\sigma_i \otimes \sigma_1}{2}, & K_4 &= \frac{i\mathbf{1} \otimes \sigma_2}{2}. \end{aligned} \tag{5}$$

Now, one notes that the angular momentum  $L_i$  commutes with  $P$  and that  $R_i = L_i P$ . Comparing then with (3) we see that  $U(1)_P$  is precisely generated by  $L_i - R_i$ , for whichever  $i = 1, 2, 3$ . Thus, parity is interpreted geometrically in terms of the new spatial direction: choosing for instance  $i = 3$ , parity  $P = U(\pi)$  consists of two simultaneous rotations by  $\pi$ : one among 3–4 generated by  $XP = R_3$ , and one among 1–2 generated by  $L_3$ . After these  $\pi$  rotations, effectively the 123 directions are reversed (and so is 4). It leads thus to spinor exchange (4) plus internal spatial reflection. Different  $i$  imply different rotation planes, but after the  $\pi$  rotation the final effect is the same.

This first example misses the fact that  $\psi_L$  and  $\psi_R$  in the SM belong to different gauge multiplets. In particular, they have different weak and hypercharge representations or, in the language of LR symmetry, they transform under different  $SU(2)_{L,R}$  groups. Therefore, a realistic example must involve at least the weak interactions, as we shall discuss now.

**$SO(1,7)$  example**

$SO(1,7)$  has a Majorana representation of real dimension 16, that can be mapped into complex dimension 8 and that, under the decomposition  $SO(1, 7) \rightarrow SO(1, 3) \times SO(0, 4)$ , leads precisely to the required pattern where  $\psi_L$  and  $\psi_R$  transform as doublets under the  $SU(2)_L, SU(2)_R$  components of  $SO(4)$ ,

$$\mathbf{16}_{\mathbb{R}} \equiv \mathbf{8}_s \rightarrow (\mathbf{2}_L, \mathbf{1}_R, \mathbf{2}_l) \oplus (\mathbf{1}_L, \mathbf{2}_R, \mathbf{2}_r). \tag{6}$$

Here the first two slots refer to  $SU(2)_{L,R}$  and the last to Lorentz, namely  $2_l, 2_r$  for left and right Weyl spinors. It is convenient to spell out the  $SO(1, 7)$  generators acting on  $\mathbf{8}_s$  as  $(2/i)\Sigma_{M,N}$ :

$$\left( \begin{array}{cc|cc} 0 & i\sigma_i \otimes \mathbf{1} \otimes \sigma_3 & i\mathbf{1} \otimes \mathbf{1} \otimes \sigma_2 & i\mathbf{1} \otimes \sigma_b \otimes \sigma_1 \\ -i\sigma_j \otimes \mathbf{1} \otimes \sigma_3 & \epsilon_{ijk} \sigma_k \otimes \mathbf{1} \otimes \mathbf{1} & -\sigma_j \otimes \mathbf{1} \otimes \sigma_1 & \sigma_b \otimes \sigma_j \otimes \sigma_2 \\ \hline -i\mathbf{1} \otimes \mathbf{1} \otimes \sigma_2 & \boxed{\sigma_i \otimes \mathbf{1} \otimes \sigma_1} & 0 & -\mathbf{1} \otimes \sigma_b \otimes \sigma_3 \\ -i\sigma_b \otimes \mathbf{1} \otimes \sigma_1 & -\sigma_i \otimes \sigma_b \otimes \sigma_2 & \mathbf{1} \otimes \sigma_a \otimes \sigma_3 & \epsilon_{abc} \mathbf{1} \otimes \sigma_c \otimes \mathbf{1} \end{array} \right)$$

where, out of the internal directions  $M, N = 0, \dots, 7$ , the first four (0123) are along Lorentz. Thus the upper-left block represents the  $SO(1, 3)$  generators and the lower-right the  $SO(4)$  ones. In the respective spaces we denoted  $i, j$  or  $a, b$

as indices from 1 to 3, thus matching  $M, N = 1, 2, 3 \rightarrow i, j = 1, 2, 3$  and  $M, N = 5, 6, 7 \rightarrow a, b = 1, 2, 3$ .

The three boxed generators 123–4 correspond precisely to  $R_i \sim \sigma_i \otimes \mathbf{1} \otimes \sigma_1$ , while just above one finds the rotations  $L_i \sim \sigma_i \otimes \mathbf{1} \otimes \mathbf{1}$ . As before, we find that for any  $i$  the combination  $R_i - L_i$  generates a  $U(1)_P$  and a rotation by  $\pi$  generates  $P = U(\pi) = \mathbf{1} \otimes \mathbf{1} \otimes \sigma_1$ . Here the novelty is that this time, under  $P$ , the swapping of left and right spinors is accompanied by the swapping of the Left and Right weak groups, as required in a realistic model.<sup>2</sup>

**Symmetric phase and breaking.**

We can turn now to describe a mechanism of symmetry breaking which preserves parity as a discrete remnant of the original continuous gauge group. The task is complicated by the unification of local Lorentz with the other gauge forces, and by the fact that spacetime symmetries must be involved in such a way that at low energy  $\mathcal{P}$  includes the spacetime inversion.

As anticipated, in the first-order approach the Lorentz symmetry is disentangled from spacetime transformations (diffeomorphisms) and treated as an internal gauge symmetry, further extended to include other interactions.

The framework is based on the first-order (Cartan) formulation of Gravity (see e.g. [25]) where the gauge field of the Lorentz group is a spin connection  $\omega_{\mu n}^m$  and the vierbein field  $e_\mu^m$  transforms as a vector under the local Lorentz group (index  $m = 0, \dots, 3$ ). A background value (VEV) of the vierbein is needed for a sensible low energy spacetime metric: for a standard Minkowski background it is  $\bar{e}_\mu^m = M_{pl} \delta_\mu^m$ , with  $M_{pl}$  the Planck mass. This VEV, regarded as the choice of a fixed (unitary) gauge, breaks both the local Lorentz group and the diffeomorphism invariance. It nevertheless leaves unbroken a joint *global* Lorentz symmetry, realized when Lorentz and diff transformations on  $\mu$  and  $m$  are matched. This is the global Lorentz invariance of the Minkowski background that we experience at low energy. The counting of degrees of freedom confirms that of the 16 independent fields in  $e_\mu^m$ , 6 fields correspond to the gauge modes of local Lorentz transformations and are set to zero with the gauge fixing, or “eaten” by the spin connection, which can be shown to acquire a mass of the order of the Planck scale  $M_{pl}$ . The other 10 degrees of freedom become propagating and carry the standard graviton. In this formulation, the vierbein acts as a Higgs field for the breaking of the local Lorentz group to a global symmetry.

It is worth stressing again that in the symmetric phase the internal (gauge) Lorentz transformations are disentangled from the spacetime (diff) ones, while in the broken phase they are glued. Accordingly, spinors are originally scalars

under spacetime transformations and just transform under internal local Lorentz. Only in the broken phase they become spinors also of spacetime transformations, being these glued to Lorentz. As an example, their fermionic kinetic term arises from a symmetric-phase lagrangian written geometrically as

$$\mathcal{L}_{\psi kin} = \bar{\psi} \gamma^m D\psi \wedge e^n \wedge e^r \wedge e^s \epsilon_{mnr s}, \tag{7}$$

where i)  $\psi$  are spinors under the gauge group and scalars under diffs; ii) the vierbein one form is  $e^m = e_\mu^m dx^\mu$ ; iii) the covariant derivative  $D$  contains the gauge connection one form  $\omega_n^m = \omega_{\mu n}^m dx^\mu$ . In the broken phase, this action reproduces the standard fermionic kinetic term, including gravitational interactions.

In this formulation, gravity is ready for enlargement of the Lorentz gauge group to a generic group  $G$ , including Lorentz and other gauge interactions. One promotes the local Lorentz index  $m$  to a larger index  $M$  in a representation of  $G$ , while space-time and its indices  $\mu$  remain four-dimensional. The extended vierbein  $e_\mu^M$  still transforms as a one-form under standard 4D diffs, but  $M$  is enlarged. The gauge field  $\omega_{\mu N}^M$  of the enlarged group  $G$  contains both the spin connection and standard gauge interactions.

Let us exemplify this construction in the case of  $G = SO(1, 3 + N)$ , which preserves the metric  $\eta_{MN} = \text{diag}\{1, -1, -1, -1, -1 \dots\}$  with  $M = 0, \dots, N + 3$ .

Notably, a vierbein VEV can be arranged again in just four directions,

$$\bar{e}_\mu^M = \begin{cases} M_{pl} \delta_\mu^M, & \text{for } 0 \leq M \leq 3 \\ 0, & \text{for } 4 \leq M \leq N + 3 \end{cases} \tag{8}$$

which does a twofold job. It breaks again diffs and the 4D part of  $G$  down to global simultaneous Lorentz transformations of  $\mu$  and the first four indices  $M$ , and in addition it leaves unbroken a local subgroup  $SO(N)$ , mixing the last  $N$  directions where the VEV vanishes. This mechanism was used in [17], with  $SO(11, 3)$  broken in this single step to a  $SO(10)$  GUT. As analyzed there, the correct fermionic, gauge, and gravitational lagrangians emerge after the symmetry breaking of the  $G$ -invariant unified theory, for instance from a direct generalization of (7).

In this work, the VEV (8) is assumed. It was shown in [17,18] that it is a solution of the connection’s equations of motion, while a dynamical mean field origin was proposed in [16]. An other interesting possibility is that the vierbein is realized as bilinear condensate of more fundamental fermions, see for instance [26–29].

In [17,18] it was also discussed how this unification respects the Coleman Mandula theorem. The point is that in the broken phase the symmetry group is indeed the direct product of an internal gauge and global Lorentz. Conversely, in the unified phase, a background metric is absent and the theorem does not apply. We refer to [16,30] for extra discussions.

<sup>2</sup> The further nine generators below the boxed ones also lead to  $P$ , modulo a  $SO(4)$  gauge rotation.

We wish here to study the action of  $U(1)_P$  and of  $P$  on the background (8). Because  $U(1)_P$  contains a gauge rotation in (e.g.) the 1–2 and 3–4 planes and the VEV  $\bar{e}_\mu^M$  is nonzero in these subspaces, the continuous  $U(1)_P$  symmetry is broken (indeed only  $SO(N)$  survives). The discrete  $P = U(\pi)$  instead has a more interesting fate. After this  $\pi$  rotation, four internal spatial directions change sign,  $e_\mu^M \rightarrow -e_\mu^M$  for  $M = 1, 2, 3, 4$ . Because the VEV is nonvanishing only in the first three directions  $M = \mu = i = 1, 2, 3$ , one can write

$$\begin{aligned} \bar{e}_\mu^{M=0,\dots,3} &= M_{pl} \text{diag}(1, 1, 1, 1) \\ &\xrightarrow{U(\pi)} M_{pl} \text{diag}(1, -1, -1, -1), \end{aligned} \tag{9}$$

or  $\bar{e}_\mu^M \rightarrow \bar{e}_\mu^M \eta^{MM}$  (no summation). Thus also  $P = U(\pi)$  is broken, as it does not preserve  $\bar{e}_\mu^M$ .

We however notice that the VEV can be restored by adding a  $I_s$  spatial inversion,  $e_\mu^M \rightarrow \eta_{\mu\mu} e_\mu^M$ , which completes the action of  $\mathcal{P}$ . We then find

$$\mathcal{P} : \bar{e}_\mu^M \rightarrow \eta_{\mu\mu} \bar{e}_\mu^M \eta^{MM} = \bar{e}_\mu^M, \tag{10}$$

i.e. the vierbein VEV is invariant under combined internal parity and spatial inversion,  $\mathcal{P} = I_s \circ P$ . This result shows that the breaking mechanism glues not only the gauge and diff Lorentz transformations but also glues internal parity with spatial inversion, to produce the standard behavior of parity in the low energy field theory.

Thus, if the Lagrangian is invariant under space inversion, then the low energy theory will be exactly  $\mathcal{P}$  invariant.

### Emergence of LRSM Yukawa terms

It is instructive to explicitly discuss, in the  $SO(1, 7)$  example, the emergence of the  $\mathcal{P}$ -invariant fermionic Yukawa terms of the LRSM. While in the symmetric phase a direct (Majorana) mass term for the fermions  $\Psi^t C \Psi$  is forbidden by the other gauge interactions, e.g.  $B - L$  and color (or  $SU(4)$ ) one can have Yukawa terms by introducing some extra bosonic field, for instance a generic (reducible)  $H \in 8_c 8_c^\dagger$ :

$$\mathcal{L}_{Yuk} = Y_H \Psi^\dagger H \Psi + h.c., \tag{11}$$

where  $Y_H$  is a generic complex Yukawa matrix.

It is also useful to spell out the decomposition of  $\Phi$  under the breaking  $SO(1, 7) \rightarrow SU(2)_L \times SU(2)_R \times SO(1, 3)$ ,

$$\begin{aligned} H &= (1_L + 3_L, 1_R, \bar{2}_l 2_l) + (1_L, 1_R + 3_R, \bar{2}_r 2_r) \\ &\quad + (2_L^*, 2_R, \bar{2}_l 2_r) + (2_L, 2_R^*, \bar{2}_r 2_l) \\ &= L_\mu (1_L, 1_R, 4_l) + L_\mu^a (3_L, 1_R, 4_l) + L_i (2_L^*, 2_R, 3_{lr}) \\ &\quad + \Phi_{LR} (2_L^*, 2_R, 1_{lr}) + L \leftrightarrow R. \end{aligned} \tag{12}$$

Thus  $H$  contains Lorentz 4-vector, 3-vector, and singlet representations transforming under the weak groups.

The last term represents a scalar bidoublet, as found in Left-Right symmetric theories, where its weak scale VEV breaks electroweak symmetry and gives standard masses to

fermions. We find actually two independent such complex bidoublets,  $\Phi_{LR} = \Phi_1$  and  $\Phi_{RL} = \Phi_2^\dagger$ . Decomposing  $Y_H$  in hermitian components,  $Y_H = Y + i\tilde{Y}$ , (11) becomes a generic Yukawa lagrangian  $\bar{\psi}_L [Y(\Phi_1 + \Phi_2) + \tilde{Y}i(\Phi_1 - \Phi_2)] \psi_R + h.c.$ . The invariance under  $\mathcal{P}$  is confirmed by noting the bidoublets transformation  $\Phi_{1,2} \leftrightarrow \Phi_{2,1}^\dagger$ .

Now, since the minimal LRSM has only one bidoublet, a fact tied to the nice model predictivity, one may be tempted to restrict the  $H$  field. However, the only possibility is to assume a hermitian representation,  $H \equiv H^\dagger$ , i.e.  $\Phi_1 \equiv \Phi_2$ , but this would lead to unrealistic fermion masses, given by the sole matrix  $Y$ . The natural possibility is instead to allow generic  $\Phi_1, \Phi_2$  fields and realize that after the  $G$  breaking at Planck scale, only one combination can (and has to) be kept light, with mass at the  $v_R$  scale, and identified with the LRSM bidoublet. The situation is parallel to what happens when embedding the SM into the LRSM: the SM Higgs doublet  $\phi$  may be rewritten as a *real* LR bidoublet  $\Phi (\equiv \epsilon \Phi^* \epsilon)$ , but then the Yukawa lagrangian would unrealistically allow just a single hermitian matrix. One considers thus a *complex* bidoublet: one real component is kept light and identified with the SM Higgs doublet at weak scale by careful choice of coupling constants ( $\mu$ -terms); without further choices, the other remains naturally heavy at the high ( $v_R$ ) breaking scale. In the present framework, one combination  $\Phi$  of the two above bidoublets shall be kept light and leads effectively to LRSM Yukawa lagrangian,

$$\mathcal{L}_{Yuk} \rightarrow \bar{\psi}_L [Y\Phi + \tilde{Y}\tilde{\Phi}] \psi_R + h.c. \tag{13}$$

The other bidoublet has a natural mass at the Planck breaking scale, disappearing from the low energy spectrum. Incidentally, the same fate can be assumed for all the other components transforming nontrivially under Lorentz in (12),  $L_\mu, L_\mu^a, L_i$ , also avoiding possible issues with the signature of their nonstandard kinetic terms, see discussion below.

Similarly, one can implement Majorana Yukawa terms for fermions as  $\mathcal{L}_{Maj} = Y_\Delta \Psi^t \Delta \Psi$  where, still in  $SO(1, 7)$ ,  $\Delta$  transforms in the  $8_s 8_s$  representation. Its decomposition contains the two  $SU(2)_{L,R}$  triplets  $\Delta_{L,R}$ , and generates the standard Yukawa terms  $Y_\Delta \psi_L^t C \Delta_L \psi_L + L \leftrightarrow R$ , leading to Majorana masses for neutrinos via type-I and II seesaw. As above, several field components which transform nontrivially under Lorentz are present and naturally have mass at the Planck scale.

A comment is in order regarding the consequences of having noncompact gauge groups, which are known not to have finite-dimensional unitary representations. Indeed, the  $L_\mu, L_\mu^a$  and  $L_i$  states appeared above in Eq. (12) are a manifestation of this fact. This is potentially a serious problem, that could make the whole approach fundamentally flawed. A possible solution argued above is that no ghost state shall have mass below the Planck scale. A complete modeling, going



**Table 1** Breaking of unifying orthogonal groups and emerging discrete symmetries

$\mathfrak{p}+\mathfrak{q} = 8$	<b>spinor</b> = $16_{\mathbb{R}}$ (Majorana)	
SO(1, 7)	SO(1, 3) $\otimes$ SO(0, 4)	$\mathcal{P}$
SO(5, 3)	SO(4, 0) $\otimes$ SO(1, 3)	$\mathcal{T}$
$\mathfrak{p}+\mathfrak{q} = 14$	<b>spinor</b> = $64_{\mathbb{R}}$ (Majorana–Weyl)	
SO(7, 7)	SO(6, 0) $\otimes$ SO(1, 3) $\otimes$ SO(0, 4)	$\mathcal{T}_{col}, \mathcal{P}$
SO(11, 3)	SO(10, 0) $\otimes$ SO(1, 3)	$\mathcal{C}$ $\mathcal{T}_{so10}$

beyond the scope of this study, should pay special attention to this requirement. It is worth recalling that states with Planck mass and seemingly negative kinetic terms appear also in generic gravitational theories with propagating torsion, regardless of unification [31]. Various proposals to circumvent this problem exist in the literature (see e.g. [32, 33]) including recent ones, where the possible metastability of ghost states is investigated [34], or their quantization with a dedicated prescription is proposed [35].

We can add on top of these possibilities, that the standard tree level analysis is hardly conclusive, as these ghosts occur with the transition to a different regime. Indeed, in the symmetric phase, as noted in [16, 17, 30], the theory does not possess a background metric and has no quadratic kinetic terms. It thus belongs to a topological nonperturbative phase of quantum gravity, where new representations may appear as bound states. Interestingly, recent proposals where the vierbein is built as from fermion biliners, may help in dealing with these issues, see e.g. [26, 27, 29, 36, 37].

These comments apply to the SO(1, 7) example and to the more general groups that we discuss now.

**Complete unifications and other symmetries**

The analysis of other groups and the inclusion of strong interactions can proceed straightforwardly: a good path is to pre-unify color SU(3) and hypercharge U(1) into SU(4)  $\approx$  SO(6) of Pati and Salam [3], ready to be included in a pseudo-orthogonal group. Considering in generality SO( $p, q$ ), we display in Table 1 the realistic cases involving weak interactions, which we briefly comment.

First, from the SO(1, 7) example above, we have seen that the  $R_i$  generator involved in  $\mathcal{P}$  is a cross rotation between one spatial Lorentz direction and one relative to SO(0, 4). It is then clear that if SO(4) were to be included with a time-like signature, like SO(4, 0) inside SO(5, 3), then  $\mathcal{P}$  could not be achieved, as the cross generators are noncompact, boost-like. Instead, one can rotate one of the SO(4) directions with internal direction 0 to obtain its inversion, and the VEV may be preserved by adding a time inversion  $I_t$ . We indicate the symmetry as  $\mathcal{T}$  in the table: it amounts to time-reversal plus exchange of the Left and Right weak groups. Its enforcement leads to *real* Yukawa matrices, thus requiring spontaneous CP

violation, which is not so appealing at least from the point of view of model minimality.

Complete unifications involving strong interactions can give rise to more general discrete symmetries. We list in the table the realistic cases, which can be implemented only by the groups SO(11, 3) (proposed in [17, 23]) or SO(7, 7) (also proposed in [21, 22]). In both cases, the minimal Majorana–Weyl spinor representation has real dimension 64 which, when mapped into 32 complex [17], leads precisely to a complete SM family,

$$64_{\mathbb{R}} \equiv 32 \rightarrow (2_l, 2_L, 1_R, 4) \oplus (2_r, 1_L, 2_R, 4), \tag{14}$$

in Lorentz  $\times$  Pati–Salam notation.

Other groups as SO(1, 13) or SO(5, 9) are not viable as they have only *symplectic* Majorana–Weyl representations, leading to extra mirror families of opposite chirality.

In the SO(7, 7) case, SO(4) is present with spatial signature and leads to  $\mathcal{P}$ , but SO(6) is included as time-like. By an analysis similar to above, one finds a new discrete symmetry, amounting to time-reversal plus SU(4) color conjugation, named  $\mathcal{T}_{col}$  in the table. This additional discrete symmetry may or may not survive the lower stages of symmetry breaking.

We stress that the breaking of SO(7, 7) has arguably to proceed in one step at Planck scale, at least to the Pati–Salam subgroup SO(4)  $\times$  SO(6), so that the noncompact generators of SO(4, 6) have mass at or above the Planck scale.

In the case SO(11, 3) we find an analogous symmetry,  $\mathcal{T}_{so10}$ , while  $\mathcal{P}$  is absent.  $\mathcal{T}_{so10}$  may lead to  $\mathcal{T}$  and/or  $\mathcal{T}_{col}$ , depending on the SO(10) breaking pattern. In the table, we list also the more standard  $\mathcal{C}$  LR-symmetry, i.e. charge conjugation plus exchange of Left and Right weak groups, which is part of SO(10).

We confirm that, in the Pati–Salam notation (14),  $\mathcal{P}$  acts linearly by exchanging the two Left and Right components, while  $\mathcal{T}$  and  $\mathcal{T}_{so10}$  act antilinearly, replacing the spinor with its complex conjugate, as is required for a time-reversal (see also [17] for a discussion of antilinearity of broken generators).

**Discussion on parity and strong CP**

We have shown that by considering the presence of a high scale gauge symmetry unifying local Lorentz and gauge interactions, the theory automatically enjoys  $\mathcal{P}$ -parity symmetry below the first stage of symmetry breaking. This motivates the framework of Left-Right symmetric theories with  $\mathcal{P}$  as exact LR symmetry (then broken spontaneously at a lower scale).

However, it is necessary to deepen and clarify our understanding of this result. We established that  $\mathcal{P}$  arises from the gluing of internal parity  $P$  and spatial inversion  $I_s$ . This leads us to consider the possibility that the theory respects  $P$ , as a gauge symmetry, but might still violate space reflection.

An example is the analog of the QCD theta term, namely  $\theta F_N^M \wedge F_M^N$ , the two-form  $F_N^M$  being the curvature of the connection one-form  $\omega_N^M$ . This term respects internal parity  $\mathcal{P}$  because it is gauge-invariant but violates space inversion. As a result, in the low-energy theory, it leads to a term such as  $\theta F \tilde{F}$ , which violates  $\mathcal{P}$ .<sup>3</sup>

The spatial inversion could thus be assumed or not to be an invariance of the theory. This is in accord with the fact that, while in 4 dimensions diffeomorphisms have two disconnected components, the proper one and the one including a reflection, General Relativity is formulated as invariance under the proper component only. One might thus assume invariance under the full diffeomorphisms as a guiding principle, and the theory would have no  $\mathcal{P}$  violating terms. This choice can be viewed as a solution to the Strong CP problem, as in [2, 6, 39], see [40].

In a more physical approach, one shall test this hypothesis by considering possible violations of spatial inversion in the theory. In the present context, the unification of the internal symmetries leads at least to the prediction that various  $\mathcal{P}$ -violating terms, regarding different interactions, will be connected.

For instance, one of the most stringent tests is the experimental bound from the electric dipole moments (e.g. of the neutron [41]). The relative bounds of the order  $\bar{\theta} < 10^{-10}$  directly translate for us into limits on the gravitational analogous,  $\theta \tilde{R}R$ . This is argued to be physical [42], and the question of how it could be measured is the subject of some recent studies, e.g. [43–45].

On the other hand, the bounds on parity-violating Chern-Simons extensions [46] would be connected with the QCD axion terms. Another example breaking spatial parity but not the gauge symmetry is the Immirzi term  $\alpha R^{MN} \wedge e_M \wedge e_N$ , although there is practically no bound on it from semiclassical effects [47]. More in general, a detailed program investigating all possible terms violating spatial parity could be undertaken, along the lines of the analysis for standard Cartan gravity [48].

### Phenomenological implications

In the LRSM, the discrete parity  $\mathcal{P}$ , among many constraints on the parameters of the model [8, 49–52], imposes that the QCD  $\theta$  is strictly zero [53]. In this case  $\bar{\theta}$  is computable, and nEDM together with other CP-violating observables was shown to put strong bounds on the right-handed scale [40, 54–57]. The RH scale is pushed beyond  $\sim 28$  TeV [52, 54].

The present framework instead motivates also the situation as described in Ref. [8, 52], namely,  $\bar{\theta}$  is free. In this case,  $\mathcal{P}$

symmetry is valid in the Yukawa sector but strong CP poses no additional constraints, in complete analogy with the case of  $\mathcal{C}$  symmetry [8]. In this scenario, the  $W_R$  scale can be lowered to  $\sim 6$  TeV.

The future LHC runs and next-generation colliders would be able to probe  $W_R$  up to  $\sim 30$  TeV [58], and the potential discovery of  $W_R$  in this range would point to the second scenario, where  $\theta$  is nonzero and determined. In this case, the striking consequence is that together with the validity of  $\mathcal{P}$  in the quark sector [8, 59], one would test predicted correlations between the various electric dipoles of neutron and nuclei, as analyzed in [55]. This would help to clarify the underlying mechanism behind  $\mathcal{P}$ .

### Cosmic strings

It is noteworthy that, although the present framework contains the breaking of a continuous symmetry to a discrete one, cosmic strings [9] do not appear. This can be understood by looking at a possible transformation of the vierbein VEV along a closed path around a string: with the gradual 3–4 plus 1–2 rotation up to final angle  $\pi$ , the result is the new VEV (9). This would be matched with the starting one by inverting the spatial coordinates  $\mu = 1, 2, 3$  as mentioned above. However, inverting them just on the final part of the path is not legitimate in a given space-time configuration, because the whole space would be nonorientable, a situation that clearly can not be generated by standard physics like gravitational collapse or phase transition. In practice, asking for space-time orientability rules out the possibility of a nontrivial  $\mathcal{P}$  around a string.

It is interesting to speculate whether one may semiclassically create such nonorientable cosmic strings *in pairs*, on the line of nonorientable gravitational instantons [60]. Traveling around one such string one would be faced with the  $\mathcal{P}$ -symmetric physics. In any case, the spontaneous breaking of  $\mathcal{P}$  at a lower scale would attach domain walls to these strings, which would preclude the view of space nonorientability, quite an exotic situation. Analogous comments apply to the emergent  $\mathcal{T}$  symmetry.

A final word is worth about topological defects that may arise at the lower scales of symmetry breaking, such as domain walls or GUT monopoles [61, 62]. In our framework, their appearance can not be cured by nonrenormalizable operators from gravity as in [63], but other ways out include low scale inflation or symmetry nonrestoration at high temperature [64, 65].

### Summary and outlook

We have proposed a framework for UV completion of  $\mathcal{P}$ -parity, and thus of the LRSM in its original formulation, where  $\mathcal{P}$  was introduced as the LR restoration of standard parity.

We have first decomposed  $\mathcal{P} = P \circ I_S$  into simple space inversion  $I_S$  plus internal LR and chirality exchange  $P$ . Then,

<sup>3</sup> The  $\mathcal{P}$ -violating term  $\theta F \tilde{F}$  may be rotated away in the quark masses via the anomaly [38]. The Yukawa couplings in (13) become non-hermitian and only preserve a new internal parity  $\mathcal{P}'$ , because the chiral rotation does not commute with  $U(1)_P$ . The model is still  $\mathcal{P}$ -violating, but the non-hermiticity in (13) lies in an overall phase  $\theta$  at most.

we have shown that  $P$  can be made continuous and gauged by embedding it into the  $SO(1, 7)$  or  $SO(7, 7)$  gauge groups, unifying Lorentz with gauge interactions and pointing respectively to gravi-weak and gravi-GUT models. In these scenarios, diffeomorphism and gauge symmetry are disentangled at high energy and are broken together at the Planck scale in such a way that the standard *global* Lorentz symmetry remains. The breaking also preserves the simultaneous  $P$  and  $I_s$  transformations. This guarantees invariance under  $\mathcal{P}$ , which is thus also protected by the gauging of  $P$ .

Space coordinate reflection  $I_s$  needed an additional discussion. Because it is not strictly included in (proper) diffeomorphisms, one can choose whether to assume it as an additional fundamental invariance or not.

By enlarging diffeomorphisms with  $I_s$ , the low-energy theory is automatically  $\mathcal{P}$  invariant. In this case, a direct implication is that there are no non-renormalizable  $\mathcal{P}$ -violating operators from gravity.

In case basic spatial inversion symmetry is not assumed, one still deals with internal  $SO(1, 7)$  or  $SO(7, 7)$  gauge groups, leading to a mostly  $\mathcal{P}$ -invariant low energy theory, save for a few  $\mathcal{P}$ -violating terms that can now appear. One notable case is a topological term  $\theta F\tilde{F}$  in the QCD lagrangian – unified with the gravitational equivalent  $\theta R\tilde{R}$ . Therefore, even if the theory enjoys  $\mathcal{P}$  symmetry in the quark and scalar sectors, it does not require a vanishing of  $\bar{\theta}$ . In other words, protecting  $P$  by gauge invariance alone does not solve the strong CP problem.

On the other hand, this scenario has direct links with the phenomenology of the LRSM. In that context, exact  $\mathcal{P}$  symmetry is at the basis of predictivity in the flavor sector but was also used to attack the strong CP problem, where the nEDM limit implies a strong lower bound on the  $W_R$  mass. As we discussed, the possibility of nonzero  $\bar{\theta}$  motivates the scenario of lower  $W_R$  accessible at forthcoming colliders.

The choice of various unifying groups has uncovered the possibility of novel low energy discrete symmetries, such as time-reversal  $\mathcal{T}$  involving LR exchange or  $\mathcal{T}_{col}$  involving color conjugation. Their analysis is left for future work.

A further property of our framework is that, although featuring a transition from continuous to a discrete symmetry, due to the role of space there is no appearance of cosmic strings, avoiding related cosmological issues.

Summarizing, in this work we have established a proof of concept for the gauge protection of  $\mathcal{P}$  as the remnant of a high energy unified gauge group and investigated the relative symmetry structure and breaking mechanism. After this stage, a viable model unifying gravitational and BSM degrees of freedom will be the next outstanding challenge. We can speculate that the large symmetry structure will pose stringent constraints on the unified model.

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