Thermodynamics of AdS planar black holes and holography

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ABSTRACT: A large AdS Schwarzschild black hole can be approximated by an AdS planar black hole. However, the temperature of a planar black hole is not well defined due to the translational invariance of the horizon. We propose to fix this arbitrariness by imposing the entropy area law. Furthermore, using the AdS/CFT holography, we propose a relationship between the temperatures of an AdS planar and large AdS Schwarzschild black holes.

Contents

1 Introduction

Anti-de Sitter (AdS) Schwarzschild black holes have a curious thermodynamic property. The horizon temperature of a large AdS Schwarzschild black hole (BH) increases with increasing horizon radius in contrast to an ordinary Schwarzschild BH the temperature of which decreases with horizon radius. Hubeny, Marolf, and Rangamani [1] argue that the thermodynamic behavior of AdS Schwarzschild BHs is tightly related to the thermodynamic properties of planar BHs. This argument is based on the fact that a large AdS Schwarzschild BH can be well approximated by an AdS planar BH [2].

To the best of our knowledge, the AdS planar BHs were first stadied by Lemos [3] and more recently were shown to appear by scaling a near-extremal brane solution [4]. Interesting properties of AdS planar BHs were found [5] in the context of thermodynamics. Besides, planar BHs have recently attracted considerable attention in non-gravitational contexts. Geometric structures in the form of a planar BH may have interesting applications in condensed matter physics [6], 2+1-dimensional superconductor [7–10], and acoustic geometry [10–12].

As we will shortly demonstrate, the horizon of an AdS planar BH is translated when a planar BH metric undergoes a rescaling transformation, so the horizon location of an AdS planar BH is not well defined. Following Hubeny, Marolf, and Rangmani [1] we will refer to this property as "translational invariance" of the BH horizon, even though this term could be somewhat misleading in this context. Based on this property, the authors of ref. [1] argue that large Schwarzschild-AdS black holes, although very hot, locally appear at most only lukewarm.

In this paper, we study the relationship between a large AdS Schwarzschild BH in $d+1$ dimensions and AdS planar BH in the context of anti-de Sitter/conformal field theory

(AdS/CFT) holography. First, we will show that the horizon temperature of a planar BH can be fixed by the requirement of the entropy area law. Then, by invoking the AdS/CFT correspondence conjecture, we will relate the local temperatures of the planar BH and large AdS Schwarzschild BH near the AdS boundary.

We divide the remainder of the paper into three sections. In section 2 we briefly review the thermal properties of AdS Schwarzschild and planar BHs. In section 3 we derive relationships between the temperatures of the boundary conformal fluid and of a large AdS BHs by employing AdS/CFT holography. Concluding remarks are given in section 4.

2 AdS Schwarzschild and planar black hole

The metric of an AdS Schwarzschild black hole in $d + 1$ dimensions is usually written in the form [2, 13]

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega_{d-1}. \tag{2.1}$$

The function f is defined as

$$f(r) = \frac{r^2}{\ell^2} + 1 - \mu \left(\frac{\ell}{r}\right)^{d-2}, \tag{2.2}$$

where ℓ is the curvature radius of AdS_{d+1} and the dimensionless parameter μ is related to the black-hole mass via [2, 14]

$$\mu = \frac{16\pi G_{d+1} M_{\mathrm{bh}}}{(d-1)\ell^{d-2}\Omega_{d-1}}. \tag{2.3}$$

Here, G_{d+1} is the $d + 1$-dimensional Newton's constant and

$$\Omega_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)} = \begin{cases} (2\pi)^{d/2}/(d-2)!!, & \text{for even } d, \\ 2(2\pi)^{(d-1)/2}/(d-2)!!, & \text{for odd } d, \end{cases} \tag{2.4}$$

is the volume of a unit $d - 1$-sphere. The parameter μ can be expressed in terms of the largest root r_+ of the equation $f(r) = 0$,

$$\mu = \left(\frac{r_+}{\ell}\right)^{d-2} \left(1 + \frac{r_+^2}{\ell^2}\right). \tag{2.5}$$

Note that the metric (2.1) with (2.2) with horizon radius involves two scales: the AdS curvature ℓ and $d + 1$-dimensional Newton's constant G_{d+1}

The horizon temperature af an AdS Schwarzschild BH can be derived in the usual way by approximating the Euclidean metric in the near horizon limit with

$$f = (r - r_+)f_+, \tag{2.6}$$

where

$$f_+ = \left.\frac{\partial f}{\partial r}\right|_{r=r_+}. \tag{2.7}$$

Defining the periodicity of the Euclidean time so to avoid the cone singularity, one finds the inverse horizon temperature

$$\beta = \frac{1}{T_+} = \frac{4\pi}{f_+} = \frac{4\pi\ell^2 r_+}{dr_+^2 + (d-2)\ell^2}. \tag{2.8}$$

Note that, the horizon temperature $T_+ \to \infty$ when either $r_+ \to 0$ (small BH) or $r_+ \to \infty$ (large BH) unlike in the Schwarzschild BH in an asymptotically flat background where $T_+ \to 0$ for a large BH. In particular, for a large AdS Schwarzschild BH we have

$$T_+ = \frac{dr_+}{4\pi\ell^2}. \tag{2.9}$$

2.1 AdS planar black hole

As pointed by Witten [2], in the limit of large AdS Schwarzschild BH the topology goes from $\mathbf{S}^1 \times \mathbf{S}^{d-1}$ to $\mathbf{S}^1 \times \mathbf{R}^{d-1}$ where a Schwarzschild BH is approximated by a planar BH with a translationally invariant horizon. To see this, consider a large Schwarzschild-AdS black hole with $\mu \gg 1$ ($r_+ \gg \ell$). By rescaling

$$r = \mu^{1/d}\frac{\ell^2}{\rho}, \tag{2.10}$$

the function f becomes

$$f = \mu^{2/d}\left(\frac{\ell^2}{\rho^2} - \frac{\rho^{d-2}}{\ell^{d-2}} + \mu^{-2/d}\right). \tag{2.11}$$

Then, by neglecting the last term in brackets and rescaling the time $t = \mu^{-1/d}\tau$, the line element squared becomes

$$ds^2 = \frac{\ell^2}{\rho^2}\left[\left(1 - \frac{\rho^d}{\ell^d}\right)d\tau^2 - \left(1 - \frac{\rho^d}{\ell^d}\right)^{-1}d\rho^2 - \mu^{2/d}\ell^2 d\Omega_{d-1}^2\right]. \tag{2.12}$$

For $M_{\mathrm{bh}} \to \infty$ (i.e., for $\mu \to \infty$) the radius $\mu^{1/d}\ell$ of $\mathbf{S}^{d-1}$ diverges and hence, $\mathbf{S}^{d-1}$ becomes flat and looks locally as $\mathbf{R}^{d-1}$. Now we introduce coordinates x_i near a point $P \in \mathbf{S}^{d-1}$ such that at P, $d\Omega_{d-1}^2 = \ell^{-2}\mu^{-2/d}d\boldsymbol{x}^2$. Then the metric becomes

$$ds^2 = \frac{\ell^2}{\rho^2}\left[\left(1 - \frac{\rho^d}{\ell^d}\right)d\tau^2 - \left(1 - \frac{\rho^d}{\ell^d}\right)^{-1}d\rho^2 - d\boldsymbol{x}^2\right], \tag{2.13}$$

where

$$d\boldsymbol{x}^2 = \sum_{i=1}^{d-1} \mathrm{d}x^i \mathrm{d}x^i. \tag{2.14}$$

The metric (2.13) describes an asymptotically AdS$_{d+1}$ geometry with a planar BH horizon located at $\rho = \ell$. Note that dependence on Newton's constant G_{d+1} has disappeared and the metric (2.13) involves only one scale: the AdS curvature ℓ.

Now, it is easy to show by simultaneously rescaling the coordinates that the metric (2.13) is equivalent to the whole class of planar BHs represented by the line element

$$ds^2 = \frac{\ell^2}{\rho^2} \left[\left(1 - \frac{\rho^d}{\rho_{\text{pl}}^d} \right) d\tau^2 - \left(1 - \frac{\rho^d}{\rho_{\text{pl}}^d} \right)^{-1} d\rho^2 - d\boldsymbol{x}^2 \right], \qquad (2.15)$$

with horizon located at an arbitrary ρ_{pl}.

As before, the horizon temperature of the Planar BH can be calculated by demanding that the periodicity β_{pl} is such that the conical singularity is removed. One finds

$$\beta_{\text{pl}} = \frac{1}{T_{\text{pl}}} = \frac{4\pi\rho_{\text{pl}}}{d}, \qquad (2.16)$$

so the temperature of the planar BH increases when the horizon shifts towards the AdS boundary at $\rho = 0$. However, since ρ_{pl} is not fixed, the temperature T_{pl} is not well defined. Hence, local observations are independent of ρ_{pl} and, since the AdS scale is the only scale in the problem, local observers see no excitations above this scale.

The fact that the temperature T_{pl} of the AdS planar BH is not well defined, is reminiscent of the case for the extreme Reissner-Nordström BH with AdS near horizon geometry. As argued, e.g., in refs. [15–17], the temperature of the extreme Reissner-Nordström BH is arbitrary since its horizon is infinitely far away.

Since small ρ corresponds to large r, one would naively conclude that the thermodynamic behavior of the AdS planar BH is consistent with that of the large AdS Schwarzschild BH. However, this conclusion could be wrong as it is not obvious that the planar BH temperature is in any way related to the temperature of the AdS Schwarzschild BH.

2.2 Entropy of AdS planar black holes

We closely follow Witten's procedure for the calculation of the BH entropy. This procedure applied to the metric (2.1) with (2.2) yields the AdS Schwarzschild BH entropy [2]

$$S_{\text{BH}} = \frac{r_+^{d-1}\Omega_{d-1}}{4G_{d+1}}, \qquad (2.17)$$

which demonstrates the area law in agreement with Bekenstein [18] and Hawking [19]. To calculate the horizon entropy of the AdS planar BH we start from the metric (2.15) in which we transform the coordinate ρ to $r = \rho_{\text{pl}}^2/\rho$ so that r goes from $r = \rho_{\text{pl}}$ to $r = \infty$ as ρ goes from $\rho = \rho_{\text{pl}}$ to $\rho = 0$. The Euclidean metric is given by

$$ds_{\text{E}}^2 = \frac{r^2}{\ell^2} \left[\left(1 - \frac{\rho_{\text{pl}}^d}{r^d} \right) d\tau^2 + \left(1 - \frac{\rho_{\text{pl}}^d}{r^d} \right)^{-1} \frac{\ell^4}{r^4} dr^2 + d\boldsymbol{x}^2 \right], \qquad (2.18)$$

with the associated horizon temperature T_{pl} given by (2.16).

Following Hawking and Page [20] and Witten [2] we describe the BH free energy as a renormalized on shell Euclidean bulk action. The on shell AdS BH action is proportional to an infinite volume of the AdS BH spacetime so it must be regularized and renormalized.

The regularization is achieved by integrating the space volume up to a large radius R. The renormalized action is then obtained by subtracting the pure AdS space time volume from the AdS BH spacetime volume, assuming equality of the local AdS temperature and the redshifted planar BH temperature at the hypersurface $r = R$. More explicitly

$$F_{\text{pl}} = -T_{\text{pl}} \ln Z = \frac{d\ell^{-2}}{8\pi G_{d+1}\beta_{\text{pl}}} \lim_{R \to \infty} (V_{\text{pl}} - V_{\text{AdS}}), \tag{2.19}$$

where

$$V_{\text{pl}} = \int_0^{\beta_{\text{pl}}} d\tau \int_0^R dr \sqrt{g} \left(\int_{-L/2}^{L/2} dx \right)^{d-1}, \tag{2.20}$$

$$V_{\text{AdS}} = \int_0^{\beta_{\text{AdS}}} d\tau \int_0^R dr \sqrt{g} \left(\int_{-L/2}^{L/2} dx \right)^{d-1}. \tag{2.21}$$

The length L is the linear size of the hyperplane $r = \text{const.}$. The inverse AdS temperature β_{AdS} is obtained from the equation

$$\beta_{\text{AdS}} \sqrt{g_{00}^{\text{AdS}}} = \beta_{\text{pl}} \sqrt{g_{00}^{\text{pl}}}, \tag{2.22}$$

where g_{00}^{AdS} and g_{00}^{pl} are the time-time components of the AdS and-black hole metric tensors, respectively, evaluated at $r = R$. We find

$$\beta_{\text{AdS}} = \beta_{\text{pl}} \left(1 - \frac{\rho_{\text{pl}}^d}{R^d} \right)^{1/2}. \tag{2.23}$$

Then, from (2.19) we obtain the free energy expressed as a function of ρ_{pl}

$$F_{\text{pl}} = \frac{L^{d-1}}{16\pi G_{d+1}\ell} \left(\frac{\rho_{\text{pl}}}{\ell} \right)^d, \tag{2.24}$$

where L^{d-1} is the area of the hyperplane $r = \text{const.}$. From this, using (2.16) and the thermodynamic relation between entropy and free energy, we find

$$S_{\text{pl}} = -\frac{\partial F_{\text{pl}}}{\partial T_{\text{pl}}} = \frac{L^{d-1}}{4G_{d+1}} \left(\frac{\rho_{\text{pl}}}{\ell} \right)^{d+1}. \tag{2.25}$$

Hence, we recover the standard area law for the planar BH entropy if $\rho_{\text{pl}} = \ell$. So, if we demand the validity of the area law, the metric (2.12) will be a natural representative of all AdS planar BHs belonging to the class described by the line element (2.13) with $0 < \rho_{\text{pl}} < \infty$.

We shall next invoke the holographic principle to relate the temperature T_{pl} of the planar BH to the temperature of the conformal fluid at the boundary of AdS spacetime.

3 AdS/CFT holography

AdS/CFT holographic principle relates the bulk geometry to the conformal field theory at the AdS boundary. To study the CFT at the boundary, it is convenient to transform both the AdS Schwarzschild metric (2.1) and the AdS planar BH metric (2.15) to the Fefferman-Graham coordinates. In Fefferman-Graham coordinates the general asymptotically AdS metric can be expressed as

$$ds^2 = \frac{\ell^2}{z^2} \left(g_{\mu\nu}(z)dx^\mu dx^\nu - dz^2 \right), \tag{3.1}$$

where the d-dimensional metric $g_{\mu\nu}$ near the boundary at $z = 0$ can be expanded as [21]

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + \ldots + z^d g_{\mu\nu}^{(d)} + h_{\mu\nu} z^d \ln z^2 + \mathcal{O}(z^{d+1}). \tag{3.2}$$

The logarithmic term appears only for d even. Using this expansion one can express the vacuum expectation value of the boundary CFT stress tensor as [21]

$$\langle T_{\mu\nu} \rangle = -\frac{d\ell^{d-1}}{16\pi G_{d+1}} g_{\mu\nu}^{(d)} + X_{\mu\nu}[g^{(n)}], \tag{3.3}$$

where $X_{\mu\nu}[g^{(n)}]$ is a function of $g_{\mu\nu}^{(n)}$ with $n < d$. Its exact form depends on the spacetime dimension and it reflects the conformal anomalies of the boundary CFT [22] (for $d = 4$ and applications in cosmology see also refs. [23, 24]). For d odd, there are no gravitational conformal anomalies and $X_{\mu\nu}$ is equal to zero. For d even, we may expect that the conformal anomaly related to the X term will correspond to the logarithmic correction to the BH entropy [25–27]. In the following, we will be interested in the dominant area law term in the BH entropy for which we expect to correspond to the traceless part of the universal term in (3.3) proportional to $g_{\mu\nu}^{(d)}$.

In the following, we detail the correspondence between the vacuum expectation value of the CFT stress tensor and the geometries of Schwarzschild and planar black holes via equation (3.3).

3.1 AdS Schwarzschild BH

Consider first the AdS Schwarzschild BH. By transforming the radial coordinate $r = \ell^2/\rho$, we rewrite the metric (2.1) in the form similar to (2.12),

$$ds^2 = \frac{\ell^2}{\rho^2} \left[\left(1 + \frac{\rho^2}{\ell^2} - \mu\frac{\rho^d}{\ell^d} \right) d\tau^2 - \left(1 + \frac{\rho^2}{\ell^2} - \mu\frac{\rho^d}{\ell^d} \right)^{-1} d\rho^2 - \ell^2 d\Omega_{d-1}^2 \right]. \tag{3.4}$$

Then, the transformation

$$\frac{dz}{z} = \frac{d\rho}{\rho} \left(1 + \frac{\rho^2}{\ell^2} - \mu\frac{\rho^d}{\ell^d} \right)^{-1/2} \tag{3.5}$$

will take us to the Fefferman-Graham form. The integration of (3.5) cannot be performed in terms of elementary functions, with the exception of $d = 2$ and $d = 4$ [23]. However, as

we are interested in the expansion (3.2) near $z = 0$ up to the order z^d, we can apply an ansatz to the metric in Fefferman-Graham coordinates:

$$ds^2 = \frac{\ell^2}{z^2} \left[H(z)d\tau^2 - F(z)\ell^2 d\Omega_{d-1}^2 - dz^2 \right]. \tag{3.6}$$

Here,

$$F(z) = 1 + \sum_{n=2}^\infty f_n \frac{z^n}{\ell^n}, \quad H(z) = 1 + \sum_{n=2}^\infty h_n \frac{z^n}{\ell^n}, \tag{3.7}$$

where the coefficients f_n and h_n are to be determined by equating the line element (3.6) with (3.4) and making use of (3.5). We find

$$F(z) = 1 - \frac{1}{2}\frac{z^2}{\ell^2} + \frac{1}{16}\frac{z^4}{\ell^4} + \frac{1}{d}\mu\frac{z^d}{\ell^d} + \dots, \tag{3.8}$$

$$H(z) = 1 + \frac{1}{2}\frac{z^2}{\ell^2} + \frac{1}{16}\frac{z^4}{\ell^4} - \frac{d-1}{d}\mu\frac{z^d}{\ell^d} + \dots, \tag{3.9}$$

so the background is the static Einstein universe $\mathbf{R} \times \mathbf{S}^{d-1}$ with line element

$$g_{\mu\nu}^{(0)}dx^\mu dx^\nu = d\tau^2 - \ell^2 d\Omega_{d-1}^2 \tag{3.10}$$

For $d \neq 4$ we find

$$g_{\mu\nu}^{(d)}dx^\mu dx^\nu = -\frac{\mu}{d\ell^d}[(d-1)d\tau^2 + \ell^2 d\Omega_{d-1}^2], \tag{3.11}$$

with trace $\mathrm{Tr}g^{(d)} = 0$. For $d = 4$ we have

$$g_{\mu\nu}^{(4)}dx^\mu dx^\nu = -\frac{1}{4\ell^4}\left[\left(3\mu - \frac{1}{4}\right)d\tau^2 + \left(\mu + \frac{1}{4}\right)\ell^2 d\Omega_3^2\right], \tag{3.12}$$

with $\mathrm{Tr}g^{(4)} = 1/(4\ell^4)$.

To find the traceless part of the holographic stress tensor (3.3) we just subtract the trace, so for any $d \geq 2$ we have

$$\langle T_{\mu\nu}^{(d)}\rangle = -\frac{d\ell^{d-1}}{16\pi G_{d+1}}\left(g_{\mu\nu}^{(d)} - \frac{1}{d}\mathrm{Tr}g^{(d)}g_{\mu\nu}^{(0)}\right). \tag{3.13}$$

By writing the BH mass parameter μ in terms of r_+ by eq. (2.5), we obtain

$$\langle T^{(d)\mu}{}_\nu\rangle = \frac{\ell^{d-1}}{16\pi G_{d+1}}\left(\frac{r_+}{\ell^2}\right)^d\left(1 + \frac{\ell^2}{r_+^2}\right)\,\mathrm{diag}\,(d-1, -1, -1, \dots). \tag{3.14}$$

This stress tensor is of the form of the stress tensor of a d-dimensional conformal fluid (CF) with the equation of state

$$p_{\mathrm{CF}} = \varepsilon_{\mathrm{CF}}/(d-1), \tag{3.15}$$

where p_{CF} and $\varepsilon_{\mathrm{CF}}$ are the fluid pressure and energy density, respectively, and

$$\varepsilon_{\mathrm{CF}} = \frac{(d-1)\ell^{d-1}}{16\pi G_{d+1}}\left(\frac{r_+}{\ell^2}\right)^d\left(1 + \frac{\ell^2}{r_+^2}\right). \tag{3.16}$$

We now assume that the conformal fluid at the boundary is equivalent to the blackbody radiation at a temperature T. For a theory with n_B massless bosons and n_F massless fermions in $d-1$ spatial dimensions, the energy density of a gas at a temperature T is given by

$$\varepsilon = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left(\frac{n_B k}{e^{k/T}-1} + \frac{g n_F k}{e^{k/T}+1} \right) = a_d T^d, \qquad (3.17)$$

the pressure by $p = \varepsilon/(d-1)$, and the entropy density by

$$s = T^{-1}(p+\varepsilon) = \frac{d a_d}{d-1} T^{d-1}, \qquad (3.18)$$

where

$$a_d = \frac{\Omega_{d-2}\Gamma(d)\zeta(d)}{(2\pi)^{d-1}} \left(n_B + (1-2^{1-d})g n_F \right). \qquad (3.19)$$

The factor g is included to account for degeneracy due to internal degrees of freedom of the fermions. For example, for Weyl fermions in 4-dimensional spacetime, $g=2$.

For a large BH ($r_+ \gg \ell$), the energy density ε_{CF} (as expressed in eq. (3.16)) takes the form (3.17) with the black-body temperature $T \propto r_+$. Then, according to (2.9), the temperature T of the conformal fluid at the AdS boundary is proportional to the horizon temperature T_+ of the large AdS Schwarzschild BH. Hence, the entropy density (3.18) of the conformal fluid at the boundary scales as T_+^{d-1} as expected [2].

3.2 AdS planar BH

Next, we apply the above procedure to a planar BH. Starting from (2.15) we transform the coordinate ρ into z by

$$\frac{dz}{z} = \frac{d\rho}{\rho} \left(1 - \frac{\rho^d}{\rho_{pl}^d} \right)^{-1/2}. \qquad (3.20)$$

Integrating this equation we find

$$\left(\frac{z}{z_{pl}} \right)^d = \frac{1 - \sqrt{1-(\rho/\rho_{pl})^d}}{1 + \sqrt{1-(\rho/\rho_{pl})^d}}, \qquad (3.21)$$

where z_{pl} is an arbitrary positive integration constant which can be interpreted as the location of the planar BH horizon in Fefferman-Graham coordinates. In the limit $\rho \to 0$ we have

$$(z/z_{pl})^d = \frac{1}{4}(\rho/\rho_{pl})^d + \mathcal{O}((\rho/\rho_{pl})^{2d}), \qquad (3.22)$$

so it is natural to identify $z_{pl}^d \equiv 4\rho_{pl}^d$. The inverse to (3.21) is easily found:

$$\rho = z \left(1 + \frac{1}{4} \left(\frac{z}{\rho_{pl}} \right)^d \right)^{-2/d}. \qquad (3.23)$$

Using this we can recast the metric (2.15) in the Fefferman-Graham form

$$ds^2 = \frac{\ell^2}{z^2} \left[\left(1 - \frac{1}{4}\frac{z^d}{\rho_{pl}^d} \right)^2 \left(1 + \frac{1}{4}\frac{z^d}{\rho_{pl}^d} \right)^{4/d-2} d\tau^2 - \left(1 + \frac{1}{4}\frac{z^d}{\rho_{pl}^d} \right)^{4/d} d\boldsymbol{x}^2 - dz^2 \right], \qquad (3.24)$$

We now compare this with the general expression for the asymptotically AdS metric (3.1). Expanding the functions in (3.24) and comparing with (3.1) and (3.2) we find

$$g^{(0)}_{\mu\nu} = \eta_{\mu\nu}, \qquad g^{(d)}_{\mu\nu} = -\frac{1}{d\rho^d_{\rm pl}} \, \mathrm{diag}\,(d-1,1,1,\ldots),$$
(3.25)

so the traceless part of the stress tensor is given by

$$\langle T^{(d)\mu}{}_\nu \rangle = \frac{\ell^{d-1}\rho^{-d}_{\rm pl}}{16\pi G_{d+1}} \, \mathrm{diag}\,(d-1,-1,-1,\ldots).$$
(3.26)

As in the case of the AdS Schwarzschild BH, this form is identical to the stress tensor of the d-dimensional conformal fluid with the equation of state $p_{\rm CF} = \varepsilon_{\rm CF}/(d-1)$, where the energy density is given by

$$\varepsilon_{\rm CF} = \langle T^{(d)0}{}_0 \rangle = \frac{(d-1)\ell^{d-1}\rho^{-d}_{\rm pl}}{16\pi G_{d+1}}.$$
(3.27)

We can compare this with the energy density (3.17) of black-body radiation and relate $\rho_{\rm pl}$ to the black-body temperature $T \propto \rho^{-1}_{\rm pl}$. As before, we find that the entropy of the conformal fluid at the boundary scales as $T^{d-1}_{\rm pl}$.

3.3 Temperature relationships

In this section, based on the correspondence between the CFT entropy at the boundary and the BH entropies, we establish relationships between the temperatures T, T_+, and $T_{\rm pl}$, of the boundary conformal fluid, the large AdS Schwarzschild BH, and AdS planar BH, respectively.

Consider first a large AdS Schwarzschild BH. Our approach relies on the assumption that the entropy of a large BH can be identified with the entropy of the conformal boundary. This assumption is quite natural since we have explicitly demonstrated that the energy density of the conformal fluid at the boundary, according to equation (3.16), scales as r^d_+ or equivalently as T^d_+. As a consequence, the entropy density of the conformal fluid at the boundary scales as T^{d-1}_+, exactly as the entropy of a large AdS Schwarzschild BH. Hence, we equate the Schwarzschild BH entropy $S_{\rm BH}$ given by (2.17) with the total entropy of the black-body radiation on the boundary.

$$S = \lim_{R\to\infty} \frac{da_d}{d-1} T^{d-1} R^{d-1} \Omega_{d-1},$$
(3.28)

where $R^{d-1}\Omega_{d-1}$ is the area of the large hypersphere of radius R, which approaches the boundary. From the equation $S = S_{\rm BH}$ we find

$$T^{d-1} = \frac{d-1}{4da_d G_{d+1}} \frac{r^{d-1}_+}{R^{d-1}}.$$
(3.29)

Then, for a large BH ($r_+ \gg \ell$), using (2.9) we find a relationship between the temperatures T and T_+

$$T = \frac{4\pi\ell}{d} \left(\frac{d-1}{4da_d G_{d+1}}\right)^{1/(d-1)} \frac{T_+}{R/\ell}.$$
(3.30)

Note that, according to Tolman's relation applied to the asymptotic AdS spacetime [30], the local temperature near the boundary is given by

$$T_{+,\text{loc}} \equiv \frac{T_+}{\sqrt{g_{00}}} = \frac{T_+}{R/\ell}, \tag{3.31}$$

and hence, the expression (3.30) relates the temperature of the holographic conformal fluid to the local Schwarzschild BH temperature near the AdS boundary at $r = R$.

Consider next the planar BH geometry. In this case, the total entropy of the black-body radiation on the boundary is given by

$$S = \lim_{\epsilon \to 0} \frac{da_d}{d-1} T^{d-1} \frac{\ell^{d-1}}{\epsilon^{d-1}} L^{d-1}, \tag{3.32}$$

where $(\ell^{d-1}/\epsilon^{d-1})L^{d-1}$ is the area of the hypersurface at $\rho = \epsilon$, which approaches the boundary as $\epsilon \to 0$. As in the case of AdS Schwarzschild BH, we assume that the total entropy (3.32) of the CFT at the holographic boundary equals the AdS planar BH entropy (2.25). Then, from the equation $S = S_{\text{pl}}$ we find

$$T^{d-1} = \frac{d-1}{4da_d G_{d+1}} \frac{\epsilon^{d-1}}{\ell^{d-1}} \frac{\rho_{\text{pl}}^{d+1}}{\ell^{d+1}}. \tag{3.33}$$

As mentioned in section 2.2, to recover the area law for the planar BH we set $\rho_{\text{pl}} = \ell$. In this case, according to (2.16), the temperature of the planar BH is $T_{\text{pl}} = d/(4\pi\ell)$. Using this and eq. (3.33), the temperature T may be expressed as

$$T = \frac{4\pi\ell}{d} \left(\frac{d-1}{4da_d G_{d+1}} \right)^{1/(d-1)} \frac{T_{\text{pl}}}{\ell/\epsilon}. \tag{3.34}$$

Here, similar to case of Schwarzschild BH geometry, the quantity $T_{\text{pl}}/(\ell/\epsilon)$ can be regarded as the local temperature near the AdS boundary at $\rho = \epsilon$,

$$T_{\text{pl,loc}} \equiv \frac{T_{\text{pl}}}{\sqrt{g_{00}}} = \frac{T_{\text{pl}}}{\ell/\epsilon}. \tag{3.35}$$

Hence, the expression (3.34) relates the temperature of the holographic conformal fluid to the planar BH local temperature near the AdS boundary.

Now, we identify the temperature T of the boundary conformal fluid defined by (3.34) with that defined by (3.30). As a consequence, we obtain a relation between the redshifted local temperatures of the planar BH and the large Schwarzschild BH measured near the AdS boundary

$$\frac{T_+}{R/\ell} = \frac{T_{\text{pl}}}{\ell/\epsilon}. \tag{3.36}$$

Here, the quantity R is a cutoff near the AdS boundary with spherically symmetric geometry, and ϵ is the corresponding cutoff with planar geometry. We introduce these cutoffs to regularize the entropy of the conformal fluid at the AdS boundary, so it is natural to assume that R/ℓ and ℓ/ϵ are of the same order of magnitude. Hence, the large AdS Schwarzschild BH appears as hot as the corresponding planar BH. Using (3.36) and (2.16) with $\rho_{\text{pl}} = \ell$

we can express the horizon temperature of the large AdS Schwarzschild BH in terms of the planar BH temperature

$$T_+ \sim T_{\rm pl} = \frac{d}{4\pi\ell}.$$ (3.37)

Equations (3.36) and (3.37), as our main result, are obtained by invoking the AdS/CFT conjecture and assuming that the temperature of an AdS planar BH can be fixed by imposing the entropy area law. The main assumption leading to equation (3.36) is the identification of the total entropy of the CFT at the holographic boundary with the horizon entropy of the BH in the bulk. In the case of spherical symmetry, this identification yields the black-body radiation temperature of the conformal fluid expressed in terms of the BH horizon temperature T_+ via equation (3.29). Similarly, in the case of planar symmetry, the same identification yields the black-body temperature of the conformal fluid expressed in terms of the planar BH temperature $T_{\rm pl}$ via equation (3.30). Combining equations (3.29) and (3.30) we obtain (3.36). Then, equation (3.37) is a direct consequence of (3.36) since the cutoffs R/ℓ and ℓ/ϵ are assumed to be of the same order of magnitude. The physical meaning of equations (3.36) and (3.37) is quite clear: the temperature of a large Schwarzschild BH and that of the corresponding planar BH, as seen by an observer near the AdS boundary, are of the same order of magnitude. This result is significant as it resolves the horizon temperature ambiguity of the large AdS Schwarzschild BH and its relation to the AdS planar BH.

4 Conclusions

By making use of the AdS/CFT correspondence we have calculated the traceless part of the vacuum expectation value of the stress tensor of the boundary CFT corresponding to the bulk geometry of AdS Schwarzschild and planar BHs. Based on the correspondence between the CFT entropy at the boundary and the BH entropies, we have established relationships between the temperatures T, T_+, and $T_{\rm pl}$ of the conformal fluid at the boundary, large Schwarzschild BH, and planar BH, respectively.

In this way, we predict that the temperatures of large AdS Schwarzschild and planar BHs are of the same order of magnitude as seen by an observer near the AdS boundary. Because of the diverging redshift near the boundary, local observers measure a large redshifted temperature of the AdS Schwarzschild BH and the actual horizon temperature T_+ is unobservable. This contrasts with the case of ordinary Schwarzschild BH where asymptotic observers see the actual horizon temperature.

Our result confirms the argument of ref. [1] based on the translational invariance of the planar BH horizon and previous assertion [28, 29] that the high temperatures associated with large values of T_+ are not locally observable.

Acknowledgments

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