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Lifetimes of singly charmed hadrons

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Abstract: We provide an extensive study of the lifetimes of singly charmed baryons and mesons, within the heavy quark expansion with all known corrections included. A special attention is devoted to the choice of the charm mass and wavefunctions of heavy baryons. We give our predictions for lifetimes, lifetime ratios, and semileptonic branching ratios of singly charmed baryons. Our results accommodate the experimentally-favoured hierarchy of singly charmed baryon lifetimes

$$\tau\left(\Xi_c^0\right) < \tau\left(\Lambda_c^+\right) < \tau\left(\Omega_c^0\right) < \tau\left(\Xi_c^+\right) ,$$

in contrast to earlier theoretical findings. Predictions for charmed meson lifetimes and semileptonic decay rates are in agreement with a recent comprehensive study and experimental results within uncertainties.

Keywords: Properties of Hadrons, Charm Flavour Violation

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1 Introduction

The recent measurements of charmed baryon lifetimes by the LHCb Collaboration [1–3] stand in marked contrast to earlier determinations. Whereas the lifetimes of the Λ_c^+ and Ξ_c^+ are compatible with previous experiments [4–8], that of the Ξ_c^0 is in roughly 3σ tension with the older measurement [9, 10]. Even more dramatically, the measured lifetime of the Ω_c^0 of 274.5 fs is four times larger than, and wholly inconsistent with, earlier results [11, 12]. Moreover, the newly-established hierarchy of experimental lifetimes,

$$\tau\left(\Xi_c^0\right) < \tau\left(\Lambda_c^+\right) < \tau\left(\Omega_c^0\right) < \tau\left(\Xi_c^+\right) , \tag{1.1}$$

is in conflict with earlier theoretical predictions [13, 14], where in particular the Ω_c^0 was expected to be the shortest-lived among the singly charmed baryons.

On the theoretical side, the approach to calculating lifetimes proceeds via the heavy quark expansion (HQE), which is an expansion of the inclusive decay width in inverse powers of the heavy quark mass, developed in the 1980s and early 1990s, eg [15–20]. This was motivated by the experimental observation that the lifetime ratio of D mesons [21],

$$\frac{\tau\left(D^+\right)}{\tau\left(D^0\right)} = 2.54(2) , \tag{1.2}$$

is significantly different from the naive prediction of unity, based upon the assumption that the charm quark decay is the dominant contribution. Once contributions sensitive to the flavour of the light valence quark were taken into account, the then-experimental hierarchy could be reproduced [22–27].[1] In response to the temporary $\tau(\Lambda_b)/\tau(B)$ lifetime puzzle (discussed for example in [29] and references therein), the focus of the HQE turned to b-quark hadrons, where the much-improved convergence of the $1/m_Q$ series motivated the analysis of higher-order terms [30–36].

The applicability of the HQE to charm decays, as well as the correct way to perform the expansion, is an open question, with some alternative approaches appearing recently in [37, 38]. In the most recent study of D meson lifetimes within the HQE, including the most complete set of contributions [39], the central value of the decay width of the D^+ was found to be extremely small or even negative, driven by a large Pauli interference contribution, an observation also made in [40]. On the other hand, the uncertainties in such predictions are sizeable, due to large hadronic and scale uncertainties. In the same paper [39], the HQE predictions for ratios of decay widths and of the semileptonic branching fraction of mesons were found to be compatible with the experimental values, notwithstanding the ratio of the lifetimes of D_s^+ and D^0 that remained in a slight tension with experiment. This seems to support the possibility that the HQE is a successful approach in understanding at least the qualitative nature of charm physics in the meson sector.

The most recent update to the theoretical prediction of singly charmed baryon lifetimes was made in 2018 [40]. That calculation considered the effects of subleading spectator corrections, but neglected the Darwin term and QCD corrections, which in the case of

[1] See [28] and [29] for further details about the history of the HQE, as well as further references.

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charm physics are sizeable [39], and did not provide an error analysis. Moreover, whilst the prediction in [40] managed to accommodate the new experimental lifetime hierarchy implied by [1], it was only able to do so by introducing an arbitrary factor suppressing certain contributions to the Ω_c^0 decay width. Such a resolution is hardly satisfying.

In this paper, we extend the analysis of [39] by revisiting the inclusive lifetime predictions for the baryon sector. As compared with previous studies [13, 14, 40], we include the Darwin contributions, recently computed in [41–43] and extended to charm hadrons in [39], and dimension-seven four-quark operator contributions, first considered in the context of B hadrons in [32, 44] and subsequently in the charm sector in [28, 40]. In addition, we include existing next-to-leading order (NLO) contributions to the Wilson coefficients of two-quark operators at dimension-three and four-quark operators [30, 45] at dimension-six. We also repeat the computations of D meson lifetimes performed in [39], verifying their results, with minor differences originating from different estimates of some of the hadronic parameters.

The paper is organized as follows. In section 2, we briefly outline the HQE, defining our notation and the contributions to be included, with current experimental results presented in section 2.1 for ease of comparison. Some comments on the charm mass schemes used are presented in section 2.4. In section 3, we discuss the two-quark contributions and values for the matrix elements for all hadrons of interest to the paper, presenting the numerical results for these "non-spectator" contributions. In section 4, we present results for inclusive observables for charmed mesons, and in section 5, we do likewise for the baryons, after an extensive discussion of the baryon wavefunctions in section 5.1. The paper ends with conclusions in section 6. Appendix A collects numerical inputs used in this work, and appendix B collates various useful analytic expressions. In appendix C we give some technical details on properly relating HQET and QCD four-quark matrix elements. Finally, in appendix D we provide supplementary tables with a detailed breakdown of contributions to meson and baryon observables.

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2 Theoretical and experimental background

2.1 Experimental status

Before proceeding to a discussion of the theoretical approach to predicting lifetimes, we briefly review the present status of experimental measurements of inclusive charmed hadron lifetimes.

2.1.1 Charmed mesons

The current experimental values of the lifetimes (largely unchanged since the early 2000s) and the semileptonic branching fractions of charmed mesons are summarized in table 1. The experimental values for the lifetime ratios are therefore

$$\frac{\tau(D^+)}{\tau(D^0)} = 2.54 \pm 0.02\,, \qquad \frac{\tau(D_s^+)}{\tau(D^0)} = 1.23 \pm 0.01. \tag{2.1}$$

Quantity	D^0	D^+	D_s^+
$\tau\,[ps]$	0.4101 ± 0.0015	1.040 ± 0.007	0.504 ± 0.004
$\Gamma\,[ps^{-1}]$	2.438 ± 0.009	0.962 ± 0.006	1.984 ± 0.0016
$BR(D_i \to Xe\nu)\,[\%]$	6.49 ± 0.16	16.07 ± 0.30	6.30 ± 0.16
$\Gamma(D_i \to Xe\nu)\,[ps^{-1}]$	0.158 ± 0.004	0.155 ± 0.003	0.125 ± 0.003

Table 1. Summary of measured values for the lifetimes and semileptonic branching fractions of charmed mesons. The quoted values are the latest PDG averages [21], with the exception of $BR(D_s \to Xe\nu)$, for which we show the recent result by BESIII [46]. We combine the statistical and systematic errors in quadrature in cases where both are given by experimental collaborations. A recent Belle II measurement [47] is compatible with the world averages.

Since, however, the heavy quark expansion does not account for the pure leptonic decay of $D_s \to \tau\nu_\tau$, one usually defines the modified width

$$\tilde{\Gamma}\left(D_s^+\right) = \Gamma\left(D_s\right)\left(1 - BR\left(D_s^+ \to \tau\nu\right)\right) . \tag{2.2}$$

Using the PDG average value $BR(D_s^+ \to \tau\nu) = (5.48 \pm 0.23)\%$ [21] and the above value for $\tau(D_s)$, we obtain

$$\tilde{\tau}(D_s^+) = 0.533 \pm 0.004\,\mathrm{ps}\,, \tag{2.3}$$

with the corresponding ratio, to be compared to theoretical estimates, of

$$\frac{\tilde{\tau}\left(D_s^+\right)}{\tau\left(D^0\right)} = 1.30 \pm 0.01. \tag{2.4}$$

As can be seen, the experimental precision for charmed meson measurements is now at a sub-% level. In particular, the most recent measurement, from Belle II [47], is compatible with earlier values, indicating that the lifetime measurements of charmed mesons are robust.

We will also consider the ratios of the inclusive semileptonic decay widths involving the electrons in the final states. Denoting $\Gamma(D \to Xe\nu) \equiv \Gamma^{(e)}(D)$, and combining the experimental results from table 1, we have:

$$\frac{\Gamma^{(e)}(D^+)}{\Gamma^{(e)}(D^0)} = 0.977 \pm 0.031\,, \tag{2.5}$$

while for the remaining ratio we adopt the value given by the BESIII Collaboration [46],

$$\frac{\Gamma^{(e)}\left(D_s^+\right)}{\Gamma^{(e)}\left(D^0\right)} = 0.790 \pm 0.026\,, \tag{2.6}$$

with the statistic and systematic uncertainties combined in quadrature.

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Collaboration	$\tau\left(\Lambda_c^+\right)$/fs	$\tau\left(\Xi_c^+\right)$/fs	$\tau\left(\Xi_c^0\right)$/fs	$\tau\left(\Omega_c^0\right)$/fs
CLEO [5, 7]	179.6 ± 8.2	503 ± 50	N/A	N/A
FOCUS [6, 8, 9, 11]	203.5 ± 4.2	439 ± 24	118^{+15}_{-13}	72 ± 16
SELEX [4, 12]	198.1 ± 9.0	N/A	N/A	65 ± 16 [2]
LHCb [1, 2]	203.5 ± 2.2	457 ± 6	154.5 ± 2.6	268 ± 26
LHCb 2021 [3]	N/A	N/A	148.0 ± 3.2	276.5 ± 14.1
PDG 2018 [10]	200 ± 6	442 ± 26	112^{+13}_{-10}	69 ± 12
PDG 2020 [21]	202.4 ± 3.1	456 ± 5	153 ± 6	$268 \pm 24 \pm 10$
Reference values	202.4 ± 3.1 [21]	456 ± 5 [21]	152.0 ± 2.0 [3]	274.5 ± 12.4 [3]

Table 2. Summary of lifetime measurements of singly charmed baryons. All results are expressed in femtoseconds. As in table 1, we combine statistical and systematic errors in quadrature. The PDG world-average (as of 2018 and 2020, showing the changes due to the LHCb results) is also included. Lifetime measurements that are not available (due to never being performed at the given experiment, or at least with no existing reference) are marked "N/A". The most recent LHCb determination, of the Ω_c^0 and Ξ_c^0 lifetimes, is given in a separate row to highlight the two separate measurements. In the last row we list the most precise current results used as the reference values for the comparisons to our theoretical predictions, taken from PDG [21] for the Λ_c^+ and Ξ_c^+, and from the LHCb averages given in [3] for the Ξ_c^0 and Ω_c^0.

2.1.2 Singly charmed baryons

For baryons, experimental results are listed in table 2. Given the new LHCb results, which significantly differ from previous measurements, it is useful to be more comprehensive about the lifetime measurements. As compared with the meson lifetimes, there are some tensions in the available data. In particular:

1. The lifetime measurement for Ξ_c^0 has significantly shifted between the two eras, representing a $\sim 3\sigma$ tension. The most recent PDG update includes LHCb's earlier results [2].

2. Likewise, the Ω_c^0 lifetime has shifted even more dramatically. PDG has in fact abandoned all earlier measurements, given that the LHCb data set is approximately five times larger than from all previous experiments.

3. We also note that the CLEO measurements are in some tension with other results, in particular in the case of $\tau\left(\Lambda_c^+\right)$. The most recent LHCb measurement is in good agreement with other results, supporting the conclusion that $\tau\left(\Lambda_c^+\right)$ is close to 200 fs.

The most notable of these is undoubtedly the shift in the Ω_c^0 lifetime, which is now almost four times longer, but was previously found to be the shortest-lived charmed baryon. This prompts a new experimental hierarchy of charmed baryon lifetimes,

$$\text{exp:} \qquad \tau\left(\Xi_c^0\right) < \tau\left(\Lambda_c^+\right) < \tau\left(\Omega_c^0\right) < \tau\left(\Xi_c^+\right) . \tag{2.7}$$

[2] Unpublished except in a preprint, not cited in PDG.

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Using the reference values of the lifetimes shown in the last row of table 2 we obtain the lifetime ratios

$$\frac{\tau\left(\Xi_c^+\right)}{\tau\left(\Lambda_c^+\right)} = 2.25 \pm 0.04\,, \qquad \frac{\tau\left(\Xi_c^0\right)}{\tau\left(\Lambda_c^+\right)} = 0.75 \pm 0.02\,, \qquad \frac{\tau\left(\Omega_c^0\right)}{\tau\left(\Lambda_c^+\right)} = 1.36 \pm 0.06\,, \tag{2.8}$$

to which we compare our theoretical predictions.

Finally, the inclusive semileptonic branching fraction of $\Lambda_c^+ \to Xe\nu$ has been measured experimentally as [48]

$$\mathrm{BR}\left(\Lambda_c^+ \to Xe\nu\right) = (3.95 \pm 0.35)\%\,. \tag{2.9}$$

The remaining three semileptonic branching fractions ($\mathrm{BR}(\Xi_c^+ \to Xe\nu)$, etc), have not yet been measured. Such measurements would provide further important checks of any specific theoretical approach. We provide our predictions of these branching fractions in section 5.2.

2.2 The heavy quark expansion and inclusive decays

In this section, we briefly overview the heavy quark expansion (HQE), and refer the reader to [29] for a more detailed review.

Via the optical theorem, the total decay width can be related to the imaginary part of the forward transition operator:

$$\frac{1}{\tau\left(H\right)} = \Gamma(H) = \frac{1}{2m_H}\langle H|\mathcal{T}|H\rangle\,, \qquad \mathcal{T} = \mathrm{Im}\; i\int d^4x\, T\left[\mathcal{H}_{\mathrm{eff}}(x)\mathcal{H}_{\mathrm{eff}}(0)\right]\,, \tag{2.10}$$

where $\mathcal{H}_{\mathrm{eff}}$ is the effective Hamiltonian describing the charged current interactions of the charm quark (eg [49])

$$\begin{aligned}\mathcal{H} = \frac{G_F}{\sqrt{2}}\Bigg[&\sum_{q,q'=d,s} V_{cq}V_{uq'}^*\left(C_1\left(\mu\right)Q_1^{(qq')} + C_2\left(\mu\right)Q_2^{(qq')}\right) - V_{ub}V_{cb}^*\sum_{k=3}^{6} C_k\left(\mu\right)Q_k \\ &+ \sum_{\substack{q=d,s\\ \ell=e,\mu}} V_{cq}Q^{(q\ell)}\Bigg]\,,\end{aligned} \tag{2.11}$$

where G_F is the Fermi constant, V_{ab} are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and

$$\begin{aligned}Q_1^{(qq')} &= \left(\bar{c}^i\gamma_\mu\left(1-\gamma_5\right)q^j\right)\left(\bar{q}'^j\gamma^\mu\left(1-\gamma_5\right)u^i\right)\,,\\ Q_2^{(qq')} &= \left(\bar{c}^i\gamma_\mu\left(1-\gamma_5\right)q^i\right)\left(\bar{q}'^j\gamma^\mu\left(1-\gamma_5\right)u^j\right)\,,\\ Q_{\mathrm{SL}}^{(q\ell)} &= \left(\bar{c}\gamma_\mu\left(1-\gamma_5\right)q\right)\left(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell\right)\,,\end{aligned} \tag{2.12}$$

where i, j are colour indices. The remaining operators $Q_{3\text{-}6}$ denote the penguin operators, which are suppressed by the CKM factor $V_{ub}V_{cb}^*$. Since the Wilson coefficients C_3-C_6 are also numerically small (eg [39, 49]), we will neglect these contributions in the present paper. Note that $Q_2^{qq'}$ denotes the colour-singlet operator in our convention, following [49] but opposite to the choice by some other authors, eg [13, 39, 40], where Q_1 is the colour-singlet.

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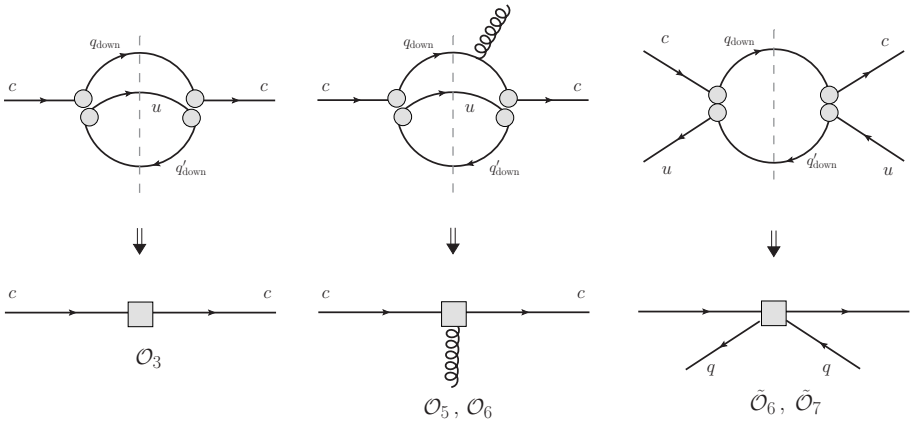

Figure 1. Representative diagrams illustrating various contributions to the inclusive width of charmed hadrons. (a) The leading contribution O_3. (b) higher-order "non-spectator" terms in the first series, generated by insertion of covariant derivatives with respect to the background gluon field. (c) An example of a four-quark "spectator" contribution, in this case Weak exchange.

The right-hand side of (2.10) can then be expanded, using the HQE, in powers of $\Lambda_{\rm QCD}/m_Q$ and α_s, where m_Q is the heavy-quark mass and $\Lambda_{\rm QCD}$ is the QCD scale [16, 19]. This yields a tower of local operators $\mathcal{O}_i$, ordered by increasing powers of the inverse heavy quark mass m_Q,

$$\mathcal{T} = \left(\mathcal{C}_3\mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2}\mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3}\mathcal{O}_6 + \ldots\right) + 16\pi^2\left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3}\tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4}\tilde{\mathcal{O}}_7 + \ldots\right), \tag{2.13}$$

where the Wilson coefficients $\mathcal{C}_i$ contain the short-distance physics, analogously to the C_i in (2.11).[3] The operators within the first bracket are each composed of heavy-quark field bilinears, with operators of increasing dimension generated by insertion of covariant derivatives, and will be referred to below as the "non-spectator" contributions. The leading term $\mathcal{O}_3$ is represented by diagram (a) in figure 1, while $\mathcal{O}_{5,6}$ is represented by diagrams similar to (b) in figure 1. The terms within the second bracket involve the contributions of four-quark operators, with one example given by diagram (c) in figure 1. These "spectator contributions" are sensitive to the flavour of the light quark in the hadron, and are one-loop enhanced relative to the non-spectator contributions by the factor $16\pi^2$. Therefore, they can result in significant lifetime splitting effects.

The $\mathcal{C}_i$ can be calculated perturbatively in powers of the strong coupling constant α_s,

$$\mathcal{C}_i = \mathcal{C}_i^{(0)}(\mu,\mu_0) + \mathcal{C}_i^{(1)}(\mu,\mu_0)\,\alpha_s(\mu) + \mathcal{C}_i^{(2)}(\mu,\mu_0)\,\alpha_s(\mu)^2 + \ldots, \tag{2.14}$$

where μ is the renormalization scale arising from evolution of the weak Hamiltonian. Both the Wilson coefficients and the operators further depend on the operator factorization scale μ_0. As will be made clear later, only a few of the $\mathcal{C}_i$ in (2.13) are known beyond leading order.

[3]The absence of the dimension-four operator, suppressed by $\Lambda_{\rm QCD}/m_Q$, was demonstrated in [16, 50].

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Having summarized some of the key ideas, we now express the decay width more explicitly, in the form

$$\Gamma(H) = \Gamma_0 \Bigg[c_3 + \frac{c_\pi \mu_\pi^2 + c_G \mu_G^2}{m_Q^2} + \frac{c_\rho \rho_D^3}{m_Q^3} + \cdots + \frac{16\pi^2}{2m_H} \Bigg(\sum_{i,q} \frac{c_{6,i}^q \langle H|O_i^q|H\rangle}{m_Q^3} + \sum_i \frac{c_{7,i}^q \langle H|P_i^q|H\rangle}{m_Q^4} + \dots \Bigg) \Bigg] , \tag{2.15}$$

with the sum over $q = u, d, s$, where $m_Q = m_c$ is the pole mass of the charm quark and m_H the mass of the heavy hadron. The normalization factor is

$$\Gamma_0 = \frac{G_F^2 m_Q^5}{192\pi^3} , \tag{2.16}$$

and all CKM contributions are included implicitly in the coefficients c_i in the equation above. The definitions of the non-perturbative parameters μ_π^2, μ_G^2, ρ_D^3, and the description of the spectator contributions, are given in the next section. The overall decay width (2.15) can be further split into contributions from semileptonic (SL) decays and nonleptonic (NL) decays, where the SL decays can also be measured separately in experiments. Note that the above form, and in particular the coefficients c_i, applies to all hadrons considered in this paper, and can also be applied to hadrons containing a b quark upon appropriate replacement of the quark/lepton masses, quark fields in the matrix elements, and CKM factors.[4] Given that the c_i are therefore universal, we now turn to briefly discussing their present status, as well as defining the operator basis of (2.15).

2.3 Contributions to the decay width and operator basis

To calculate the lifetimes, we apply the following expansions, given schematically as:

$$\begin{aligned}
\Gamma =& \Gamma^{\rm NL} + \Gamma^{\rm SL} , \\
\Gamma^{\rm NL} =& \, g_3^{(0)} + \alpha_s g_3^{(1)} + \frac{1}{m_c^2} \left(g_\pi^{(0)} + g_G^{(0)} \right) + \frac{1}{m_c^3} g_{\rm Darwin}^{(0)} + \frac{16\pi^2}{m_c^3} \left(\tilde{g}_6^{(0)} + \alpha_s \tilde{g}_6^{(1)} + \frac{1}{m_c} \tilde{g}_7^{(0)} \right) , \\
\Gamma^{\rm SL} =& \, g_3^{(0)} + \alpha_s g_3^{(1)} + \frac{1}{m_c^2} \left(g_\pi^{(0)} + \alpha_s g_\pi^{(1)} + g_G^{(0)} + \alpha_s g_G^{(1)} \right) + \frac{1}{m_c^3} g_{\rm Darwin}^{(0)} \\
& + \frac{16\pi^2}{m_c^3} \left(\tilde{g}_6^{(0)} + \alpha_s \tilde{g}_6^{(1)} + \frac{1}{m_c} \tilde{g}_7^{(0)} \right) ,
\end{aligned} \tag{2.17}$$

where the individual contributions will be described in this section. g_3 is the leading non-spectator contribution, while g_G, g_π and $g_{\rm Darwin}$ are $1/m_c^n$ suppressed contributions; $\tilde{g}_{6,7}$ label the four-quark spectator contributions. Superscripts (0) and (1) denote leading order (LO) and next-to-leading order (NLO) contributions respectively.

[4] In [37], it was argued that, when applying the HQE to inclusive charm decays, the strange quark mass should also be considered as an expansion parameter with the same status as $\Lambda_{\rm QCD}/m_Q$. We return to this point in section 2.3.1, but note here that in the present approach we follow eg [28, 39, 40] in handling m_s contributions.

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2.3.1 Non-spectator contributions

The non-spectator terms are given by the first series in (2.15):

$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{c_\pi \mu_\pi^2 + c_G \mu_G^2}{m_Q^2} + \frac{c_\rho \rho_D^3}{m_Q^3} + \dots \right] \tag{2.18}$$

where the matrix elements [20, 34, 39, 51, 52]

$$\begin{aligned} \mu_\pi^2(H) &= \frac{-1}{2m_H} \langle H | \bar{c}_v (iD)^2 c_v | H \rangle \,, \\ \mu_G^2(H) &= \frac{1}{2m_H} \langle H | \bar{c}_v \frac{1}{2} \sigma \cdot (g_s G) c_v | H \rangle \,, \\ \rho_D^3(H) &= \frac{1}{2m_H} \langle H | \bar{c}_v (iD_\mu)(iv \cdot D)(iD^\mu) c_v | H \rangle \,, \end{aligned} \tag{2.19}$$

are the kinetic, chromomagnetic, and Darwin operators respectively ($\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$).[5] In some conventions, an additional operator arises at dimension-six, the spin-orbit term ρ_{LS}^3, but in the basis above its coefficient in the total decay rate vanishes. The above expressions are defined in terms of the heavy charm QCD field, with the large momentum fraction removed, ie [53, 54]

$$c_v(x) = e^{i m_c v \cdot x} c(x) \,, \tag{2.20}$$

where v^μ is the four-velocity of the hadron. The leading term c_3 is the coefficient of the matrix element $\langle H | \bar{c}_v c_v | H \rangle / (2m_H)$, normalized to 1 at leading order in the $1/m_c$ expansion. In fact, at dimension-five, diagrams of the type exemplified by figure 1(b) do not contribute to c_π, meaning that it is only non-zero owing to the expansion of the dimension-three matrix element [34, 55]

$$\frac{1}{2m_H} \langle H | \bar{c}_v c_v | H \rangle = 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_Q^2} + \mathcal{O}\left(1/m_Q^4\right) . \tag{2.21}$$

Consequently, $c_\pi = -c_3/2$. On the other hand, figure 1(b) does contribute to c_G, alongside the contribution generated by the expansion (2.21).

To LO, the general form of the coefficients is

$$c_n^{(0)} = N_C C_1^2 \mathcal{K}_{n,11}^{(0)} + 2 C_1 C_2 \mathcal{K}_{n,12}^{(0)} + N_C C_2^2 \mathcal{K}_{n,22}^{(0)} + \mathcal{K}_{n,\mathrm{SL}}^{(0)} \,, \tag{2.22}$$

where $N_C = 3$ is the number of colours, and we remind the reader that in our conventions C_2 is the Wilson coefficient for the colour-singlet operator. Defining the mass ratios as

$$x_s = \frac{m_s^2}{m_c^2} \,, \qquad x_\mu = \frac{m_\mu^2}{m_c^2} \,, \tag{2.23}$$

then the LO results for c_3 are [29, 40, 56, 57]

$$\begin{aligned} \mathcal{K}_{3,ij}^{(0)} &= |V_{cs}|^2 \left(|V_{ud}|^2 I_0(x_s, 0, 0) + |V_{us}|^2 I_0(x_s, x_s, 0) \right) + |V_{cd}|^2 |V_{ud}|^2 I_0(0, 0, 0) \,, \\ \mathcal{K}_{3,\mathrm{SL}}^{(0)} &= |V_{cs}|^2 \left(I_0(x_s, 0, 0) + I_0(x_s, x_\mu, 0) \right) + |V_{cd}|^2 \left(I_0(0, 0, 0) + I_0(0, x_\mu, 0) \right) \,, \end{aligned} \tag{2.24}$$

[5] In some early literature, eg [51], the alternative notation $\lambda_{1,2}$ was employed for the first two matrix elements, with $\mu_\pi^2 = -\lambda_1$ and $\mu_G = d_H \lambda_2$, where d_H is a spin factor given explicitly in (3.3).

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where we have included the singly CKM-suppressed contributions, while $I_0(x,y,z)$ is a phase-space function defined explicitly in appendix B. At LO, all the Wilson coefficient structures in c_3 are identical, while the SL part can be recovered from the C_2^2 coefficient by applying the replacements $C_2 \to 1$, $N_C \to 1$, $|V_{ud}|^2 \to 1$, and by appropriate redefinition of masses. For c_G we have

$$\begin{aligned}
\mathcal{K}^{(0)}_{G,ii} &= -|V_{cs}|^2 \left(|V_{ud}|^2 I_1\left(x_s,0,0\right) + |V_{us}|^2 I_1\left(x_s,x_s,0\right)\right) - |V_{cd}|^2|V_{ud}|^2 I_1\left(0,0,0\right) \,, \\
\mathcal{K}^{(0)}_{G,12} &= |V_{cs}|^2 \left(|V_{ud}|^2 \left(4I_2\left(x_s,0,0\right) - I_1\left(x_s,0,0\right)\right) + |V_{us}|^2 \left(4I_2\left(x_s,x_s,0\right) - I_1\left(x_s,x_s,0\right)\right)\right) \\
&\quad + |V_{cd}|^2|V_{ud}|^2 \left(4I_2\left(0,0,0\right) - I_1\left(0,0,0\right)\right) \,, \\
\mathcal{K}^{(0)}_{G,SL} &= -|V_{cs}|^2 \left(I_1\left(x_s,0,0\right) + I_1\left(x_s,x_\mu,0\right)\right) - |V_{cd}|^2 \left(I_1\left(0,0,0\right) + I_1\left(0,x_\mu,0\right)\right) \,,
\end{aligned} \tag{2.25}$$

where $I_{1,2}(x,y,z)$ are additional phase-space functions given in appendix B [17, 18, 58, 59]. It is worth noting that the scale dependence of c_G at LO is quite significant, and can even cause c_G to change sign, going negative at larger values of μ [39].

As for c_ρ, no such compact expressions are available, but we provide the analytic expressions, taken from [39] (see also [41–43, 60, 61]), in appendix B. The SL parts of $c_{G,\rho}$ can again be recovered from the C_2^2 coefficient by applying the same replacements listed below eq. (2.24).

In terms of the α_s expansion, c_3 has been computed to NLO for NL decays in [62–65], with a partial result at NNLO in [66]. For SL decays, results are available at NLO in [67], NNLO in [68–72], and recently to N^{3}LO in [73, 74]. However, since many results beyond NLO are partial, and in view of the divergent nature of the c_3 series, we restrict our analysis to include only the NLO contributions in the present work, and will treat the α_s series as asymptotic.[6]

For c_G, results are available at NLO [75–77] only for SL decays. We include these results in our analysis. For the c_ρ contribution, the NLO result was computed, for $b \to c\tau\nu_\tau$ decays, in [78, 79]. It is worth stressing, however, that in handling the ρ_D^3 contributions, care must be taken in handling the mixing effects with the other dimension-six operators [37, 39], with a different treatment required for $c \to s$ decays as compared with $b \to c$ decays. As a result, the results of [78, 79] cannot be naively applied to $c \to s$ decays, and so we do not include them in our analysis.

In summary, we include the available NLO contributions for all non-spectator terms apart from in the Darwin contribution, but do not include contributions beyond NLO. A list of relevant contributions and references is given in table 4.

Before proceeding to discuss the contributions arising from four-quark operators, we wish to discuss two alternative approaches to the HQE in the charm sector advanced recently, specifically those in [37] and [38]. The work of the former is particularly interesting, and may not yet have received enough attention in the inclusive HQE literature. The principal idea in that paper is that, for consistency, the parameter m_s/m_c should be treated as an

[6] Some further comments on the c_3 series can be found in section 2.4.2.

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expansion parameter in the HQE, on the same footing as $\Lambda_{\rm QCD}/m_c$. This differs from the approach of [13, 39, 40], whereby results obtained traditionally in the setting of inclusive $b \to c$ decays are assumed to apply to $c \to s$ decays, with appropriate replacement of quark masses and CKM factors, eg [13, 40]. In the context of the Darwin contribution, the work of [39] partially confirms this (compare with [41–43] computing the same terms for $b \to c$ decays), but in fact the modified HQE in the charm sector goes beyond just the Darwin contribution, as can be confirmed by comparing the expressions in eqs. (5.6) and (A.1) in [37]. To what extent this represents a genuine difference that cannot be reconciled with the standard approach, as opposed to merely a re-ordering of the same expression, remains to be seen. Since, however, the factor m_s^2/m_c^2 is fairly small, it is reasonable to assume that any errors in the handling of the strange quark in the present approach are negligible compared with other uncertainties. Further studies of this point would be welcome, particularly if determinations of other parameters in the HQE come with reduced uncertainties.

In [38], it was argued that the contribution to inclusive decays of four-quark operators should be re-summed, and considered part of the leading term, in order to render the HQE a true expansion with parameters of order unity. This, however, presents the difficulty of evaluating non-local matrix elements, and while this could perhaps be done on the lattice in the future, the approach has thus far only been tested in an extremely simplified setting. Further work exploring these questions would also be welcome, especially as an alternative to merely evaluating further terms in the slowly-converging $1/m_c$ series.

In any case, the validity of the current approach, or equivalently the urgency of re-formulating the HQE for charm decays, can be assessed most strongly by comparing the results obtained with experimental data. The debate over the proper application of the HQE to charm decays is hardly new, eg [28, 29, 80, 81], and is likely to continue for some time. Applying the approaches advocated in [37, 38] of tailoring the HQE more suitably for charm decays in a more concrete setting may serve to clarify the issue.

2.3.2 Spectator contributions

The remaining contributions to the decay width (2.15) arise from four-quark operators, and can be described, for mesons, by the topologies in figure 2. The three topologies are typically referred to as weak exchange (WE), Pauli interference (PI), and weak annihilation (WA). As compared with the terms discussed in the previous section, these are enhanced by the factor $16\pi^2$, being one-loop effects, and are primarily responsible for the lifetime splitting between heavy hadrons. For baryons, the equivalent topologies are represented in figure 3, and are referred to as weak exchange (exc), constructive Pauli interference (int^+), and destructive Pauli interference (int^-). The correspondence to the equivalent meson contributions (WE $\leftrightarrow$ int^-, PI $\leftrightarrow$ exc, WA $\leftrightarrow$ int^+) is clearly visible by comparing figure 2 and figure 3. Practically, this means that the expressions are the same at the operator level, although they differ at the level of the resulting matrix elements, as discussed in sections 4 and 5. Spectator contributions are also present in semileptonic decays of both mesons and baryons, as indicated in figures 2(c) and 3(c) respectively, with the identical topology to WA/int^+. For mesons, such contributions are helicity-suppressed, but this does not apply

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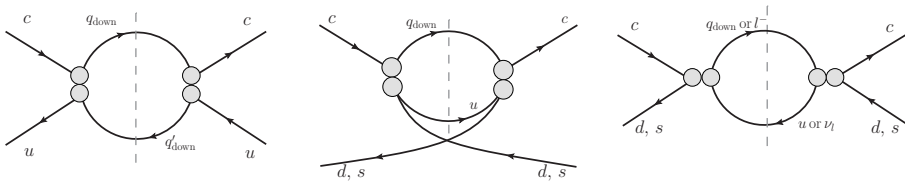

Figure 2. Diagrammatic representations of the spectator contributions resulting in the four-quark operators in *mesons*. From left to right: (a) weak exchange (WE), (b) Pauli interference (PI), (c) weak annihilation (WA).

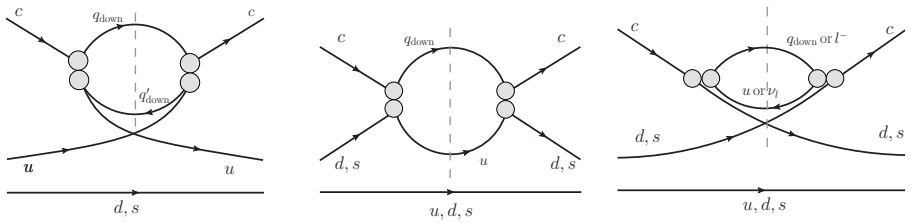

Figure 3. Diagrammatic representations of the spectator contributions resulting in the four-quark operators in *baryons*. From left to right: (a) destructive Pauli interference, labelled 'int$^-$'; (b) Weak exchange, labelled 'exc'; (c) constructive Pauli interference, labelled 'int$^+$'. The non-participating light quark is also indicated. The correspondence to the equivalent meson diagrams is clearly visible by comparison with figure 2.

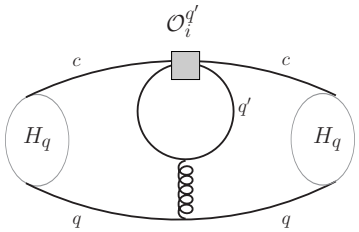

Figure 4. Diagrammatic representation of "eye contractions", which serve as a further contribution relevant in the four-quark operators [39, 82, 83]. In particular, the quark running in the loop q' need not match the valence quark in the hadron, q, leading to further non-valence contributions to the decay widths. Such contributions are numerically small [39, 83], but allow, for example, for a PI contribution to the D^0 lifetime.

in baryon decays, where these contributions are relevant. The effect of these contributions in semileptonic decays was first noticed by Voloshin in [84].

A further relevant topology, known as the "eye contraction" [39, 82, 83], is represented in figure 4. The eye contractions allow for contributions where the light valence quark in the hadron does not necessarily match the quark involved in the short-distance interaction.

The relevant operator basis varies depending on whether we are considering mesons or baryons, although, as can be seen by comparing figure 2 and figure 3, the different bases are

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related. Beginning with the mesons, the dimension-six operators are, following [28, 39, 85],[7]

$$
\begin{aligned}
O_1^q &= (\bar{c}_i\gamma_\mu\,(1-\gamma_5)\,q_i)\,(\bar{q}_j\gamma^\mu\,(1-\gamma_5)\,c_j)\,, & O_2^q &= (\bar{c}_i\,(1-\gamma_5)\,q_i)\,(\bar{q}_j\,(1+\gamma_5)\,c_j)\,,\\
T_1^q &= \left(\bar{c}_i\gamma_\mu\,(1-\gamma_5)\,t^a_{ij}q_j\right)(\bar{q}_k\gamma^\mu\,(1-\gamma_5)\,t^a_{kl}c_l)\,, & T_2^q &= \left(\bar{c}_i\,(1-\gamma_5)\,t^a_{ij}q_j\right)(\bar{q}_k\,(1+\gamma_5)\,t^a_{kl}c_l)\,,
\end{aligned}
\tag{2.26}
$$

where colour indices are denoted by i, j, the t^a are colour matrices, and $q = u, d, s$ are light-quark flavours. This basis is most suited for mesons, because within the vacuum insertion approximation (VIA) the matrix elements $\langle D|T_i^q|D\rangle$ vanish [85]. Note that these operators are defined with full QCD fields. To obtain a consistent expansion in $1/m_Q$, one uses instead the basis

$$
\begin{aligned}
\mathcal{O}_1^q &= \left(\bar{h}_{v,i}\gamma_\mu\,(1-\gamma_5)\,q_i\right)(\bar{q}_j\gamma^\mu\,(1-\gamma_5)\,h_{v,j})\,, & \mathcal{O}_2^q &= \left(\bar{h}_{v,i}\,(1-\gamma_5)\,q_i\right)(\bar{q}_j\,(1+\gamma_5)\,h_{v,j})\,,\\
\mathcal{T}_1^q &= \left(\bar{h}_{v,i}\gamma_\mu\,(1-\gamma_5)\,t^a_{ij}q_j\right)(\bar{q}_k\gamma^\mu\,(1-\gamma_5)\,t^a_{kl}h_{v,l})\,, & \mathcal{T}_2^q &= \left(\bar{h}_{v,i}\,(1-\gamma_5)\,t^a_{ij}q_j\right)(\bar{q}_k(1+\gamma_5)t^a_{kl}h_{v,l})\,,
\end{aligned}
\tag{2.27}
$$

where h_v is the heavy-quark field, and again the colour-octet contributions $\langle D|\mathcal{T}_i^q|D\rangle$ vanish in the VIA. This basis will be referred to as the Heavy Quark Effective Theory (HQET) basis.

The corresponding LO coefficients $c^q_{6,i}$ in (2.15) have been known for some time [15, 26, 27, 84, 85], while the NLO contributions were first reported in [28, 30, 31, 45]. We adapt the NLO results from [45], which were computed for b decays, to the charm sector. The explicit results in that reference correspond to the basis

$$
\begin{aligned}
\bar{\mathcal{O}}_1^q &= \left(\bar{h}_{v,i}\gamma_\mu\,(1-\gamma_5)\,h_i\right)(\bar{q}_j\gamma^\mu\,(1-\gamma_5)\,q_j)\,,\\
\bar{\mathcal{O}}_2^q &= \left(\bar{h}_{v,i}\gamma_\mu\,(1+\gamma_5)\,h_{v,i}\right)(\bar{q}_j\gamma^\mu\,(1-\gamma_5)\,q_j)\,,\\
\bar{\mathcal{O}}_3^q &= \left(\bar{h}_{v,i}\gamma_\mu\,(1-\gamma_5)\,t^a_{ij}h_{v,j}\right)\left(\bar{q}_i\gamma^\mu\,(1-\gamma_5)\,t^a_{ij}q_j\right)\,,\\
\bar{\mathcal{O}}_4^q &= \left(\bar{h}_{v,i}\gamma_\mu\,(1+\gamma_5)\,t^a_{ij}h_{v,j}\right)\left(\bar{q}_i\gamma^\mu\,(1-\gamma_5)\,t^a_{ij}q_j\right)\,,
\end{aligned}
\tag{2.28}
$$

with the operators denoted by bars, in order to avoid overlap with the notation in eq. (2.27). The two bases are related by the Fierz transformation

$$
\vec{\mathcal{O}} = F\vec{\mathcal{Q}}\,, \tag{2.29}
$$

where $\vec{\mathcal{O}} = (\bar{\mathcal{O}}_1^q, \bar{\mathcal{O}}_2^q, \bar{\mathcal{O}}_3^q, \bar{\mathcal{O}}_4^q)$ and $\vec{\mathcal{Q}} = (\mathcal{O}_1^q, \mathcal{O}_2^q, \mathcal{T}_1^q, \mathcal{T}_2^q)$, with the transformation matrix

$$
F = \begin{pmatrix}
\frac{1}{3} & 0 & 2 & 0\\
0 & -\frac{2}{3} & 0 & -4\\
\frac{4}{9} & 0 & -\frac{1}{3} & 0\\
0 & -\frac{8}{9} & 0 & \frac{2}{3}
\end{pmatrix}. \tag{2.30}
$$

The matching between the flavour nonsinglet dimension-six four quark operators defined in HQET $\vec{\mathcal{Q}}$ and the corresponding QCD operators $\vec{Q}$ is performed using the relation

$$
\vec{\mathcal{Q}}(\mu_0) = F^{-1}\widehat{C}^{-1}(\mu_h, \mu_0, m_c)F\vec{Q}(\mu_h)\,, \tag{2.31}
$$

[7] A useful dictionary relating this parametrization to other historical parametrizations of the four-quark matrix elements is given in appendix C of [86].

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with the matrix [45]

$$\widehat{C}\left(\mu_h, \mu_0, m_c\right) = \left(W^T\left(\mu_h, m_c\right)\right)^{-1} \hat{C}\left(m_c\right) \widetilde{W}^T\left(\mu_0, m_c\right) \,, \tag{2.32}$$

where $\widetilde{W}(\mu_0, m_c)$ and $W(\mu_h, m_c)$ denote the renormalization group evolution matrices within HQET and QCD, respectively, and $\hat{C}(m_c)$ is the matrix used to match between the two frameworks at the scale m_c. The scale $\mu_h \sim 1\,\text{GeV}$ denotes the low hadronic scale. If one remains within the HQET framework, the matrix $\widehat{C}(\mu_h, \mu_0, m_c)$ reduces to the usual HQET evolution matrix implementing the so-called hybrid renormalization (eg [85]). In general, this evolution to the lower scale μ_h will also bring penguin operators into play [27, 80]. There are two types of penguin contributions: those from the penguin operators themselves, and those arising from penguin-like diagrams. Both of these can be safely neglected, since on the one hand the penguin diagrams are neglible, while on the other hand the penguin operator contributions bring at most a few % corrections, which are not relevant compared to other uncertainties in the computation.

For our numerical evaluation, we use the fixed value $\mu_0 = 1.5\,\text{GeV}$, and we also set $m_c = 1.5\,\text{GeV}$ in the formulas in eqs. (2.31) and (2.32). For the case of charmed mesons, we use the explicit numerical results for the corresponding matrix elements of the four-quark operators evaluated within the framework of HQET sum rules [39, 83]. Therefore, in this case we have $\widehat{C}$ equal to the identity matrix. For the case of baryons, we have assumed that the matrix elements of the four-quark operators evaluated within the nonrelativistic constituent quark model (NRCQM) correspond to the matrix elements of the QCD operators renormalized at the low hadronic scale, which we set to $\mu_h = 1\,\text{GeV}$. Note that the above running and matching matrices are currently known only for the nonsinglet flavour operators, eg $\mathcal{O}_i^u - \mathcal{O}_i^d$, that do not mix under renormalization with flavour singlet operators involving penguin contractions. The matrix elements of flavour nonsinglet operators appear in the differences between the baryon lifetimes within a given $\text{SU}(3)_F$ multiplet. However, we have applied these results for evaluations of the absolute values of the decay widths, due to the expectation that the neglected missing contributions are small compared with other uncertainties in the matrix elements.

At the next order in the $1/m_c$ expansion, the dimension-seven operators in the meson basis are

$$\begin{aligned}
P_1^q &= m_q \left(\bar{c}_i \left(1-\gamma_5\right) q_i\right) \left(\bar{q}_j \left(1-\gamma_5\right) c_j\right) \,, \\
P_2^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu \left(1-\gamma_5\right) D^\rho q_i\right) \left(\bar{q}_j \gamma^\mu \left(1-\gamma_5\right) c_j\right) \,, \\
P_3^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \left(1-\gamma_5\right) D^\rho q_i\right) \left(\bar{q}_j \left(1+\gamma_5\right) c_j\right) \,, \\
S_1^q &= m_q \left(\bar{c}_i \left(1-\gamma_5\right) t^a_{ij} q_j\right) \left(\bar{q}_k \left(1-\gamma_5\right) t^a_{kl} c_l\right) \,, \\
S_2^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu \left(1-\gamma_5\right) t^a_{ij} D^\rho q_j\right) \left(\bar{q}_k \gamma^\mu \left(1-\gamma_5\right) t^a_{kl} c_l\right) \,, \\
S_3^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \left(1-\gamma_5\right) t^a_{ij} D^\rho q_j\right) \left(\bar{q}_k \left(1+\gamma_5\right) t^a_{kl} c_l\right) \,.
\end{aligned} \tag{2.33}$$

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Note that an earlier basis for the dimension-seven operators, used in [28], contained an additional operator $P_2^{q\,[28]} = m_q(\bar{c}_i(1+\gamma_5)q_i)(\bar{q}_j(1+\gamma_5)c_j)$, but this operator can be omitted, since it is related to P_1^q above by hermitian conjugation [39]. Again, within this basis the matrix elements $\langle D|S_i^q|D\rangle$ can be expected to vanish within the VIA. Likewise, for a consistent $1/m_Q$ expansion, one should re-express the operators in terms of heavy-quark fields. However, unlike at dimension-six, this time new operators emerge:

$$\begin{aligned}
\mathcal{P}_1^q &= m_q \left(\bar{h}_v^i (1-\gamma_5) q^i\right) \left(\bar{q}^j (1-\gamma_5) h_v^j\right) , \\
\mathcal{P}_2^q &= \left(\bar{h}_v^i \gamma_\mu (1-\gamma_5)\, iv\cdot D q_i\right) \left(\bar{q}_j \gamma^\mu (1-\gamma_5) h_v^j\right) , \\
\mathcal{P}_3^q &= \left(\bar{h}_v^i (1-\gamma_5)\, iv\cdot D q^i\right) \left(\bar{q}^j (1+\gamma_5) h_v^j\right) , \\
\mathcal{R}_1^q &= \left(\bar{h}_v^i \gamma^\mu (1-\gamma_5) q^i\right) \left(\bar{q}^j \gamma_\mu (1-\gamma_5) \not{D} h_v^j\right) , \\
\mathcal{R}_2^q &= \left(\bar{h}_v^i (1-\gamma_5) q^i\right) \left(\bar{q}^j (1-\gamma_5) \not{D} h_v^j\right) ,
\end{aligned} \tag{2.34}$$

supplemented with the colour-octet operators, $\mathcal{S}_i^q$ corresponding to $\mathcal{P}_i^q$, and $\mathcal{U}_{1,2}^q$ corresponding to $\mathcal{R}_{1,2}^q$:

$$\begin{aligned}
\mathcal{S}_1^q &= m_q \left(\bar{h}_v^i (1-\gamma_5) t^a_{ij} q_j\right) \left(\bar{q}^k (1-\gamma_5) t^a_{kl} h_v^l\right) , \\
\mathcal{S}_2^q &= \left(\bar{h}_v^i \gamma_\mu (1-\gamma_5)\, iv\cdot D t^a_{ij} q_j\right) \left(\bar{q}_k \gamma^\mu (1-\gamma_5) t^a_{kl} h_v^l\right) , \\
\mathcal{S}_3^q &= \left(\bar{h}_v^i (1-\gamma_5)\, iv\cdot D t^a_{ij} q_j\right) \left(\bar{q}^k (1+\gamma_5) t^a_{kl} h_v^l\right) , \\
\mathcal{U}_1^q &= \left(\bar{h}_v^i \gamma^\mu (1-\gamma_5) t^a_{ij} q_j\right) \left(\bar{q}^k \gamma_\mu (1-\gamma_5) \not{D} h_v^j t^a_{kl} h_v^l\right) , \\
\mathcal{U}_2^q &= \left(\bar{h}_v^i (1-\gamma_5) t^a_{ij} q_j\right) \left(\bar{q}^k (1-\gamma_5) \not{D} t^a_{kl} h_v^l\right) ,
\end{aligned} \tag{2.35}$$

Finally, for a complete dimension-seven HQE basis, one should include non-local operators, defined explicitly in eqs. (2.66)-(2.69) of [39] (see also section 3.6 in [53] and eqs. (30), (31) in [87]). However, as was shown in [53, 80, 87] (see also [39] and appendix C), in meson decay widths the effect of the non-local dimension-seven matrix elements can be re-absorbed into the dimension-six matrix elements, up to higher-order corrections in $1/m_c$ and α_s. For our purposes, therefore, we do not quote the definitions of the non-local elements.

For baryons, the equivalent dimension-six basis is

$$\begin{aligned}
O_1^q &= (\bar{c}_i \gamma_\mu (1-\gamma_5) q_i) (\bar{q}_j \gamma^\mu (1-\gamma_5) c_j) , & O_2^q &= (\bar{c}_i (1-\gamma_5) q_i) (\bar{q}_j (1+\gamma_5) c_j) , \\
\tilde{O}_1^q &= (\bar{c}_i \gamma_\mu (1-\gamma_5) q_j) (\bar{q}_j \gamma^\mu (1-\gamma_5) c_i) , & \tilde{O}_2^q &= (\bar{c}_i (1-\gamma_5) q_j) (\bar{q}_j (1+\gamma_5) c_i) ,
\end{aligned} \tag{2.36}$$

which is related to the meson basis, at the operator level, owing to the colour identity

$$t^a_{ij} t^a_{kl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_C}\delta_{ij}\delta_{kl} , \tag{2.37}$$

which implies

$$T_{1,2}^q = \frac{1}{2}\tilde{O}_{1,2}^q - \frac{1}{2N_C} O_{1,2}^q , \tag{2.38}$$

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and similarly for all colour-octet operators. The reason for choosing a different basis for baryons is that, in this case, the totally antisymmetric colour wavefunction imposes the relation between the matrix elements

$$\langle \mathcal{B}_c | \tilde{O}_i^q | \mathcal{B}_c \rangle = -\tilde{B} \langle \mathcal{B}_c | O_i^q | \mathcal{B}_c \rangle \,, \tag{2.39}$$

where $\mathcal{B}_c$ denotes a singly charmed baryon, and $\tilde{B} \simeq 1$, with equality in the valence quark approximation [85]. The dimension-seven basis for baryons is

$$\begin{aligned}
P_1^q &= m_q \left(\bar{c}_i \left(1-\gamma_5\right) q_i\right) \left(\bar{q}_j \left(1-\gamma_5\right) c_j\right) \,, \\
P_2^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu \left(1-\gamma_5\right) D^\rho q_i\right) \left(\bar{q}_j \gamma^\mu \left(1-\gamma_5\right) c_j\right) \,, \\
P_3^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \left(1-\gamma_5\right) D^\rho q_i\right) \left(\bar{q}_j \left(1+\gamma_5\right) c_j\right) \,, \\
\tilde{P}_1^q &= m_q \left(\bar{c}_i \left(1-\gamma_5\right) q_j\right) \left(\bar{q}_j \left(1-\gamma_5\right) c_i\right) \,, \\
\tilde{P}_2^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu \left(1-\gamma_5\right) D^\rho q_j\right) \left(\bar{q}_j \gamma^\mu \left(1-\gamma_5\right) c_i\right) \,, \\
\tilde{P}_3^q &= \frac{1}{m_Q} \left(\bar{c}_i \overleftarrow{D}_\rho \left(1-\gamma_5\right) D^\rho q_j\right) \left(\bar{q}_j \left(1+\gamma_5\right) c_i\right) \,,
\end{aligned} \tag{2.40}$$

which is again related to the meson basis (2.33) by the colour identity (2.38). As before, to obtain a consistent $1/m_c$ expansion one should use a basis with the heavy-quark field, equivalent to that in eqs. (2.27) and (2.34), supplemented by the non-local operators. However, there is no currently-available parametrization for the non-local baryon matrix elements. Furthermore, since the re-absorption of the non-local dimension-seven contributions, following the arguments of [39, 53, 80, 87], cannot be readily applied to baryons, we prefer to use QCD matrix elements throughout in the analysis of baryon lifetimes.

Having defined the operator basis, we are now in a position to quote expressions for the spectator contributions to decay width. At leading order, the expressions for mesons have been computed in [15, 26–28, 39, 40, 84, 85], and are provided in appendix B. The contributions to the lifetimes of specific charmed mesons are then obtained from the general expressions in eq. (B.12) by inserting the appropriate CKM factors according to figure 2, and evaluation of the appropriate matrix elements. In practice, several simplifications could be invoked. In the valence approximation, only those contributions in which the light quark q' within the operator coincides with the meson's valence quark q are included. Within this approximation, the contributions to the decay widths of D mesons from the four-quark

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decay / H_c	CE NL $c \to s\bar{d}u$	SCS-s NL $c \to s\bar{s}u$	SCS-d NL $c \to d\bar{d}u$	DCS NL $c \to d\bar{s}u$	CE SL $c \to s\bar{l}\nu_l$	CS SL $c \to d\bar{l}\nu_l$
$D^0(\bar{u}c)$	$\tilde{\Gamma}_{\rm WE}$	$\tilde{\Gamma}_{\rm WE}$	$\tilde{\Gamma}_{\rm WE}$	$\tilde{\Gamma}_{\rm WE}$	-	-
$D^+(\bar{d}c)$	$\tilde{\Gamma}_{\rm PI}$	-	$\tilde{\Gamma}_{\rm PI} + \tilde{\Gamma}_{\rm WA}$	$\tilde{\Gamma}_{\rm WA}$	-	$\tilde{\Gamma}^{\rm SL}_{\rm WA}$
$D_s^+(\bar{s}c)$	$\tilde{\Gamma}_{\rm WA}$	$\tilde{\Gamma}_{\rm PI} + \tilde{\Gamma}_{\rm WA}$	-	$\tilde{\Gamma}_{\rm PI}$	$\tilde{\Gamma}^{\rm SL}_{\rm WA}$	-
$\Lambda_c^+(udc)$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^-}$	$\tilde{\Gamma}_{\rm int^-}$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^-} + \tilde{\Gamma}_{\rm int^+}$	$\tilde{\Gamma}_{\rm int^-} + \tilde{\Gamma}_{\rm int^+}$	-	$\tilde{\Gamma}^{\rm SL}_{\rm int^+}$
$\Xi_c^+(usc)$	$\tilde{\Gamma}_{\rm int^-} + \tilde{\Gamma}_{\rm int^+}$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^-} + \tilde{\Gamma}_{\rm int^+}$	$\tilde{\Gamma}_{\rm int^-}$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^-}$	$\tilde{\Gamma}^{\rm SL}_{\rm int^+}$	-
$\Xi_c^0(dsc)$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^+}$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^+}$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^+}$	$\tilde{\Gamma}_{\rm exc} + \Gamma_{\rm int^+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int^+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int^+}$
$\Omega_c^0(ssc)$	$\tilde{\Gamma}_{\rm int^+}$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int^+}$	-	$\tilde{\Gamma}_{\rm exc}$	$\tilde{\Gamma}^{\rm SL}_{\rm int^+}$	-

Table 3. Synopsis of various contributions of the four-quark operators to the lifetimes of singly charmed hadrons in the valence approximation, whereby the hierarchies stemming from various CKM scalings are denoted by CE (Cabibbo enhanced), SCS (singly Cabibbo suppressed), DCS (doubly Cabibbo suppressed). As before, NL and SL respectively denote nonleptonic and semileptonic contributions. The decay processes in the first row schematically denote the kinds of weak transitions appearing in the diagrams in figure 2, and are to be rearranged according to the quark involved in the decay.

operators at dimension-six are

$$\begin{aligned}
\tilde{\Gamma}_{6,D^0} &= |V_{cs}|^2|V_{ud}|^2\langle\widehat{\Gamma}^u_{6,\rm WE}(x_s,0)\rangle_{D^0} + |V_{cs}|^2|V_{us}|^2\langle\widehat{\Gamma}^u_{6,\rm WE}(x_s,x_s)\rangle_{D_0} \\
&\quad + |V_{cd}|^2|V_{ud}|^2\langle\widehat{\Gamma}^u_{6,\rm WE}(0,0)\rangle_{D^0}\,, \\
\tilde{\Gamma}_{6,D^+} &= |V_{cs}|^2|V_{ud}|^2\langle\widehat{\Gamma}^d_{6,\rm PI}(x_s,0)\rangle_{D_+} + |V_{cd}|^2|V_{ud}|^2\langle\widehat{\Gamma}^d_{6,\rm WA}(0,0)\rangle_{D^+} \\
&\quad + |V_{cd}|^2|V_{ud}|^2\langle\widehat{\Gamma}^d_{6,\rm PI}(0,0)\rangle_{D^+} + |V_{cd}|^2\sum_{\ell=e,\mu}\langle\widehat{\Gamma}^{d,\rm SL}_{6,\rm WA}(x_\ell,0)\rangle_{D^+}\,, \\
\tilde{\Gamma}_{6,D_s} &= |V_{cs}|^2|V_{ud}|^2\langle\widehat{\Gamma}^s_{6,\rm WA}(0,0)\rangle_{D_s} + |V_{cs}|^2|V_{us}|^2\langle\widehat{\Gamma}^s_{6,\rm WA}(x_s,0)\rangle_{D_s} \\
&\quad + |V_{cs}|^2|V_{us}|^2\langle\widehat{\Gamma}^s_{6,\rm PI}(x_s,0)\rangle_{D_s} + |V_{cs}|^2\sum_{\ell=e,\mu}\langle\widehat{\Gamma}^{s,\rm SL}_{6,\rm WA}(x_\ell,0)\rangle_{D_s}\,,
\end{aligned} \tag{2.41}$$

where $\langle\widehat{\Gamma}^q(x_1,x_2)\rangle_{M_q}$ is a shorthand for $\langle M_q|\widehat{\Gamma}^q(x_1,x_2)|M_q\rangle$, and we have neglected the doubly Cabibbo-suppressed terms. The contributions for dimension-seven, using the analytic expressions in eq. (B.13), are exactly analogous. To include non-valence contributions, we insert terms arising from $\langle\widehat{\Gamma}^{q'}(x_1,x_2)\rangle_{M_q}$, where $q' \neq q$, which would for example generate WA and PI contributions to $\tilde{\Gamma}_{6,D^0}$. The resulting modification to (2.41) is systematic, since, by including non-valence terms, all possible topologies contribute to any given meson [39, 83]. The expressions above reflect the clear hierarchy of contributions, presented in table 3: for example, the D^+ width receives a large PI contribution and a small, Cabibbo-suppressed, semileptonic contribution, whereas the D_s receives a smaller, Cabibbo-suppressed PI contribution alongside the semileptonic and WA contributions.

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Considering only the valence contributions, and neglecting doubly Cabibbo-suppressed terms, the analogous expressions to (2.41) for baryons are

$$
\begin{aligned}
\tilde{\Gamma}_{6,\Lambda_c^+} =& |V_{cs}|^2|V_{ud}|^2\left(\langle\widehat{\Gamma}^u_{6,\text{int}^-}(x_s,0)\rangle_{\Lambda_c^+}+\langle\widehat{\Gamma}^d_{6,\text{exc}}(x_s,0)\rangle_{\Lambda_c^+}\right)+|V_{cs}|^2|V_{us}|^2\langle\widehat{\Gamma}^u_{6,\text{int}^-}(x_s,x_s)\rangle_{\Lambda_c^+}\\
&+|V_{cd}|^2|V_{ud}|^2\left(\langle\widehat{\Gamma}^u_{6,\text{int}^-}(0,0)\rangle_{\Lambda_c^+}+\langle\widehat{\Gamma}^d_{6,\text{int}^+}(0,0)\rangle_{\Lambda_c^+}+\langle\widehat{\Gamma}^d_{6,\text{exc}}(0,0)\rangle_{\Lambda_c^+}\right)\\
&+|V_{cd}|^2\sum_{\ell=e,\mu}\langle\widehat{\Gamma}^{d,\text{SL}}_{6,\text{int}^+}(x_\ell,0)\rangle_{\Lambda_c^+},\\
\tilde{\Gamma}_{6,\Xi_c^+} =& |V_{cs}|^2|V_{ud}|^2\left(\langle\widehat{\Gamma}^u_{6,\text{int}^-}(x_s,0)\rangle_{\Xi_c^+}+\langle\widehat{\Gamma}^s_{6,\text{int}^+}(0,0)\rangle_{\Xi_c^+}\right)+|V_{cd}|^2|V_{ud}|^2\langle\widehat{\Gamma}^u_{6,\text{int}^-}(0,0)\rangle_{\Xi_c^+}\\
&+|V_{cs}|^2|V_{us}|^2\left(\langle\widehat{\Gamma}^s_{6,\text{exc}}(x_s,0)\rangle_{\Xi_c^+}+\langle\widehat{\Gamma}^s_{6,\text{int}^+}(x_s,0)\rangle_{\Xi_c^+}+\langle\widehat{\Gamma}^u_{6,\text{int}^-}(x_s,x_s)\rangle_{\Xi_c^+}\right)\\
&+|V_{cs}|^2\sum_{\ell=e,\mu}\langle\widehat{\Gamma}^{s,\text{SL}}_{6,\text{int}^+}(x_\ell,0)\rangle_{\Xi_c^+},\\
\tilde{\Gamma}_{6,\Xi_c^0} =& |V_{cs}|^2|V_{ud}|^2\left(\langle\widehat{\Gamma}^s_{6,\text{int}^+}(0,0)\rangle_{\Xi_c^0}+\langle\widehat{\Gamma}^d_{6,\text{exc}}(x_s,0)\rangle_{\Xi_c^0}\right)\\
&+|V_{cd}|^2|V_{ud}|^2\left(\langle\widehat{\Gamma}^d_{6,\text{int}^+}(0,0)\rangle_{\Xi_c^0}+\langle\widehat{\Gamma}^d_{6,\text{exc}}(0,0)\rangle_{\Xi_c^0}\right)\\
&+|V_{cs}|^2|V_{us}|^2\left(\langle\widehat{\Gamma}^s_{6,\text{int}^+}(x_s,0)\rangle_{\Xi_c^0}+\langle\widehat{\Gamma}^s_{6,\text{exc}}(x_s,0)\rangle_{\Xi_c^0}\right)\\
&+|V_{cs}|^2\sum_{\ell=e,\mu}\langle\widehat{\Gamma}^{s,\text{SL}}_{6,\text{int}^+}(x_\ell,0)\rangle_{\Xi_c^0}+|V_{cd}|^2\sum_{\ell=e,\mu}\langle\widehat{\Gamma}^{d,\text{SL}}_{6,\text{int}^+}(x_\ell,0)\rangle_{\Xi_c^0},\\
\tilde{\Gamma}_{6,\Omega_c^0} =& |V_{cs}|^2|V_{ud}|^2\langle\widehat{\Gamma}^s_{6,\text{int}^+}(0,0)\rangle_{\Omega_c^0}+|V_{cs}|^2|V_{us}|^2\left(\langle\widehat{\Gamma}^s_{6,\text{int}^+}(x_s,0)\rangle_{\Omega_c^0}+\langle\widehat{\Gamma}^s_{6,\text{exc}}(x_s,0)\rangle_{\Omega_c^0}\right)\\
&+|V_{cs}|^2\sum_{\ell=e,\mu}\langle\widehat{\Gamma}^{s,\text{SL}}_{6,\text{int}^+}(x_\ell,0)\rangle_{\Omega_c^0},
\end{aligned}
\tag{2.42}
$$

and likewise for dimension-seven, where analytic forms at tree level for the various contributions are given in eqs. (B.14) and (B.15). Again, one could in principle insert non-valence contributions, resulting in longer expressions. Unlike in mesons, however, no reliable estimate of the matrix elements for non-valence contributions exist, and we therefore do not include these contributions in our analysis of the baryon lifetimes. Given that non-valence contributions in mesons are small, albeit with significant uncertainties [83], this is justified to the present degree of accuracy. As is the case with mesons, the contributions exhibit a clear hierarchy, also presented in table 3, although for baryons there are more relevant contributions, owing to the additional spectator quark, than there are for mesons. It is apparent from (2.42) that four-quark operator contributions in semileptonic decays, which always accompany the nonleptonic Γ_{int^+} contributions, are pronounced in all singly charmed baryon decays, except in Λ_c^+. In particular, the semileptonic decay of Ω_c^0 is expected to get a significant four-quark contribution due to the interference of two s quarks [84].[8]

A summary of all the contributions considered in this paper, along with useful references, is available in table 4.

2.4 Charm quark mass

The major input parameter influencing the decay rate is the mass of the charm quark itself, m_c, since in the leading term it enters the expression for the decay width as m_c^5. Consequently, it is important to define this mass precisely.

[8] Although we do not include them in our analysis, penguin contributions in baryons accompany Γ_{int^-}, meaning that one could only observe such contributions in eq. (2.42) in Λ_c^+ and Ξ_c^+.

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		α_s^0 (LO)	α_s^1 (NLO)	α_s^2 (NNLO)	α_s^3 (N^{3}LO)
c_3	SL		[67]	([68–72])	([73, 74])
	NL		[62–65]	([66])	
$c_{\pi,G}$	SL	[17, 18, 58, 59]	[75–77]		
	NL				
c_ρ	SL	[60, 61]	([78, 79])		
	NL	[39, 41–43]			
$c_{6,i}^q$	SL	[15, 26, 27, 84, 85]	[28, 30, 31, 45]		
	NL				
$c_{7,i}^q$	SL	[28, 32, 39, 40]			
	NL				

Table 4. Summary of main references for contributions to the heavy hadron decay rates, arranged by order in the $1/m_c$ and α_s expansions, with the upper three rows referring to non-spectator contributions (ie two-quark matrix elements), and the lower two rows referring to spectator contributions (ie four-quark matrix elements). The notation in the leftmost column corresponds to eq. (2.15). If a contribution is not available, the cell is left empty (the exception is the leading Γ_3 term, which in effect has been known since the 1950s). Contributions that have not been included in our computations, but may be useful for future studies, are indicated in brackets (). More details, and other key references, are also provided in the main text. Some exploratory work on higher-order non-spectator contributions in semileptonic channels is available in, for example, [34–37, 88], but since little is known about matrix elements at this order, we do not include such contributions in our analysis.

Typically, the starting point for all analytic expressions derived earlier is to treat m_c as the pole mass, which is defined as a pole in the complex p^2 plane of the heavy-quark propagator and can be perturbatively related to other renormalized masses order-by-order. The pole mass is suitable for processes with nearly on-shell heavy quarks, such as heavy hadron decays. However, the pole mass of a quark is not a physical parameter. Moreover, any perturbative definition of the pole mass suffers from a divergence due to infrared (IR) renormalons, which impose a minimal uncertainty of $\mathcal{O}(\Lambda_{\rm QCD})$ (see, for example, [89, 90] and references therein for a detailed discussion).[9] This can be seen explicitly in the relation between the $\overline{\rm MS}$ and pole masses, which to third order reads [91, 92]

$$\begin{aligned} m_c^{\rm pole} &= \overline{m}_c\left(\overline{m}_c\right)\left[1+\frac{4}{3}\frac{\alpha_s\left(\overline{m}_c\right)}{\pi}+10.3\left(\frac{\alpha_s\left(\overline{m}_c\right)}{\pi}\right)^2+116.5\left(\frac{\alpha_s\left(\overline{m}_c\right)}{\pi}\right)^3+\ldots\right] \\ &= \overline{m}_c(\overline{m}_c)(1+0.16+0.15+0.21+\ldots)\,, \end{aligned} \tag{2.43}$$

where we have used $\alpha_s(\overline{m}_c)=0.38$. Assuming that the value of $\overline{m}_c$ can be extracted without ambiguities from lattice QCD, as has been done in [93–97], this demonstrates an instability in the determination of $m_c^{\rm pole}$ starting already from two-loop corrections.[10]

[9] The authors of [55] were the first to point out this important drawback in the concept of a pole mass, which becomes apparent as soon as one addresses leading non-perturbative corrections to order $1/m_Q$.

[10] As expected, heavier quark pole masses are less influenced by the problem of the non-convergence of the α_s series. Thus, the series for the b-quark pole mass starts to diverge from fourth order, and for the top quark mass at the eighth order in the expansion [90] (note that there are some numerical inconsistencies in tables 2 and 3 therein, confirmed by the author in a private communication).

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To circumvent this issue, various alternative renormalon-free mass definitions have been proposed. A common alternative definition for the heavy quark mass is the above-mentioned $\overline{\rm MS}$ mass ($m^{\overline{\rm MS}} \equiv \overline{m}$), which is the short-distance, perturbatively-defined mass appearing as a parameter in the Lagrangian. Being the running mass defined at the scale μ, it includes only effects of momenta higher than μ [90]. The scale at which the $\overline{\rm MS}$ mass is evaluated is usually taken to be the mass itself, $\overline{m}_c(\mu_m) = \overline{m}_c(\overline{m}_c)$, the default value which is used throughout this paper. Although $\overline{\rm MS}$ is a good scheme for quantities that involve energies much larger than m, it is not an appropriate choice for physical processes such as heavy quark (practically on-shell) decays, since it is a strictly defined short-distance object and is therefore quite far away from the on-shell pole mass by $O(\overline{m}\alpha_s)$. On the other hand, it was shown in [98] that the characteristic normalization scale for the mass in inclusive heavy quark decays is $m_Q/5$. Whereas, for the b-quark decay, this becomes $m_b/5 \sim 1\,\text{GeV}$, for inclusive c-quark decays, the equivalent scale $m_c/5 \sim 0.3\,\text{GeV}$ is nonperturbative.

There are several other renormalon-free short-distance masses, $m_c^X(\mu_f)$ (also called low-scale short-distance masses, since $\mu_f \ll m$), each constructed in a way to be more appropriate for a particular process. Here, X labels the given scheme. All of them use a perturbative relation to the $\overline{\rm MS}$ mass and are defined such that the leading IR renormalon divergence is subtracted by construction [90]:

$$\begin{aligned} m_c^X(\mu_f) &= m_c^{\rm pole} - \delta m_c^X(\mu_f) \\ &= \overline{m}_c(\overline{m}_c) + \overline{m}_c(\overline{m}_c) \sum_{n=1}^{\infty} \left[c_n(\mu, \overline{m}_c(\overline{m}_c)) - \frac{\mu_f}{\overline{m}_c(\overline{m}_c)} s_n^X(\mu/\mu_f) \right] \alpha_s^n(\mu) \,, \end{aligned} \tag{2.44}$$

where both coefficients diverge as $c_n, s_n^X \sim (-2\beta_0)^n n! n^a$; β_0 is the leading coefficient of the QCD β-function. The s_N^X are constructed so that the leading IR renormalon divergence cancels in the bracket. This introduces a new, arbitrary, mass scale μ_f in the range $\Lambda_{\rm QCD} \ll \mu_f \ll m$. To have a perturbative expansion the scale μ_f has to be somewhat large, but on the other hand, it must also be sufficiently small so as not to run into the same problem faced by the $\overline{\rm MS}$ mass in the description of the heavy hadron decays, discussed above. In practice, we should construct renormalon-free masses which are numerically close to the pole mass. Since the difference between the pole mass and an arbitrary renormalon-free mass is $\delta m^X \sim \mu_f \alpha_s(\mu_f)$, the arbitrary parameter μ_f is typically chosen to minimize this relation and to ensure the convergence of the expansion (2.44).[11]

Theoretically, the most appropriate mass in semileptonic heavy hadron decays is the kinetic mass [52, 98], defined from the relation between the heavy quark and heavy meson masses

$$m_Q(\mu) = \overline{M}_H - \overline{\Lambda}(\mu) - \frac{\mu_\pi^2(\mu)}{2m_Q(\mu)} + \ldots \tag{2.45}$$

where $\overline{M}_H = (M_H + 3M_{H^*})/4$, the spin-averaged mass of H and H^* mesons, is introduced to cancel μ_G dependence that would otherwise enter (2.45). The short-distance, renormalon-

[11] For the $\overline{\rm MS}$ mass, $\mu_f = \overline{m}$.

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free, mass $m_c^{\rm kin}$ is then defined by perturbative loop-momentum calculations below the scale μ_f:

$$m_c^{\rm kin}(\mu_f) = m_c^{\rm pole} - \overline{\Lambda}(\mu_f)_{\rm pert} - \frac{\mu_\pi^2(\mu_f)_{\rm pert}}{2m_c^{\rm kin}(\mu_f)} + \dots \,, \tag{2.46}$$

and in such a way the leading IR renormalon of the pole mass is subtracted order-by-order in α_s and μ_f/m_Q. The kinetic mass scheme was defined for the description of the inclusive b-quark decays and therefore, again, it might not be fully appropriate for the c-quark decays, although there is a good convergence of the perturbative series $m^{\rm kin}(\overline{m})$ for both heavy quark masses [99, 100].

In addition, we will consider the MSR mass scheme [101, 102], which uses μ_f to interpolate between the $\overline{\rm MS}$ mass ($\mu_f = \overline{m}$) and the pole mass ($\mu_f = 0$), and avoids the drawback (see discussion in [102], also in [90]) of the Renormalon-Subtracted (RS) mass scheme, the simplest renormalon-free mass scheme [103]. There are two versions of the MSR mass, the "natural" and "practical", as defined in [102], which differ conceptually, but are numerically close; we will use the practical definition. Its simplicity is in the chosen scales, such that $\mu = \mu_f = \overline{m}$, and therefore the subtraction coefficients s_n^X in (2.44) are simply the same coefficients of the pole-$\overline{\rm MS}$ mass conversion,

$$s_n^{\rm MSR}{}_{|\,\mu=\mu_f} = c_n\left(\overline{m}_c, \overline{m}_c\left(\overline{m}_c\right)\right) . \tag{2.47}$$

2.4.1 Application of mass schemes in inclusive decays

Since the analytic results of section 2.3 are expressed initially in terms of the pole mass, they must then be rearranged to match the new definition. The mass in the new scheme is then treated as an input parameter, determined as precisely as possible. To leading order in α_s, this has little consequence beyond changing the value of the mass, but when including NLO contributions, one must also take into account the α_s relation between $m_c^{\rm pole}$ and m_c^X for a given choice of scheme. Specifically, supposing that a mass scheme at NLO can be related to the pole scheme as

$$m_c^{\rm pole} = m_c^X \left(1 + \frac{\alpha_s}{\pi} a_X^{(1)}\right) , \tag{2.48}$$

then the leading dimension-three term can be rearranged as

$$\Gamma_3^X = \frac{G_F^2}{192\pi^3}\left(m_c^X\right)^5 \left(c_3^{(0)} + \left(c_3^{(1)} + 5\, a_X^{(1)}\right)\frac{\alpha_s}{\pi}\right) , \tag{2.49}$$

where X denotes the scheme of interest, and similar relations hold for all other terms in the decay width.

We now spell out the details of our usage of different mass schemes for the charm quark mass. For definiteness, we consider the dimension-three contribution with the Wilson coefficient c_3 evaluated to the first order in α_s.

1. *Pole scheme.* For the presentation of the results in the pole scheme for the charm quark mass we adopt the one-loop result for the pole mass

$$m_c^{\rm pole} = 1.49\,{\rm GeV} . \tag{2.50}$$

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2. $\overline{\text{MS}}$ *scheme.* The expression,

$$\Gamma_3^{\text{pole}} = \frac{G_F^2}{192\pi^3} m_{c\,\text{pole}}^5 \left(c_3^{(0)} + c_3^{(1)} \alpha_s(\mu) + \ldots\right), \tag{2.51}$$

is translated to the $\overline{\text{MS}}$ scheme using the relation between the masses to the same order in α_s,

$$m_c^{\text{pole}} = \overline{m}_c(\mu_m) \left[1 + \frac{\alpha_s(\mu_m)}{\pi} \left(\frac{4}{3} + \log \frac{\mu_m^2}{\overline{m}_c(\mu_m)^2}\right) + \ldots\right]. \tag{2.52}$$

Our strategy is to set $\mu_m = \overline{m}_c$ as already discussed at the beginning of this section, and to expand α_s about $\mu = \overline{m}_c$. Expanding the fifth power of the mass to first order in α_s we have

$$(m_c^{\text{pole}})^5 = \overline{m}_c(\mu_m)^5 \left[1 + 5\,\frac{\alpha_s(\mu)}{\pi}\left(\frac{4}{3}\right)\right], \tag{2.53}$$

resulting in

$$\Gamma_3^{\overline{\text{MS}}} = \frac{G_F^2}{192\pi^3} \overline{m}_c^5 \left[c_3^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(5 \cdot \frac{4}{3} + c_3^{(1)}\right) + \ldots\right]. \tag{2.54}$$

For any individual contribution c_i corresponding to a given order in m_c^{-1}, evaluated to a specific order in α_s, we use the relation (2.52) to this same order of α_s. Hence, for the coefficients c_i known only to leading order in α_s, we simply perform the replacement $m_c^{\text{pole}} \to \overline{m}_c(\overline{m}_c)$. The numerical value of the $\overline{\text{MS}}$ mass is [104, 105]

$$\overline{m}_c(\overline{m}_c) = 1.28\,\text{GeV}\,. \tag{2.55}$$

3. *Kinetic scheme.* Expressing the results in terms of kinetic scheme proceeds in analogy to the above case of $\overline{\text{MS}}$. Pole mass is expressed in terms of the kinetic mass with the $\mathcal{O}(\alpha_s)$ relation

$$m_c^{\text{pole}} = m_c^{\text{kin}}(\mu_f) \left[1 + \frac{\alpha_s(\mu)}{\pi}\left(\frac{16}{9}\frac{\mu_f}{m_c^{\text{kin}}} + \frac{2}{3}\frac{\mu_f^2}{m_c^{\text{kin}\,2}}\right)\right]. \tag{2.56}$$

The relation between the kinetic and $\overline{\text{MS}}$ mass is known to three loops [99, 100]. We evaluate the numerical three-loop value for m_c^{kin} at $\mu_f = 0.5\,\text{GeV}$ using `RunDec` [106, 107],

$$m_c^{\text{kin}} = 1.40\,\text{GeV}\,, \tag{2.57}$$

with the input $\overline{m}_c(\overline{m}_c) = 1.28\,\text{GeV}$, see table 5. The scale $\mu_f = 0.5\,\text{GeV}$ is chosen to be as close to the expected value of $\mu_f = m_c/5$, without going below the scale Λ_{QCD} [98].

4. *MSR scheme.* For the MSR scheme, we have [102]

$$m_c^{\text{pole}} = m_c^{\text{MSR}}(\mu_f)\left(1 + \frac{4}{3}\frac{\alpha_s(\mu)}{\pi}\frac{\mu_f}{m_c^{\text{MSR}}(\mu_f)} + \ldots\right). \tag{2.58}$$

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$\overline{m}_c(\overline{m}_c) = 1.28\,\text{GeV}$	1-loop	2-loop	3-loop	4-loop
m_c^{pole}	1.49	1.68	1.95	2.43
m_c^{kin}	1.36	1.39	1.40	-
m_c^{MSR}	1.33	1.35	1.36	1.36

Table 5. Illustration of convergences of the charm quark masses for different mass schemes, expressed in units of GeV. In all cases the masses are expanded around the $\overline{\text{MS}}$ mass $\overline{m}_c(\overline{m}_c) = 1.28\,\text{GeV}$ [104, 105]. The scale μ_f, defined in (2.44), is $\mu_f = 0.5\,\text{GeV}$ for the kinetic mass, and $\mu_f = 1\,\text{GeV}$ for the MSR mass, with the values taken from [90]. The numerical values for m_c^{pole} and m_c^{kin} are evaluated using the Mathematica package `RunDec` [106, 107].

For the numerical value of the MSR mass we use the four-loop result [90]

$$m_c^{\text{MSR}} = 1.36\,\text{GeV}\,. \tag{2.59}$$

at the scale $\mu_f = 1\,\text{GeV}$.

In table 5 we illustrate the behaviour of higher-order α_s contributions for the charm mass schemes used in the paper.[12]

For the light masses, which enter phase space mass corrections as $(m_q/m_c)^n$, we will use their $\overline{\text{MS}}$ masses, since these corrections are relatively small and change of mass schemes there bring numerically no difference to the results. We neglect the m_u, m_d masses, and for the strange quark mass we use $\overline{m}_s(\mu = 1.5\,\text{GeV}) = 0.1\,\text{GeV}$.

Finally, it is important to emphasise that various renormalon-free mass schemes yield different, renormalon-free definitions of the $\overline{\Lambda}$ parameter that appear in the HQET expression of the heavy hadron masses (see also eq. (3.1)):

$$\begin{aligned} M_H &= m_c^{\text{pole}} + \overline{\Lambda} + \cdots \\ &= \left[m_c^{\text{pole}} - \delta m_c^X(\mu_f)\right] + \left[\overline{\Lambda} + \delta m_c^X(\mu_f)\right] + \cdots \\ &= m_c^X(\mu_f) + \overline{\Lambda}^X(\mu_f) + \cdots \end{aligned} \tag{2.60}$$

which then becomes a scale-dependent quantity. In our calculation, $\overline{\Lambda}$ parametrizes meson matrix elements of the four-quark operators in dimension-seven contributions, cf. section 4.1. Since the estimation of these contributions is anyhow plagued by large uncertainties, we refrain from using different values of $\overline{\Lambda}^X$ when applying different mass schemes, but instead associate the $\overline{\Lambda}$ parameter with a large uncertainty, made explicit below in eq. (4.6).

[12] Another, physically-motivated, mass definition is the $1S$ scheme, where the mass of the c (b) quark is extracted from the J/ψ (Υ) resonance [108–110]. This has, however, received criticisms in its applicability to heavy-light systems due to an inconsistent α_s expansion [111, 112], and also a large non-perturbative correction from the gluon condensate, calculated in [110] for the m_b^Υ mass, which could be even more significant in the charm sector. Therefore, the $1S$ mass should be seen as unsuitable for such D (or B) decays, despite its use in recent papers on charm lifetimes [39]. However, it could still be relevant to inclusive decays of the B_c meson, which is more 'quarkonium-like' [113].

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2.4.2 Effect of different mass schemes on the α_s convergence of the results

We also wish to comment on the α_s series, which is (partially) known for semileptonic decays up to N^{3}LO in the leading Γ_3 term [73]. Numerically, one finds in the pole scheme[13]

$$\frac{c_3^{\text{pole}}(X_c \to X_d e\nu_e)}{|V_{cd}|^2\Gamma_0} = \begin{cases} 1 - 0.25 - 0.26 - 0.37\,, & m_c = 1.68\,\text{GeV} \\ 1 - 0.27 - 0.28 - 0.43\,, & m_c = 1.49\,\text{GeV} \end{cases}, \tag{2.61}$$

where the respective corrections on the right-hand side are the first-, second-, and third-order $\alpha_s(m_c)$ contributions respectively. It can be seen that the series is divergent, with the divergent behaviour already emerging at second order. This again reflects the unsuitability of the pole mass, and shows that care must be taken to define an appropriate scheme. Likewise, in the $\overline{\text{MS}}$ scheme (at $\overline{m}_c(\overline{m}_c) = 1.28\,\text{GeV}$), we obtain

$$\frac{c_3^{\overline{\text{MS}}}(X_c \to X_d e\nu_e)}{|V_{cd}|^2\Gamma_0} = 1 + 0.52 + 0.44 + 0.45\,, \tag{2.62}$$

which also shows signs of divergence, and represents only a mild improvement on the pole scheme result. Note that the definition of Γ_0 here is adapted to the $\overline{\text{MS}}$ scheme.

As a further example, in the kinetic scheme, the behaviour of the perturbative series is heavily dependent on the choice of the scale μ_f. For illustration, we present the behaviour of the series for values of $\mu_f = (0.3, 0.5, 1)\,\text{GeV}$, with α_s evaluated at the scale m_c^{kin} (with Γ_0 adapted to the kinetic scheme):

$$\frac{c_3^{\text{kin}}(X_c \to X_d e\nu_e)}{|V_{cd}|^2\Gamma_0} = \begin{cases} 1 - 0.06 + 0.00 + 0.06\,, & \mu_f = 0.3\,\text{GeV} \\ 1 + 0.13 + 0.19 + 0.24\,, & \mu_f = 0.5\,\text{GeV} \\ 1 + 1.08 + 1.26 + 1.30\,, & \mu_f = 1\,\text{GeV} \end{cases}, \tag{2.63}$$

where the perturbative nature of the series is particularly poor for larger values of μ_f. On the other hand, the excellent behaviour of the α_s series for $\mu_f = 0.3\,\text{GeV}$ is artificial, since this does not reflect the limiting value of $\mu_f \gg \Lambda_{\text{QCD}}$ [114].[14] However, merely improving the perturbative behaviour of c_3 is not sufficient to motivate a mass scheme, as the entire HQE should be considered: to some extent, the renormalon divergence in c_3 motivates the presence of the non-perturbative contributions from μ_π^2, μ_G^2, ρ_D^3 etc [55]. One could expect similarly divergent behaviour in all the coefficients that have yet to be computed, such as $c_{G,\rho}$ and the coefficients of four-quark operators. It is therefore important to consider the series as a whole, rather than the behaviour of a single coefficient in the HQE.[15]

[13] To reproduce these results, one needs to set $n_b(N_H) = 1$, $n_c(N_C) = 0$, and $n_l(N_L) = 3$ in [73] ([71]), as well as $\delta = 1$. Since the results in [73] are presented as a series expansion in $\delta = 1 - m_c/m_b$, the value of the α_s^3 coefficient is not exact, although the behaviour of the series suggests that higher-order corrections are of order no more than a few %.

[14] It should be noted that the value of c_3 in the kinetic scheme is quite stable at N^{3}LO, with little dependence on the value of the mass, and so on the value of μ_f. However, this is likely to be an accident, and cannot be expected to hold at higher orders in the α_s series.

[15] The point can be illustrated by considering the behaviour of the dimension-eight contribution: since the leading m_c dependence cancels here, owing to $\Gamma_8 = \Gamma_0 c_8 O_8/m_c^5 \sim m_c^5 c_8 O_8/m_c^5 = c_8 O_8$, every mass scheme is equivalent in this contribution, and any divergent behaviour therefore cannot be addressed separately by a given mass scheme.

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In our work, we circumvent these problems by neglecting the available higher-order contributions in the semileptonic c_3 coefficient, preferring to treat the α_s expansion as asymptotic and keeping only the NLO contribution.

3 Matrix elements for the non-spectator parameters

The three inputs relevant to the non-spectator series are the kinetic, chromomagnetic and Darwin parameters μ_π^2, μ_G^2, and ρ_D^3, arising from the matrix elements defined in eq. (2.19). The first two can be defined in terms of the heavy-quark expansion of the hadron mass [115–117],

$$m_H = m_c + \bar{\Lambda} + \frac{\mu_\pi^2(H)}{2m_c} - \frac{\mu_G^2(H)}{2m_c} + \mathcal{O}\left(\frac{1}{m_c^2}\right) . \tag{3.1}$$

where $\bar{\Lambda} \sim 0.5\,\text{GeV}$ for charmed hadrons, and all parameters in the expansion are formally independent of the heavy quark mass.

The ρ_D^3 parameter enters the series (3.1) at the next order in the $1/m_c$ expansion. However, it does so alongside other, non-local, parameters, and cannot be disentangled from them. As discussed in section 3.3, we will instead extract ρ_D^3 by applying the equation of motion, and so do not give the explicit dependence of m_H on ρ_D^3 in (3.1).

3.1 Chromomagnetic parameter $\mu_G^2(H)$

By also considering resonances H^*, it is possible to extract the value of μ_G^2 for a given hadron, with the result

$$\mu_G^2(H) = d_H 2m_c \frac{m_{H^*} - m_H}{d_H - d_{H^*}} , \tag{3.2}$$

where d_H denotes the spin factor

$$d_H = -2\left(S_H\left(S_H+1\right) - S_h\left(S_h+1\right) - S_l\left(S_l+1\right)\right) , \tag{3.3}$$

which follows from the fact that the chromomagnetic operator describes the interaction of the spin between the heavy quark and light quark(s) in the hadron. S_H is the spin of the heavy hadron, S_h that of the heavy quark, and S_l that of the light quark system. H^* is an excited state with analogous definition of d_{H^*} to (3.3). In mesons, $d_H = 3$ and $d_{H^*} = -1$, while in baryons, only $d_{\Omega_c^0}$ is non-zero, with $d_{\Omega_c^0} = 4$ and $d_{\Omega_c^{0*}} = -2$ (see also table 6). Here, it has been assumed that the other parameters in the expansion (3.1), $\bar{\Lambda}$ and μ_π^2, are identical for a given hadron H and the excited state H^*. Formally, μ_G^2 and all other parameters in (3.1) are independent of the heavy quark mass, and to reflect this it is common to apply the relation $2m_c \approx m_{H^*} + m_H$, so that

$$\mu_G^2(H) \equiv d_H \lambda_2 = d_H \frac{m_{H^*}^2 - m_H^2}{d_H - d_{H^*}} . \tag{3.4}$$

The two determinations (3.2) and (3.4) differ by $1/m_Q$ corrections, which for charm quarks could be significant. We will use the relation (3.4) to determine values of μ_G^2. Using the latest values for the masses in PDG [21], which are also quoted in appendix A, we obtain the values in table 7. Note that the chromomagnetic operator obtains a non-vanishing

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H	D	D^*	Λ_c^+, Ξ_c^+, Ξ_c^0	Ω_c^0	Ω_c^{0*}
S_H	0	1	1/2	1/2	3/2
S_l	1/2	1/2	0	1	1
d_H	3	−1	0	4	−2

Table 6. Values of spins and of the parameter d_H for all relevant particles. In all cases $S_h = 1/2$, as this is merely the spin of a single quark, and is therefore not included in the table. Values of S_H, S_l and d_{H^*} for the triplet baryons Λ_c^+, Ξ_c^0, and Ξ_c^+ are not indicated, since d_H is zero for all three of these baryons, making μ_G^2 identically zero.

anomalous dimension, known to three loops [118]. Since, however, we assume that μ_G^2 for the charm hadrons is renormalized at the charm mass scale, the running of μ_G^2 does not play any role.

3.2 Kinetic parameter $\mu_\pi^2(H)$

Unlike $\mu_G^2(H)$, the value of μ_π^2 cannot be so easily extracted in terms of known hadron masses, and is therefore less well-determined than μ_G^2. Because of this, various attempts have been made to evaluate μ_π^2 using sum rules, lattice QCD, or experimental fits, with several different values available throughout the literature, (see eg table I of [119], eq. (3.133) of [29], and table 10 from [39]). Some experimental fits, in the case of B mesons, give $\mu_\pi^2 = 0.465(68)\,\text{GeV}^2$ [120] and $\mu_\pi^2 = 0.432(68)\,\text{GeV}^2$ [36]. For the charm sector, no such analyses have been performed. Hence, previous studies have tended to assume that $\mu_\pi^2(D) = \mu_\pi^2(B) = \mu_\pi^2(\Lambda_c^+)$, etc, as used in [39, 40]. In addition, there have been attempts to estimate SU(3)$_F$-breaking corrections, finding them to be no more than 20% [28, 121].

There does exist the theoretical lower bound, derived in [52, 122], that

$$\mu_\pi^2 \geq \mu_G^2\,, \tag{3.5}$$

which in principle provides a constraint on its value. Some other estimates can be derived by imposing the heavy-quark symmetry relation $\mu_\pi^2(D^{(*)}) = \mu_\pi^2(B^{(*)})$, and repeated application of (3.1). In [17, 119, 123], for example, μ_π^2 was related to the pole mass difference $m_b - m_c$ as

$$m_b - m_c \simeq \left(\frac{m_B + 3m_{B^*}}{4} - \frac{m_D + 3m_{D^*}}{4}\right) + \mu_\pi^2(D)\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right). \tag{3.6}$$

In [119], the further replacement on the right-hand side was made of $m_c \to (m_D + 3m_{D^*})/4$, and $m_b \to (m_B + 3m_{B^*})/4$, the spin-averaged masses. This is not the only possible choice, and one can also approximate $m_c \to m_D$, $m_b \to m_B$, which is valid to leading order in the heavy quark mass when taking the difference $m_b - m_c$. Since there is a further uncertainty in the value of the pole masses, we also replace $m_b - m_c \to m_B - m_D$ on the left-hand side. Regardless of the approach, one must accept uncertainties due to neglected higher-order terms in the $1/m_c$ expansion. With this replacement, and using the equation as originally presented in [119], we find a central value of $\mu_\pi^2(D) = 0.45\,\text{GeV}^2$. On the other hand, making the approximations $m_c \to m_D$, $m_b \to m_B$ throughout, we obtain a central value

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$\mu_\pi^2(B) = 0.42\,\text{GeV}^2$. Remarkably, both are compatible with the experimental fits given above, although any estimate for μ_π^2 obtained in this manner is highly sensitive to the choice of how to express $m_b - m_c$ in terms of known quantities.

In [17], it was shown how to extract the difference $\mu_\pi^2(\Lambda_c^+) - \mu_\pi^2(D)$, again assuming that $\mu_\pi^2(D^{(*)}) = \mu_\pi^2(B^{(*)})$ and $\mu_\pi^2(\Lambda_c^+) = \mu_\pi^2(\Lambda_b^0)$. Correcting a typo, the relationship reads

$$\left(m_D+3m_{D^*}-4m_{\Lambda_c^+}\right)-\left(m_B+3m_{B^*}-4m_{\Lambda_b^0}\right)\simeq 2\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\left(\mu_\pi^2(D)-\mu_\pi^2\left(\Lambda_c^+\right)\right)+\mathcal{O}\left(\frac{1}{m_Q}\right). \tag{3.7}$$

Using the latest particle mass values from PDG [21], we obtain

$$\mu_\pi^2(D) - \mu_\pi^2(\Lambda_c^+) \approx -0.05\,\text{GeV}^2\,, \tag{3.8}$$

which suggests that, to this order in $1/m_Q$, the kinetic parameter is approximately identical for baryons and mesons. This argument extends to the Ξ_c^0 and Ξ_c^+, with appropriate replacement of masses. For the Ω_c^0 and Ω_b^-, the contribution from μ_G^2 in this case does not cancel in the relation (3.7), but one can use the combination

$$\mu_\pi^2(\Omega_c^0)\left(\frac{1}{2m_b}-\frac{1}{2m_c}\right) = m_c - m_b + \frac{1}{3}\left(\left(m_{\Omega_b^-}+2m_{\Omega_b^{-*}}\right)-\left(m_{\Omega_c^0}+2m_{\Omega_c^{0*}}\right)\right). \tag{3.9}$$

In our numerical analysis, we will use the spectroscopic estimates from above, applying also the replacement

$$\frac{1}{m_c}-\frac{1}{m_b} \to \frac{4}{m_{D_{(s)}}+3m_{D^*_{(s)}}} - \frac{4}{m_{B_{(s)}}+3m_{B^*_{(s)}}}\,. \tag{3.10}$$

The central values so obtained are compatible with previous studies and the experimental fits [36, 120], and are given in table 7. These relations rely on the heavy-quark symmetry limit, are affected by $1/m_Q$ corrections, and are unlikely to capture all SU(3)$_F$-breaking effects. In particular, our central estimate of the SU(3)$_F$-breaking in mesons is

$$\mu_\pi^2\left(D_s\right) - \mu_\pi^2\left(D^0\right) = 0.03\,\text{GeV}^2\,, \tag{3.11}$$

which can be compared with previous estimates of up to $\mu_\pi^2(D_s) - \mu_\pi^2(D^0) = 0.10\,\text{GeV}^2$ from the literature, eg [28, 121]. Our estimate of SU(3)$_F$-breaking should not therefore be taken as definitive, and in analysing observables sensitive to the difference (3.11), the two parameters will be treated as uncorrelated. In light of these considerations, we assign a 30% uncertainty to all central values for μ_π^2. A more precise determination of these parameters, either from the lattice or from experimental studies, in the spirit of [36, 120] but applied to the charm sector, would serve to clarify these issues.

3.3 Darwin parameter $\rho_D^3(H)$

Several attempts have been made to fit to experimental data to extract the Darwin parameter, again for B mesons only, in [36, 120, 124]. The resulting values fall within the approximate range $0.15\,\text{GeV}^3 \le \rho_D^3(B_{(s)}) \le 0.2\,\text{GeV}^3$.

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	D^0	D^+	D_s^+	Λ_c^+	Ξ_c^+	Ξ_c^0	Ω_c^0
$\mu_G^2/\,\mathrm{GeV}^2$	0.41(12)	0.41(12)	0.44(13)	0	0	0	0.26(8)
$\mu_\pi^2/\,\mathrm{GeV}^2$	0.45(14)	0.45(14)	0.48(14)	0.50(15)	0.55(17)	0.55(17)	0.55(17)
$\rho_D^3/\,\mathrm{GeV}^3$	0.056(12)	0.056(22)	0.082(33)	0.04(1)	0.05(2)	0.06(2)	0.06(2)

Table 7. Non-perturbative parameters in the non-spectator contributions used in our analysis, including assigned ∼30% uncertainties to all the entries, by applying the methods discussed in sections 3.1, 3.2 and 3.3. The values for μ_π^2 for the Ξ baryons are derived by replacing $m_{\Lambda_c^+}$ in (3.7) with $m_{\Xi_c^+}$, and similarly for the Ξ_c^0, which has the effect of lifting the values slightly and accounts for SU(3)$_F$-breaking. The central values of the Darwin parameters for the mesons are adopted from [39]. We estimate the central values of the Darwin parameters for the baryons using the equation of motion for the gluon field with $\alpha_s = 1$, see section 5, and our constituent quark model estimates of the dimension-six four-quark matrix elements in eqs. (5.17) and (5.18)).

An alternative approach [20] is to relate the Darwin parameter to the matrix elements of the dimension-six spectator contributions by applying the equations of motion for a gluon field, $[D^\mu, G_{\mu\nu}] = -g_s\, t_a\, \bar{q}_f t_a \gamma_\nu q_f$, where all the repeated indices are summed over, including the index f that denotes the light quark flavours, and the gluon field strength is $ig_s G^{\mu\nu} \equiv [iD^\mu, iD^\nu]$. In addition, applying the equation of motion $(iv\cdot D)h_v = 0$, one obtains the leading order relation between the matrix elements

$$\begin{aligned} 2m_H\rho_D^3 &= \langle H|\bar{h}_v\,(iD_\mu)\,(iv\cdot D)\,(iD^\mu)\,h_v|H\rangle + \mathcal{O}\,(1/m_c) \\ &= g_s^2\langle H|\left(-\frac{1}{9}\mathcal{O}_1^q + \frac{2}{9}\mathcal{O}_2^q + \frac{1}{12}\mathcal{T}_1^q - \frac{1}{6}\mathcal{T}_2^q\right)|H\rangle + \mathcal{O}\,(1/m_c)\,, \end{aligned} \tag{3.12}$$

where $\mathcal{O}_i^q, \mathcal{T}_i^q$ are the HQET four-quark operators defined in eq. (2.27). This approach has been recently employed for D mesons beyond the VIA in [39]. Using this approximation for charmed mesons we have

$$\rho_D^3\,(D_q) = \frac{g_s^2}{18} f_{D_q}^2 m_{D_q} + \mathcal{O}\,(1/m_c)\,, \tag{3.13}$$

where f_{D_q} denotes the decay constant of D_q, also defined in eq. (4.3).

For the charmed baryons, the same expression (3.12) is rewritten in terms of the operators (2.36) in the baryon basis:

$$2m_H\rho_D^3 = g_s^2\langle H|\left(-\frac{1}{8}O_1^q + \frac{1}{24}\tilde{O}_1^q + \frac{1}{4}O_2^q - \frac{1}{12}\tilde{O}_2^q\right)|H\rangle + \mathcal{O}\,(1/m_c)\,. \tag{3.14}$$

In (3.12) and (3.14), $g_s^2 = 4\pi\alpha_s$ represents a non-perturbative scale; following eg [20], we set $\alpha_s = 1$. With explicit values of the matrix elements of operators from (3.12) given in section 5.1, we obtain the Darwin parameters of charmed baryons given in table 7.

3.4 Results for non-spectator contributions

In table 8 we present the numerical results for the contribution Γ_3 at LO (NLO). In accord with our discussion in section 2.4.1, the quantity $\Gamma_3^{(1)}$ involves terms proportional to α_s that

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Mass scheme	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$
Pole	$1.49^{+0.17}_{-0.14}$	$1.62^{+0.26}_{-0.22}$
$\overline{\text{MS}}$	$0.69^{+0.08}_{-0.07}$	$1.28^{+0.37}_{-0.29}$
Kinetic	$1.10^{+0.13}_{-0.11}$	$1.65^{+0.41}_{-0.32}$
MSR	$0.93^{+0.11}_{-0.09}$	$1.54^{+0.41}_{-0.32}$

Table 8. Values of $\Gamma_3\,[\text{ps}^{-1}]$ contribution to the total decay widths at LO ((0)) and NLO((1)) for various schemes of the charm-quark mass. For the central values, we set $\mu = 1.5\,\text{GeV}$, while the errors result from the variation of the scale μ in the range $(1\,\text{GeV}, 3\,\text{GeV})$. For the pole scheme, we use $m_c^{\text{pole}} = 1.49\,\text{GeV}$, evaluated using the relation (2.52) to order α_s; see table 5 for the values of charm quark mass in other schemes. We adopt the values m^{MSR} and m^{kin} evaluated to four and three loops, respectively, see eqs. (2.57) and (2.59).

originate from the evaluation of the leading coefficient $c_3^{(0)}$ using $C_{1,2}^{(1)}$, the order-α_s term from the conversion formula to a given mass scheme, as well as the genuine $c_3^{(1)}$ contribution evaluated with $C_{1,2}^{(0)}$.

Values of the $\Gamma_3\,[\text{ps}^{-1}]$ contribution evaluated using different schemes for the charm-quark mass are listed in table 8. For these evaluations, we set $\mu_m = \overline{m}_c$ within the expression in eq. (2.53). We used the value of the pole mass $m_c^{\text{pole}} = 1.49\,\text{GeV}$, found using the relation (2.52) to order α_s. The uncertainties are estimated by varying the scale μ in the range $(1\,\text{GeV}\,, 3\,\text{GeV})$, while the central values correspond to fixing $\mu = 1.5\,\text{GeV}$. The scale μ_f in eq. (2.58) is set to $\mu_f = 1\,\text{GeV}$, the same value used for the evaluation of the MSR mass in terms of $\overline{m}_c$. The results in the kinetic scheme are for $\mu_f = 0.5\,\text{GeV}$.

Values of the Γ_5 contributions Γ_π and Γ_G are listed in table 9. The procedure for the evaluations follows the description for Γ_3 shown in the caption in table 8. For consistency of the power-counting in the perturbative expansion, we keep only the leading part of Γ_3 in the expansion $\Gamma_\pi^{(0)} = \Gamma_3^{(0)}(-\mu_\pi^2/(2m_Q^2))$, in the absence of complete α_s-corrections to the operators at the dimension-five. Note, however, that in the case of the semileptonic contribution, the complete α_s-corrections to the dimension-five operators are retained for our final numerical results presented in sections 4.2 and 5.2. However, the effects of including these additional corrections turn out to be small.

One can notice several features of the dimension-five and dimension-six non-spectator contributions, previously described in [39]. Firstly, the Wilson coefficient of the chromomagnetic operator c_G shows strong dependence on the renormalization scale, but nonetheless stays relatively small. The coefficient of the Darwin operator turns out unexpectedly large. This hints at a bad convergence of the $1/m_c$ expansion in the non-spectator sector. However, this can be determined only with the knowledge of the unknown higher-dimension terms. We also note that the above estimate of the Darwin matrix element involves unusually large SU(3)$_F$-breaking, ie $\rho(D_s^+)/\rho(D^+) \simeq (f_{D_s}^2 m_{D_s})/(f_D^2 m_D) = 1.5$. Finally, our results agree with those given in [39].

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Mass scheme	$\Gamma_\pi^{(0)}$	$\Gamma_G^{(0)}$	$\Gamma_{\text{Darwin}}^{(0)}$
Pole	$(-0.17^{+0.02}_{-0.02})\frac{\mu_\pi^2}{0.5\,\text{GeV}^2}$	$(0.01^{+0.07}_{-0.07})\frac{\mu_G^2}{0.25\,\text{GeV}^2}$	$(0.44^{+0.09}_{-0.09})\frac{\rho_D^3}{0.1\,\text{GeV}^3}$
$\overline{\text{MS}}$	$(-0.10^{+0.01}_{-0.01})\frac{\mu_\pi^2}{0.5\,\text{GeV}^2}$	$(0.00^{+0.05}_{-0.05})\frac{\mu_G^2}{0.25\,\text{GeV}^2}$	$(0.43^{+0.08}_{-0.08})\frac{\rho_D^3}{0.1\,\text{GeV}^3}$
Kinetic	$(-0.14^{+0.01}_{-0.02})\frac{\mu_\pi^2}{0.5\,\text{GeV}^2}$	$(0.01^{+0.06}_{-0.06})\frac{\mu_G^2}{0.25\,\text{GeV}^2}$	$(0.44^{+0.09}_{-0.08})\frac{\rho_D^3}{0.1\,\text{GeV}^3}$
MSR	$(-0.13^{+0.01}_{-0.01})\frac{\mu_\pi^2}{0.5\,\text{GeV}^2}$	$(0.01^{+0.06}_{-0.06})\frac{\mu_G^2}{0.25\,\text{GeV}^2}$	$(0.44^{+0.09}_{-0.08})\frac{\rho_D^3}{0.1\,\text{GeV}^3}$

Table 9. LO values of dimension-five and dimension-six non-spectator contributions to the total decay widths, $\Gamma_{\pi,G}$ and Γ_{Darwin}, respectively, for several schemes of the charm-quark mass. The numerical coefficients are in units ps^{-1}, and the non-perturbative parameters μ_π^2, μ_G^2 and ρ_D^3 in units GeV^2, GeV^2 and GeV^3, respectively. See the caption of table 8, and table 5, for further details about the input parameters. The expressions are valid for both mesons and baryons, after the suitable replacements of the hadronic parameters.

4 Charmed mesons

In this section we present results for the charmed mesons D^0, D^+, and D_s^+. A similar computation was already presented in [39], which serves as a useful cross-check of our method and analytic inputs.

4.1 Matrix elements of spectator contributions

The final set of inputs to the lifetime are the four-quark matrix elements, whose coefficients were discussed in section 2.3.2, and are provided explicitly at LO in eqs. (B.12) and (B.13). As previously noted, these elements can be defined in terms of full QCD quark fields, or, more consistently for the power-counting in $1/m_c$, with heavy quark fields instead. For mesons, since HQET estimates of the parameters are available [39, 83], we choose to work within HQET.

At dimension-six, the relevant matrix elements can be parametrized, following [39], as

$$\langle D_q|\mathcal{O}_i^q|D_q\rangle = F_{D_q}(\mu)^2 m_{D_q} B_i^q\,, \tag{4.1}$$

$$\langle D_q|\mathcal{O}_i^{q'}|D_q\rangle = F_{D_q}(\mu)^2 m_{D_q} \delta_i^{q'q}\,, \quad q \neq q'\,, \tag{4.2}$$

where $F_{D_q}(\mu)$ is the static decay constant, which, in the $m_c \to \infty$ limit, is given by $F_{D_q} = f_{D_q}\sqrt{m_{D_q}}$. The first line describes valence quark contributions, and the second line describes non-valence contributions, with the parameters $\delta_i^{q'q}$ denoting the "eye contractions" (schematically represented in figure 4) [39, 82, 83].[16] The HQET bag parameters for the operators $\mathcal{O}_{1,2}^q$ are denoted by $B_{1,2}^q$, while $B_{3,4}^q \equiv \epsilon_{1,2}^q$ denote the bag parameters of the colour octet operators $\mathcal{T}_{1,2}^q$. The isospin relations $B_i^u = B_i^d \equiv B_i^q$, $\delta_i^{ud} = \delta_i^{du} \equiv \delta_i^{qq}$, $\delta_i^{su} = \delta_i^{sd} \equiv \delta_i^{sq}$ and $\delta_i^{us} = \delta_i^{ds} \equiv \delta_i^{qs}$, are applied throughout. Recall that in the VIA $B_{1,2} = 1$, while all other parameters vanish. Eye contractions that serve as corrections to the valence-quark matrix elements (4.1), δ_i^{qq}, are implicitly included in the bag parameters B_i^q. The δ_i^{qq} and

[16] Non-valence four-quark operator contributions were extensively discussed in [86].

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$\delta_i^{q'q}$ parameters were calculated, using HQET sum rules, in [83], but have not been evaluated in the most recent lattice estimates [125]. At tree level, eye contractions vanish for the octet operators, and the non-vanishing contributions come only from singlet operators. For charm quark decays, non-valence contributions with the s quark into the loop can be non-negligible. However, since these cannot be reliably calculated, and the penguin contributions, of a similar size, are usually neglected (which is justified for c-quark decays), these contributions have been neglected. For the non-valence contributions, we use the HQET evaluations provided in [39, 83], including the available eye contraction parameters.

The QCD (HQET) definitions of the decay constants of D_q mesons, f_{D_q} and F_{D_q}, respectively, are

$$\begin{aligned}\langle 0|\bar{q}\gamma^\mu\gamma_5 c|D_q(p)\rangle &= i f_{D_q} p^\mu\,,\\ \langle 0|\bar{q}\gamma^\mu\gamma_5 h_v|D_q(v)\rangle &= i F_{D_q}(\mu)\sqrt{m_{D_q}}v^\mu\,.\end{aligned} \tag{4.3}$$

Employing the result, valid at the scale $\mu = m_c$, that the non-local dimension-seven matrix elements can be reabsorbed into the dimension-six matrix elements (see [39, 53, 80, 87] and appendix C), we further apply the replacement

$$F_{D_q}(\mu)^2 \to f_{D_q}^2 m_{D_q}\left(1+\frac{4}{3}\frac{\alpha_s(m_c)}{\pi}\right) \tag{4.4}$$

to express the dimension-six matrix elements (4.1) and (4.2) in terms of physical parameters. Values for all HQE parameters are taken from [39, 83].

The dimension-seven matrix elements are parametrized, again following [39], by

$$\begin{aligned}\langle D_q|\mathcal{P}_1^q|D_q\rangle &= -m_q F_{D_q}^2 m_{D_q} B_1^P\,,\\ \langle D_q|\mathcal{P}_2^q|D_q\rangle &= -\bar{\Lambda}_q F_{D_q}^2 m_{D_q} B_2^P\,,\\ \langle D_q|\mathcal{P}_3^q|D_q\rangle &= -\bar{\Lambda}_q F_{D_q}^2 m_{D_q} B_3^P\,,\\ \langle D_q|\mathcal{R}_1^q|D_q\rangle &= -F_{D_q}^2 m_{D_q}(\bar{\Lambda}_q - m_q) B_1^R\,,\\ \langle D_q|\mathcal{R}_2^q|D_q\rangle &= F_{D_q}^2 m_{D_q}(\bar{\Lambda}_q - m_q) B_2^R\,,\end{aligned} \tag{4.5}$$

with the colour-octet operators having equivalent parametrizations on replacement of $B_i^{P,R} \to \epsilon_i^{P,R}$. Since there is no available computation of these parameters in HQET, we apply the VIA, so that $B_i^{P,R} = 1$ and $\epsilon_i^{P,R} = 0$. Parametrizations of the non-local matrix elements, which formally contribute but here have been absorbed into the dimension-six matrix element via the replacement (4.4), are available in appendix C of [39]. We also apply the replacement $F_{D_q} \to f_{D_q}\sqrt{m_{D_q}}$. The parameter Λ_q is of order the QCD scale; for the numerical evaluation we use the numbers reported in [39], namely

$$\bar{\Lambda} = 0.5 \pm 0.1\,\text{GeV}\,, \qquad \bar{\Lambda}_s = 0.6 \pm 0.1\,\text{GeV}\,. \tag{4.6}$$

4.2 Final numerical predictions for mesons

Our final numerical predictions for meson decay widths and lifetime ratios are presented in table 10, while semileptonic branching fractions and ratios are presented in table 11. A

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detailed breakdown of various contributions, for two choices of the mass scheme for the charm quark, is shown in tables 22 and 23, provided in appendix D. Following [39], we express lifetime ratios via the differences of the theoretical decay widths (denoted "th"), scaled by the experimental lifetimes (denoted "exp"):

$$\frac{\tau\left(D_{(s)}^{+}\right)}{\tau\left(D^{0}\right)}=1+\left(\Gamma^{\mathrm{th}}\left(D^{0}\right)-\Gamma^{\mathrm{th}}\left(D_{(s)}^{+}\right)\right)\tau^{\mathrm{exp}}\left(D_{(s)}^{+}\right)\,. \tag{4.7}$$

The benefit of such a definition lies in the cancellation between the, universal, non-spectator contributions. This is especially beneficial in the ratio $\tau(D^+)/\tau(D^0)$, where, due to isospin symmetry, there is even cancellation of the dimension-five and -six non-spectator contributions.

For the semileptonic contributions, we consider the inclusive decay channels involving electrons in the final states, eg $\Gamma^{(e)}(D^+)\equiv\Gamma(D^+\to Xe\nu)$, with the definitions [39]

$$BR^{(e)}(D)=\Gamma^{(e)}(D)\tau^{\mathrm{exp}}(D)\,, \tag{4.8}$$

and

$$\frac{\Gamma^{(e)}\left(D_{(s)}^{+}\right)}{\Gamma^{(e)}\left(D^{0}\right)}=1+\left(\Gamma^{(e)\,\mathrm{th}}\left(D_{(s)}^{+}\right)-\Gamma^{(e)\,\mathrm{th}}\left(D^{0}\right)\right)\left(\frac{\tau\left(D^{0}\right)}{BR^{(e)}(D^{0})}\right)^{\mathrm{exp}}\,. \tag{4.9}$$

The results in tables 10 and 11 are compatible with those in [39] and with experiment. We also agree with the criticism in [39] of the results presented in [40]. There is some minor difference in the uncertainty estimates, which can be attributed to a more conservative approach to uncertainties in our study. In our approach, the varied parameters have not been interpreted as following a probability distribution. Rather, the upper and lower errors are simply the maximal and minimal distances from the central values. For the estimate of the scale uncertainties we fixed hadronic parameters to their central values.

As with [39], we also observe a slight tension with experiment in the theoretical value of the ratio $\tilde{\tau}(D_s^+)/\tau(D^0)$. This is a long-standing problem, to which several solutions have been proposed in the literature, such as large non-perturbative or non-valence WA contributions, albeit without a clear conclusion [123].

To discuss the spectator contributions in more detail, table 12 presents central values of *valence* dimension-six spectator contributions to the decay widths of charmed mesons, evaluated in the $\overline{\mathrm{MS}}$ scheme at the scale $\mu=\mu_0=1.5\,\mathrm{GeV}$. The factor $x=1$ multiplies the contributions of order α_s. Evident from the expressions shown in table 12 is the well-known helicity suppression of the weak exchange (WE) and weak annihilation (WA) contributions within the VIA, while Pauli interference (PI) drives a large suppression of the decay width of D^+ relative to that of D^0. The large value of the coefficient multiplying ϵ_1^q gives rise to a strong sensitivity to this hadronic parameter. While ϵ_1^q is close to zero [83], it comes with a large uncertainty. Sizable perturbative α_s corrections boost PI even further, driving the prediction of the D^+ lifetime towards the unphysical region for some choices of the hadronic parameters, see table 10. This observation has been also made in [39, 40]. As a result, the prediction of the D^+ lifetime is particularly problematic. To resolve this problem, a lattice QCD determination of the dimension-six matrix elements will be necessary.

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Observable	Pole	$\overline{\text{MS}}$	Kinetic	MSR	Experiment
$\Gamma(D^0)$	$1.71^{+0.41+0.39}_{-0.47-0.36}$	$1.43^{+0.36+0.48}_{-0.40-0.40}$	$1.77^{+0.40+0.53}_{-0.45-0.45}$	$1.68^{+0.38+0.53}_{-0.43-0.44}$	2.44 ± 0.01
$\Gamma(D^+)$	$-0.07^{+0.76+0.31}_{-0.68-0.20}$	$-0.27^{+0.66+0.03}_{-0.88-0.04}$	$-0.07^{+0.73+0.20}_{-0.66-0.14}$	$-0.13^{+0.71+0.13}_{-0.64-0.11}$	0.96 ± 0.01
$\tilde{\Gamma}(D_s^+)$	$1.71^{+0.49+0.44}_{-0.60-0.40}$	$1.43^{+0.42+0.49}_{-0.52-0.41}$	$1.77^{+0.47+0.55}_{-0.58-0.47}$	$1.67^{+0.46+0.55}_{-0.56-0.46}$	1.88 ± 0.02
$\tau(D^+)/\tau(D^0)$	$2.85^{+0.68+0.10}_{-0.81-0.17}$	$2.78^{+0.63+0.47}_{-0.73-0.37}$	$2.91^{+0.68+0.35}_{-0.80-0.32}$	$2.89^{+0.66+0.42}_{-0.78-0.35}$	2.54 ± 0.02
$\tilde{\tau}(D_s^+)/\tau(D^0)$	$1.00^{+0.24+0.02}_{-0.22-0.02}$	$1.00^{+0.21+0.01}_{-0.19-0.00}$	$1.00^{+0.23+0.01}_{-0.21-0.01}$	$1.00^{+0.23+0.01}_{-0.21-0.01}$	1.30 ± 0.01

Table 10. The total decay widths in units ps^{-1}, and their ratios for charmed mesons compared to the experimental values, using HQET parameters, see table 1 for the references to the experimental papers. The first uncertainties are coming from independent variations of the hadronic matrix elements within the corresponding ranges, while the second uncertainties result from the variation of the renormalization scale μ in the range $[1, 3]$ GeV. The references to the sources of the experimental data are given in section 2.1.1.

Observable	Pole	$\overline{\text{MS}}$	Kinetic	MSR	Experiment
$BR^{(e)}(D^0)$ [%]	$4.07^{+2.21+0.84}_{-2.53-0.97}$	$5.18^{+1.59+0.63}_{-1.82-0.55}$	$5.87^{+1.94+0.22}_{-2.23-0.19}$	$5.86^{+1.80+0.48}_{-2.07-0.41}$	6.49 ± 0.16
$BR^{(e)}(D^+)$ [%]	$10.34^{+5.69+2.12}_{-6.52-2.44}$	$13.15^{+4.10+1.61}_{-4.73-1.40}$	$14.92^{+5.00+0.57}_{-5.75-0.49}$	$14.90^{+4.67+1.22}_{-5.37-1.06}$	16.07 ± 0.30
$BR^{(e)}(D_s^+)$ [%]	$5.42^{+3.02+0.96}_{-3.44-1.10}$	$6.86^{+2.42+0.83}_{-2.83-0.72}$	$7.67^{+2.80+0.34}_{-3.23-0.29}$	$7.67^{+2.67+0.65}_{-3.10-0.56}$	6.30 ± 0.16
$\Gamma^{(e)}(D^+)/\Gamma^{(e)}(D^0)$	$1.00^{+0.02+0.00}_{-0.02-0.00}$	$1.00^{+0.01+0.00}_{-0.01-0.00}$	$1.00^{+0.02+0.00}_{-0.02-0.00}$	$1.00^{+0.02+0.00}_{-0.01-0.00}$	0.977 ± 0.031
$\Gamma^{(e)}(D_s^+)/\Gamma^{(e)}(D^0)$	$1.05^{+0.29+0.01}_{-0.31-0.01}$	$1.06^{+0.24+0.01}_{-0.27-0.01}$	$1.07^{+0.28+0.01}_{-0.30-0.01}$	$1.06^{+0.26+0.01}_{-0.29-0.01}$	0.790 ± 0.026

Table 11. Semileptonic decay widths in inclusive channel $Xe\nu$ in units ps^{-1}, and their ratios for charmed mesons compared to the experimental values, using HQET parameters. The first uncertainties are coming from independent variations of the hadronic matrix elements within the corresponding ranges, while the second uncertainties result from the variation of the renormalization scale μ in the range $[1, 3]$ GeV. The references to the sources of the experimental data are given in section 2.1.1.

The dimension-seven PI contribution to the D^+ decay width also turns out to be sizable, but positive, providing some cancellation of the dimension-six terms. The corresponding matrix elements are, however, currently estimated using the VIA only. Hence, as with the dimension-six operators, a lattice determination of matrix elements of dimension-seven operators, and separately an NLO computation at this order in the HQE, would be welcome. It is possible that higher-order α_s spectator contributions would play a significant role in a more complete assessment of the PI contribution. However, the issue of poor convergence of the α_s expansion, seen also in Γ_3^{SL} (cf. section 2.4.2), can also be expected to appear in the four-quark contribution. In this case, the theoretical precision would not necessarily be improved with further contributions, which would possibly motivate studies of some alternative approaches to the inclusive charm decays.

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D^0	
WE	$-(0.01+0.22\,x)B_1^q+(0.01+0.21\,x)B_2^q-(2.27+0.93\,x)\epsilon_1^q+(2.30+0.68\,x)\epsilon_2^q$

D^+	
PI	$-(1.25+1.02\,x)B_1^q-0.17\,x\,B_2^q+(7.46+5.24\,x)\epsilon_1^q-0.28\,x\,\epsilon_2^q$
WA	$-(0.13+0.05\,x)B_1^q+(0.13+0.05\,x)B_2^q-(0.02+0.06\,x)\epsilon_1^q+(0.02+0.06\,x)\epsilon_2^q$
SL	$-(0.08+0.01\,x)B_1^q+(0.08+0.01\,x)B_2^q-0.03\,x\,\epsilon_1^q+0.02\,x\,\epsilon_2^q$

D_s^+	
PI	$-(0.09+0.08\,x)B_1^s-0.01\,xB_2^s+(0.55+0.39\,x)\epsilon_1^s-0.02\,x\epsilon_2^s$
WA	$(-3.66-1.38\,x)B_1^s+(3.66+1.38\,x)B_2^s-(0.47+1.74\,x)\epsilon_1^s+(0.47+1.60\,x)\epsilon_2^s$
SL	$-(2.08+0.26\,x)B_1^s+(2.09+0.27\,x)B_2^s-0.72\,x\epsilon_1^s+0.67\,x\epsilon_2^s$

Table 12. Contributions of valence dimension-six spectator operators to the decay widths of charmed mesons, in units ps^{-1}, in the $\overline{\text{MS}}$ scheme. The different contributions are separated by the topologies defined in figure 2. The HQET bag parameters have been left unevaluated, but are assumed to be renormalized at the scale $\mu_0 \sim 1.5\,\text{GeV}$. Their coefficients correspond to the scale $\mu = 1.5\,\text{GeV}$. The factor $x = 1$ denotes the $\mathcal{O}(\alpha_s)$ contributions. Semileptonic contributions involve both e and μ channels.

5 Singly charmed baryons

5.1 Matrix elements of spectator contributions and baryonic wavefunctions

The dimension-six spectator matrix elements for the operators $O_{1,2}^{u,d,s}$ between the baryon states, introduced in eq. (2.36),

$$M_i^q(\mathcal{B}_c) \equiv \frac{\langle \mathcal{B}_c | O_i^q | \mathcal{B}_c \rangle}{2m_{\mathcal{B}_c}}\,, \qquad i=1,2 \quad \text{and} \quad q=u,d,s\,, \tag{5.1}$$

can be parametrized in constituent quark models as given in table 13. There, (N)RCQM denotes the expressions in nonrelativistic and relativistic constituent quark models. The remaining dimension-six matrix elements are related by (2.39), so that $\tilde{M}_i^q(\mathcal{B}_c) = -\tilde{B} M_i^q(\mathcal{B}_c)$, where we will take $\tilde{B} = 1$ throughout [85].

In a relativistic constituent model, a_q, b_q, and c_q are the overlap integrals

$$\begin{aligned} a_q &= \int d^3r \left[u_q^2(r)\,u_c^2(r) + v_q^2(r)\,v_c^2(r)\right]\,, \\ b_q &= \int d^3r \left[u_q^2(r)\,v_c^2(r) + v_q^2(r)\,u_c^2(r)\right]\,, \\ c_q &= \int d^3r \left[u_q(r)\,u_c(r)\,v_q(r)\,v_c(r)\right]\,, \end{aligned} \tag{5.2}$$

where $u_q(r)$ and $v_q(r)$ are the upper and lower components of the relativistic Dirac spinor. In the nonrelativistic limit, $b_q = c_q = 0$ and $a_q = |\psi_{cq}^{\mathcal{B}_c}(0)|^2$. A popular relativistic model

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$M_i^q(\mathcal{B}_c)\,,\ \mathcal{T}_c = \Lambda_c^+, \Xi_c^+, \Xi_c^0$	RCQM	NRCQM
$M_1^q(\mathcal{T}_c) \equiv \dfrac{\langle \mathcal{T}_c \vert O_1^q \vert \mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}}$	$-(a_q + b_q)$	$-\vert\Psi_{cq}^{\mathcal{T}_c}(0)\vert^2$
$M_2^q(\mathcal{T}_c) \equiv \dfrac{\langle \mathcal{T}_c \vert O_2^q \vert \mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}}$	$\frac{1}{2}(a_q + b_q)$	$\frac{1}{2}\,\vert\Psi_{cq}^{\mathcal{T}_c}(0)\vert^2$
$M_1^s(\Omega_c^0) \equiv \dfrac{\langle \Omega_c^0 \vert O_1^s \vert \Omega_c^0\rangle}{2m_{\Omega_c^0}}$	$-\frac{1}{3}(18a_s + 2b_s + 32c_s)$	$-6\,\vert\Psi_{cs}^{\Omega_c^0}(0)\vert^2$
$M_2^s(\Omega_c^0) \equiv \dfrac{\langle \Omega_c^0 \vert O_2^s \vert \Omega_c^0\rangle}{2m_{\Omega_c^0}}$	$-(a_s - \frac{5}{3}b_s - \frac{16}{3}c_s)$	$-\vert\Psi_{cs}^{\Omega_c^0}(0)\vert^2$

Table 13. Generalized parametrizations of dimension-six matrix elements for baryons in relativistic (RCQM) and nonrelativistic (NRCQM) constituent quark models [26, 40]. The definitions of a_q, b_q, and c_q are in (5.2).

was the MIT bag model [22, 126–130], with some updated parameters for heavy baryons in [131–133]. The advantage of the MIT bag model is that with the only a few adjustable parameters the model can be easily applied for qualitative and quantitative predictions of mesonic and baryonic wavefunctions. The main source for the improvement of such QCD models is heavy hadron spectroscopy, but, despite much experimental progress, it is difficult to make a meaningful assessment of bag model parameters and so have a clear guidance for improvement to the model. Indeed, there are several versions of the MIT bag model, which are not compatible with each other [131]. Moreover, the standard MIT bag model problems, like the inclusion of center-of-mass motion or the value of the quark masses in the bag, are still not fully resolved and reliably treated in the models, while an estimation of the uncertainties in such models is questionable. However, we have checked that for the hydrogen-like MIT model, which could describe a singly charmed baryon configuration, the spectator matrix elements have too small values, leading to results that are not compatible with experimental values. We therefore turn to the NRCQM approach.

The dynamics of a baryon state (Qq_1q_2) is more complex than that of a meson $Q\bar{q}$. However, in the case of heavy baryons some simplification arises due to the heaviness of one quark, Q. The heavy quark is expected to have a very weak coupling to the light quarks, which themselves couple together as a light diquark system [134–136]. In such a picture, baryons can be treated as a quasi two-body system, and show similar dynamics to heavy-light mesons.

The extraction of the wavefunction in the NRCQM is based on the application of the seminal work by de Rujula, Georgi and Glashow [137], where the expression for heavy hadron masses is obtained by considering a two-body potential and the spin-spin interaction between the constituent quarks. For ground states we have

$$M_H = \sum_i m_i^H + \langle H_{\text{spin,H}} \rangle\,, \tag{5.3}$$

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where

$$H_{\text{spin, mesons}} = \frac{32\pi\alpha_s}{9}\frac{(\vec{s}_i\cdot\vec{s}_j)}{m_i^M m_j^M}\delta_M^3(\vec{r}_{ij}) \,,$$
$$H_{\text{spin, baryons}} = \sum_{i>j}\frac{16\pi\alpha_s}{9}\frac{(\vec{s}_i\cdot\vec{s}_j)}{m_i^{\mathcal{B}} m_j^{\mathcal{B}}}\delta_{\mathcal{B}}^3(\vec{r}_{ij}) \,, \tag{5.4}$$

are the spin-spin interactions for mesons and baryons respectively. By combining the expression (5.3) for hadrons in different spin states and taking mass differences, the wavefunction $|\Psi(0)|^2_{ij} \sim \delta^3(0)$ is extracted. The spectator contribution is then proportional to the squared modulus of the wavefunction for two quarks at the origin, as seen in the rightmost column of table 13. The masses which appear in (5.4) are constituent masses (sometimes also called effective masses), rather than bare masses, so for example $m_u^{M,\mathcal{B}} \neq 0$ is non-negligible. Constituent masses of quarks in baryons and mesons differ from one another, with m_q^M and $m_q^{\mathcal{B}}$ denoting, respectively, the constituent mass of the quark q in a meson and baryon. The values of $m_q^{M,\mathcal{B}}$ are obtained from the fits to experimentally determined hadron masses [138, 139]. We have for the constituent quark masses [139] in mesons

$$m_{u,d}^M = 310\,\text{MeV}\,, \qquad m_s^M = 483\,\text{MeV}\,, \qquad m_c^M = 1663.3\,\text{MeV}\,, \qquad m_b^M = 5003.8\,\text{MeV}\,, \tag{5.5}$$

and in the baryons

$$m_{u,d}^{\mathcal{B}} = 363\,\text{MeV}\,, \qquad m_s^{\mathcal{B}} = 538\,\text{MeV}\,, \qquad m_c^{\mathcal{B}} = 1710.5\,\text{MeV}\,, \qquad m_b^{\mathcal{B}} = 5043.5\,\text{MeV}\,. \tag{5.6}$$

As expected, the constituent quark masses are somewhat smaller in mesons.

There are several possibilities to build suitable heavy hadron mass differences in order to extract the wavefunction of a heavy baryon. The first method of extraction of the Λ_c^+ wavefunction, which, to the best of our knowledge, was first proposed by Barger et al. in [140], was driven by the knowledge of the experimentally measured mass of Σ_c, ie the mass difference $M_{\Sigma_c} - M_{\Lambda_c}$. This exploits the fact that Σ_c has the same quark content as Λ_c^+, although it is a member of the $\text{SU}(3)_F$ sextet rather than of the $\text{SU}(3)_F$ antitriplet, and that the light quarks in Λ_c^+ are coupled to zero spin, so that their hyperfine interaction with the heavy quark is therefore zero. By accounting for different spins and spin interactions of sextet and antitriplet baryons, one arrives at

$$M_{\Sigma_c} - M_{\Lambda_c^+} = \frac{16\pi\alpha_s}{9}\frac{1}{m_c^{\mathcal{B}} m_u^{\mathcal{B}}}\left(\frac{m_c^{\mathcal{B}} - m_u^{\mathcal{B}}}{m_u^{\mathcal{B}}}\right)|\Psi_{cq}^{\Lambda_c^+}(0)|^2 \,, \tag{5.7}$$

where it has been assumed that the spatial wavefunctions of the baryons are equal.

This relation, however, suffers from large uncertainty due to the value of the α_s coupling in the baryon, and is also dependent on the values of the constituent quark masses, which historically were not well-determined. As first recognized by Cortes and Sanches-Guillen [23], this uncertainty can be reduced by exploiting a similar relation between charmed meson states from (5.4), and by relating the baryon and meson wavefunctions as

$$|\Psi_{cq}^{\Lambda_c^+}(0)|^2 = \frac{2m_u^{\mathcal{B}}}{(M_D - m_u^{\mathcal{B}})}\frac{M_{\Sigma_c} - M_{\Lambda_c^+}}{M_{D^*} - M_D}|\Psi_{cq}^{D_q}(0)|^2 \,, \tag{5.8}$$

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where $m_c^{\mathcal{B}}$ has been replaced by M_D, and

$$|\Psi_{cq}^{D_q}(0)|^2 = \frac{f_D^2 M_D}{12} \tag{5.9}$$

is the mod-square wavefunction of the meson. This expression relies on the value of α_s being identical for meson and baryon states.

The formula (5.8), which uses the mass differences of baryons with the same quark content and having the same spin, was extensively used for the extraction of singly charmed baryon wavefunctions, until it was proposed by Rosner in [141] to exploit the hyperfine splittings in Σ_c^* and Σ_c, with the same assumptions as above. Using

$$M_{\Sigma_c^*} - M_{\Sigma_c} = \frac{16\pi\alpha_s}{9} \frac{1}{m_c^{\mathcal{B}} m_u^{\mathcal{B}}} \frac{3}{2} |\Psi_{cq}^{\Lambda_c^+}(0)|^2 \tag{5.10}$$

this leads to

$$|\Psi_{cq}^{\Lambda_c^+}(0)|^2 = \frac{4}{3} \frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{D^*} - M_D} |\Psi_{cq}^{D_q}(0)|^2 \,. \tag{5.11}$$

This construction enables removal of the a priori unknown constituent mass $m_u^{\mathcal{B}}$ in (5.8). In addition, by taking the difference of $M_{\Sigma_c^*} - M_{\Sigma_c}$, one effectively performs the spin-weighted average of the hyperfine interactions in $\Sigma_c^*(3/2)$ and $\Sigma_c(1/2)$.

Although the two formulas (5.8) and (5.11) do not look the same, they are derived from the same mass formula in (5.4) and so should be numerically equivalent, which was not the case in the past, since the constituent quark masses were not known precisely. By inserting explicitly the constituent quark masses given in (5.6), we obtain for the difference of these wavefunctions

$$\frac{|\Psi_{cq}^{\Lambda_c^+}(0)|^2}{|\Psi_{cq}^{D_q}(0)|^2}_{|\ (5.11)} - \frac{|\Psi_{cq}^{\Lambda_c^+}(0)|^2}{|\Psi_{cq}^{D_q}(0)|^2}_{|\,(5.8)} \sim 0.04 \,, \tag{5.12}$$

which represents an approximately 6% difference, and is therefore negligible at the present level of uncertainty.

It is worth emphasizing that the derivation of the formulas (5.8) and (5.11) has relied on several assumptions, including that

(i) the wavefunctions of baryons with the same quark content are the same, even if they belong to different $\mathrm{SU}(3)_F$ mutliplets or spin states;

(ii) strong couplings α_s for all interactions are approximately equal;

(iii) constituent quark masses in mesons and baryons are equal.

Although all these assumptions seem to be plausible for such systems (Qq_1q_2) with one heavy quark, they have to be critically examined. In particular, whereas the last assumption about the equality of the constituent masses in mesons and baryons has been used in previous studies [13, 40], it is not justified in view of the values in (5.5) and (5.6). By using these values, the formulas (5.8) and (5.11) above should be multiplied by the correction factors

$$y = \frac{m_c^{\mathcal{B}} m_u^{\mathcal{B}}}{m_c^M m_u^M} \simeq 1.20 \,, \qquad y_s = \frac{m_c^{\mathcal{B}} m_s^{\mathcal{B}}}{m_c^M m_s^M} \simeq 1.15 \,, \tag{5.13}$$

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where the second factor is relevant for baryons containing an s quark. We include the above correction factors in our numerical calculation.

Bearing in mind all the considerations above, we will consider charmed baryon wavefunctions only in the NRCQM approach, using the hyperfine mass-splitting relations [137] and the method of [141] exemplified in (5.11). The relevant baryon wavefunctions are then given by

$$\begin{aligned}
|\Psi_{cq}^{\Lambda_c^+}(0)|^2 &= \frac{4}{3}\,\frac{m_{\Sigma_c^*}-m_{\Sigma_c}}{m_{D^*}-m_D}\,|\Psi_{cq}^{D_q}(0)|^2\,R_{cq}^{\Lambda_c^+}\,, \quad \text{for } q=u,d,\\
|\Psi_{cq}^{\Xi_c}(0)|^2 &= \frac{4}{3}\,\frac{m_{\Xi_c^*}-m_{\Xi_c'}}{m_{D_q^*}-m_{D_q}}\,|\Psi_{cq}^{D_q}(0)|^2 R_{cq}^{\Xi_c}\,, \quad \text{for } q=u,d,s\\
|\Psi_{cs}^{\Omega_c^0}(0)|^2 &= \frac{4}{3}\,\frac{m_{\Omega_c^{0*}}-m_{\Omega_c^0}}{m_{D_s^*}-m_{D_s}}\,|\Psi_{cs}^{D_s}(0)|^2 R_{cs}^{\Omega_c}\,,
\end{aligned} \tag{5.14}$$

with the overall scaling coefficients $R_{cq}^{\mathcal{B}}$, such that $R_{cq}^{\mathcal{B}}=y$, for $q=u,d$ and $R_{cs}^{\mathcal{B}}=y_s$, with the values in (5.13). Note that we also consider SU(3)$_F$-breaking in the $|\psi_{cq}^{\Xi_c}(0)|^2$ wavefunction. The relations above are taken to be valid at a low hadronic scale μ_H, taken to be of order 1 GeV.

The remaining question is how to treat the meson wavefunction in (5.14). The nonrelativistic D-meson wavefunction is given by[17]

$$|\Psi_{cq}^{D_q}(0)|^2 = \frac{1}{12} f_{D_q}^2 m_{D_q}\,, \tag{5.15}$$

in terms of the mesonic decay constant f_{D_q}. But, one has to keep in mind that the mesonic decay constant has its own $1/m_c$ expansion [39, 87, 144]. In [80], it was suggested to use the meson wavefunction defined in terms of the static decay constant F_{D_q}, for consistency within the HQE, while the hadron mass differences in (5.14) attain their static rather than physical values. The idea is that the renormalization of F_{D_q} to f_{D_q} in the dimension-six matrix elements by the non-local dimension-seven contributions, observed in mesons and presented in appendix C, would also occur for the Ω_c^0, owing to the spin structure of its constituent s-quarks binded in a spin 1 diquark, but would not occur in the antitriplet of baryons (Λ_c^+, Ξ_c^+, Ξ_c^0). This conjecture led also to the suggestion that Ω_c^0 might be far longer-lived than was measured to be the case at the time of [80]. In light of the new LHCb measurements increasing the Ω_c^0 lifetime, this conjecture deserves further attention. A preliminary numerical test does suggest that predictions for the Λ_c^+ and Ξ_c^0 lifetimes are brought closer to their experimental values, at the expense of a worse prediction for $\tau\,(\Xi_c^+)$, but any more concrete analysis at higher orders in the $1/m_c$ expansion will also require considering the non-local matrix elements, about which nothing is known for baryons. For

[17] In general, four-quark operators are (re)normalized at the heavy quark scale. Their evolution from m_c down to a hadronic scale $\mu_{\rm had}\sim O(0.7-1\,\text{GeV})$ scale brings hybrid renormalization into account [85, 142, 143], usually denoted by $\kappa(\mu)=(\alpha_s(\mu)/\alpha_s(m_c))^{3N_C/2\beta_0}$, and the factor is sometimes in the literature explicitly associated with the wavefunctions as $|\Psi_{cq}^{D_q}(0)|^2=\frac{1}{12}f_{D_q}^2 m_{D_q}\kappa^{-4/9}$. Here, we perform explicit HQET and QCD matching using eqs. (2.31) and (2.32), so that the hybrid anomalous dimension is already included in the operators.

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this reason, we prefer to restrict to considering only the QCD matrix elements, for which all inputs attain their physical values.

We parametrize the dimension-seven matrix elements in the NRCQM by relating them to those of dimension-six as follows:

$$\begin{aligned}
\frac{\langle \mathcal{T}_c|P_1^q|\mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}} &\simeq m_q \frac{\langle \mathcal{T}_c|O_2^q|\mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}} = \frac{1}{2} m_q |\Psi_{cq}^{\mathcal{T}_c}(0)|^2 \rho_1 \,, \\
\frac{\langle \mathcal{T}_c|P_2^q|\mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}} &\simeq \Lambda_{\rm QCD} \frac{\langle T_c|O_1^q|\mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}} = -\Lambda_{\rm QCD} |\Psi_{cq}^{\mathcal{T}_c}(0)|^2 \rho_2 \,, \\
\frac{\langle \mathcal{T}_c|P_3^q|\mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}} &\simeq \Lambda_{\rm QCD} \frac{\langle \mathcal{T}_c|O_2^q|\mathcal{T}_c\rangle}{2m_{\mathcal{T}_c}} = \frac{1}{2} \Lambda_{\rm QCD} |\Psi_{cq}^{\mathcal{T}_c}(0)|^2 \rho_3 \,,
\end{aligned}$$

$$\begin{aligned}
\frac{\langle \Omega_c^0|P_1^q|\Omega_c^0\rangle}{2m_{\Omega_c^0}} &\simeq m_q \frac{\langle \Omega_c^0|O_2^q|\Omega_c^0\rangle}{2m_{\Omega_c^0}} = -m_q |\Psi_{cq}^{\Omega_c^0}(0)|^2 \rho_1 \,, \\
\frac{\langle \Omega_c^0|P_2^q|\Omega_c^0\rangle}{2m_{\Omega_c^0}} &\simeq \Lambda_{\rm QCD} \frac{\langle \Omega_c^0|O_1^q|\Omega_c^0\rangle}{2m_{\Omega_c^0}} = -6\Lambda_{\rm QCD} |\Psi_{cq}^{\Omega_c^0}(0)|^2 \rho_2 \,, \\
\frac{\langle \Omega_c^0|P_3^q|\Omega_c^0\rangle}{2m_{\Omega_c^0}} &\simeq \Lambda_{\rm QCD} \frac{\langle \Omega_c^0|O_2^q|\Omega_c^0\rangle}{2m_{\Omega_c^0}} = -\Lambda_{\rm QCD} |\Psi_{cq}^{\Omega_c^0}(0)|^2 \rho_3 \,,
\end{aligned} \tag{5.16}$$

where $\mathcal{T}_c = \Lambda_c^+, \Xi_c^+$ and Ξ_c^0 as before, and we expect the parameters ρ_{1-3} to be of order 1. The remaining dimension-seven matrix elements are, analogously to those of dimension-six, related by eq. (2.39), ie $\tilde{P}_i^q(B) = -\tilde{B} P_i^q(B)$, where we will again take $\tilde{B} = 1$ throughout [85]. We note that our expectation for the matrix element of the operator P_1 differs from previous parametrizations in the literature [32, 40, 44]. We keep the explicit scaling with the light quark mass m_q, and use the quark model result for the matrix element $\langle P_1^q \rangle = m_q \langle O_2^q \rangle$, with $m_{u,d}$ set to 0 GeV and $\overline{m}_s(\mu = 1.5\,\text{GeV}) = 0.1\,\text{GeV}$. We also apply the relation $\langle P_{2,3}^q \rangle \simeq p_c \cdot p_q / m_c \langle O_{1,2}^q \rangle$, and estimate that $p_c \cdot p_q \sim m_c \Lambda_{\rm QCD}$. For our central values we use $\Lambda_{\rm QCD} = 0.33\,\text{GeV}$, evaluated for $n_f = 3$ [106]. This again differs from previous parametrizations [32, 40, 44], but the resulting numerical difference between our parametrization of $P_{2,3}$ and that in previous literature is not much more than 20%, and so falls within the range of uncertainties due to the wavefunctions in (5.14).

Note, however, that there is currently no first-principles evaluation of dimension-seven four-quark matrix elements for baryons, and further scrutiny of this approach, and that in [32, 40, 44], will be needed in the future, especially in view of the importance of such contributions in inclusive decay widths.

We finally evaluate the matrix elements of the Darwin operator, using (3.14) and (2.36) with the matrix elements expressed in terms of baryon wavefunctions in the NRCQM. Using the relations in (2.39) with the fixed value $\tilde{B} = 1$, we obtain

$$2m_{\mathcal{B}_c} \rho_D^3(\mathcal{B}_c) = g_s^2 \langle \mathcal{B}_c| -\frac{1}{6} O_1^q + \frac{1}{3} O_2^q |\mathcal{B}_c\rangle + \mathcal{O}(1/m_c) \,, \tag{5.17}$$

where, with respect to (3.14), we have expressed the result only in terms of $O_{1,2}^q$. Using the relations in table 13, the wavefunctions from (5.14), and the value $g_s^2 \equiv 4\pi\alpha_s = 4\pi$, leads

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to the values, already presented in table 7,

$$\begin{aligned} \rho_D\left(\Lambda_c^+\right) &= 0.04\,(1)\ \text{GeV}^3\,, \qquad \rho_D\left(\Xi_c^+\right) = 0.05\,(2)\ \text{GeV}^3\,, \\ \rho_D\left(\Xi_c^0\right) &= 0.06\,(2)\ \text{GeV}^3\,, \qquad \rho_D\left(\Omega_c^0\right) = 0.06(2)\,\text{GeV}^3\,, \end{aligned} \tag{5.18}$$

including the uncertainties, which we conservatively set to 30%.

5.2 Final numerical predictions for baryons

We present predictions for the following baryon observables:

(i) the lifetimes of each baryon, $\tau\left(\mathcal{B}_c\right)$;

(ii) ratios compared with the experimental Λ_c^+ lifetime, defined as

$$\frac{\tau\left(\mathcal{B}_c\right)}{\tau\left(\Lambda_c^+\right)} \equiv \frac{1}{1+\left(\Gamma^{\text{th}}\left(\mathcal{B}_c\right)-\Gamma^{\text{th}}\left(\Lambda_c^+\right)\right)\tau^{\text{exp}}\left(\Lambda_c^+\right)}\,; \tag{5.19}$$

(iii) inclusive semileptonic branching fractions involving the electrons in the final states, defined as

$$BR(\mathcal{B}_c \to X e\nu) \equiv \Gamma(\mathcal{B}_c \to X e\nu)\,\tau^{\text{exp}}(\mathcal{B}_c)\,. \tag{5.20}$$

As in the case of mesons, ratios are defined via the differences of the theoretical widths, which results in cancellations of the universal non-spectator terms, leading to reduction of theoretical uncertainties. Our final predictions are presented in table 14, while central values of individual contributions are given in tables 24 and 25, serving as an illustration of their relative sizes. In figure 5 we show a comparison of all our predictions in singly charm baryon sector, normalized to the corresponding experimental central values, similar to the one for charmed mesons in [39].

Central values are obtained using the NRCQM expressions for the baryon wavefunctions given in eq. (5.14), with the remaining hadronic parameters given in table 7. The corresponding uncertainties are estimated by allowing for 30% variations around these values. The matrix elements of the dimension-seven operators involve the overall scaling coefficients ρ_i, which we set to 1, so that all hadronic uncertainties from the dimension-seven contribution follow from the corresponding uncertainties of the wavefunctions.

Our values of baryon lifetimes turn out consistent, in all mass schemes and within sizable theoretical uncertainties, with experimental measurements. The preferred value for $\tau\left(\Omega_c^0\right)$ is larger than previous theoretical estimates [13, 14], and favours the most recent LHCb results [1, 3]. However, our central value for the lifetime of Λ_c^+ is 50% larger than the measured value, and we similarly overestimate the lifetime of Ξ_c^0, although in both cases the measured lifetimes fall within our estimate of theoretical uncertainties. On the other hand, we observe a tension in the lifetime ratios $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ and $\tau(\Omega_c^0)/\tau(\Lambda_c^+)$, both of which are smaller than the corresponding experimental values, which can be attributed to our larger-than-measured lifetime prediction of Λ_c^+. We nevertheless can accommodate the newly-established hierarchy of experimental lifetimes

$$\tau\left(\Xi_c^0\right) < \tau\left(\Lambda_c^+\right) < \tau\left(\Omega_c^0\right) < \tau\left(\Xi_c^+\right)\,, \tag{5.21}$$

although our results do not rule out $\tau(\Xi_c^0)/\tau(\Lambda_c^+) > 1$ or $\tau(\Omega_c^0)/\tau(\Lambda_c^+) < 1$ with certainty.

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Observable	Pole	$\overline{\text{MS}}$	Kinetic	MSR	Experiment
$\tau\left(\Lambda_c^+\right)/10^{-13}\text{s}$	$3.04^{+0.72+0.78}_{-0.51-0.62}$	$3.50^{+0.87+1.26}_{-0.61-0.95}$	$3.00^{+0.68+0.95}_{-0.49-0.74}$	$3.12^{+0.73+1.05}_{-0.52-0.81}$	2.02 ± 0.03
$\tau\left(\Xi_c^+\right)/10^{-13}\text{s}$	$4.25^{+0.79+0.93}_{-0.63-0.78}$	$4.82^{+0.97+1.38}_{-0.76-1.22}$	$4.03^{+0.73+1.16}_{-0.59-0.93}$	$4.21^{+0.78+1.28}_{-0.63-1.02}$	4.56 ± 0.05
$\tau\left(\Xi_c^0\right)/10^{-13}\text{s}$	$2.31^{+0.66+0.52}_{-0.43-0.41}$	$2.50^{+0.75+0.82}_{-0.49-0.63}$	$2.21^{+0.62+0.63}_{-0.41-0.50}$	$2.28^{+0.65+0.69}_{-0.43-0.54}$	1.52 ± 0.02
$\tau\left(\Omega_c^0\right)/10^{-13}\text{s}$	$2.59^{+0.72+0.73}_{-0.49-0.50}$	$2.62^{+0.79+1.05}_{-0.52-0.70}$	$2.33^{+0.66+0.81}_{-0.44-0.56}$	$2.37^{+0.68+0.88}_{-0.45-0.60}$	2.74 ± 0.12
$\tau\left(\Xi_c^+\right)/\tau\left(\Lambda_c^+\right)$	$1.23^{+0.24+0.10}_{-0.14-0.07}$	$1.19^{+0.20+0.11}_{-0.12-0.07}$	$1.21^{+0.23+0.11}_{-0.14-0.07}$	$1.20^{+0.22+0.12}_{-0.13-0.07}$	2.25 ± 0.04
$\tau\left(\Xi_c^0\right)/\tau\left(\Lambda_c^+\right)$	$0.83^{+0.16+0.02}_{-0.17-0.01}$	$0.81^{+0.15+0.03}_{-0.16-0.03}$	$0.81^{+0.16+0.03}_{-0.17-0.03}$	$0.81^{+0.29+0.03}_{-0.17-0.03}$	0.75 ± 0.02
$\tau\left(\Omega_c^0\right)/\tau\left(\Lambda_c^+\right)$	$0.90^{+0.34+0.03}_{-0.19-0.03}$	$0.84^{+0.28+0.05}_{-0.17-0.04}$	$0.84^{+0.31+0.05}_{-0.18-0.04}$	$0.83^{+0.30+0.05}_{-0.18-0.04}$	1.36 ± 0.06
$BR(\Lambda_c^+ \to Xe\nu)/\%$	$3.80^{+0.49+0.31}_{-0.39-0.41}$	$3.71^{+0.45+0.47}_{-0.35-0.36}$	$4.42^{+0.48+0.25}_{-0.38-0.20}$	$4.28^{+0.47+0.39}_{-0.37-0.30}$	3.95 ± 0.35
$BR(\Xi_c^+ \to Xe\nu)/\%$	$12.74^{+2.51+0.38}_{-2.24-1.00}$	$13.46^{+2.70+1.89}_{-2.42-1.69}$	$15.20^{+2.80+1.12}_{-2.52-1.17}$	$14.95^{+2.66+1.59}_{-2.45-1.50}$	not measured
$BR(\Xi_c^0 \to Xe\nu)/\%$	$4.31^{+0.86+0.12}_{-0.77-0.33}$	$4.56^{+0.92+0.64}_{-0.83-0.58}$	$5.13^{+0.96+0.38}_{-0.86-0.40}$	$5.06^{+0.91+0.54}_{-0.84-0.51}$	not measured
$BR(\Omega_c^0 \to Xe\nu)/\%$	$7.59^{+2.49+0.00}_{-2.23-0.21}$	$10.40^{+2.98+2.28}_{-2.71-2.30}$	$10.93^{+3.07+1.53}_{-2.81-1.75}$	$11.19^{+3.01+1.94}_{-2.89-2.09}$	not measured

Table 14. Results for baryons in different mass schemes, including the lifetimes, lifetime ratios compared to the Λ_c^+, and semileptonic branching fractions. The lifetime ratios are determined using eq. (5.19), and the semileptonic branching fractions using eq. (5.20). The first and second errors correspond to hadronic and renormalization scale uncertainties, respectively. As for mesons, central values correspond to the scale choice $\mu = \mu_0 = 1.5\,\text{GeV}$, while the scale uncertainties are estimated for fixed values of the hadronic parameters by varying the scale μ in the range $[1, 3]\,\text{GeV}$.

Our value for the semileptonic branching fraction $BR(\Lambda_c^+ \to Xev)$ is consistent with experiment. We also give predictions for the semileptonic branching fractions of the remaining baryons, which are yet to be measured experimentally.

As shown in tables 24 and 25, NLO contributions improve agreement of the Λ_c^+ and Ξ_c^+ lifetimes with experiment,[18] and moreover help to stabilize the dependence of our results on the mass scheme, which would otherwise be significant if only the LO results were used in predicting lifetimes. It can be expected that higher order α_s contributions, with the caveat that the question of convergence of the α_s series beyond NLO must be addressed in charm decays, could improve the agreement still further. Likewise, the missing NLO corrections of dimension-seven spectator contributions might be important for improving this picture.

Tables 24 and 25 also show the relative sizes of different contributions to the decay widths. It can be seen, for example, that the Λ_c^+ and Ξ_c^0 widths receive large contributions due to weak exchange, $\tilde{\Gamma}_{6,\text{exc}}$. If this contribution were to be enhanced by some unknown mechanism, then it is possible that the Λ_c^+ and Ξ_c^0 lifetimes would be more consistent with their measured values. It is noteworthy that this contribution is related, by comparing figures 2 and figures 3, to the PI contribution to the D^+ decay width, and appears at the same level in the CKM hierarchy. In that case, as noted in section 4.2, the large PI contribution drove the width towards unphysical negative values. It could be speculated

[18] There was an earlier concern by experimentalists that the $\Omega_c^0 \to \Xi_c^+\pi^-$ decay might generate a substantial systematic error in measurements of $\tau\left(\Xi_c^+\right)$ and enhance the lifetime, which was theoretically not confirmed in [145].

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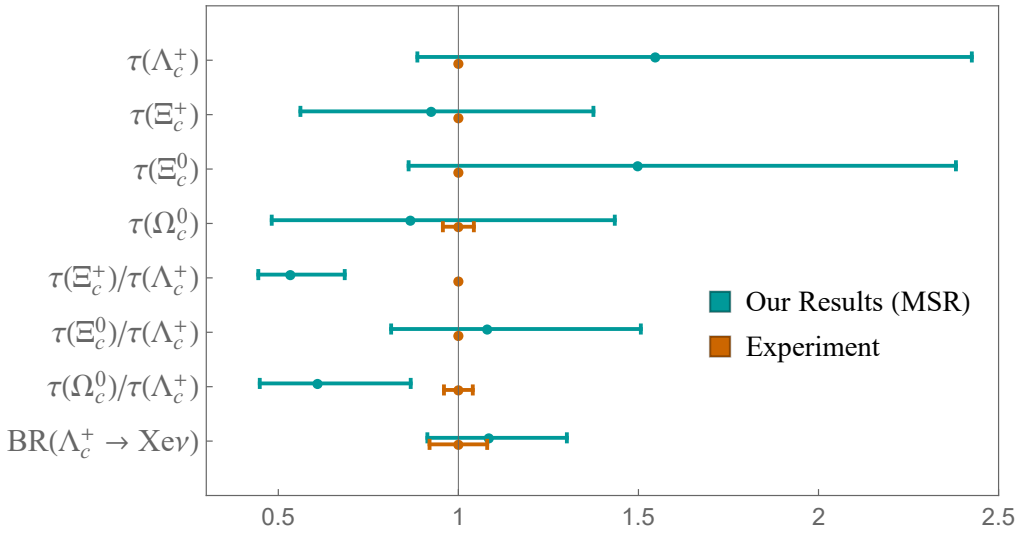

Figure 5. Predictions for observables in the singly-charmed baryon sector, normalized to the corresponding experimental values, in the MSR scheme. Experimental values (lower of each pair) are in orange, with uncertainties provided when they are larger than $\sim 1\%$. Our predictions are in bluegreen, with the uncertainties, from table 14, added linearly. Tensions in the ratios $\tau\left(\Xi_c^+\right)/\tau\left(\Lambda_c^+\right)$ and $\tau\left(\Omega_c^0\right)/\tau\left(\Lambda_c^+\right)$ are clearly visible, whereas predictions for the remaining observables are compatible with experimental values within uncertainties. A similar figure for meson observables is provided in figure 7 of [39].

that better theoretical control of the contributions arising from these topologies would alleviate tensions in the charmed hadron lifetimes, but such a simple resolution, without affecting other contributions, seems implausible.

We now compare our results to the most recent previous study of singly charmed baryon lifetimes, which was performed in [40] and subsequently reviewed in [146, 147]. Firstly, we should stress that in these calculations, several contributions, that we have found to be numerically significant, were not included. The Darwin term was, at the time, unavailable, being first computed for charm decays in [39], but represents a $\sim 20\%$ enhancement of the non-spectator contributions at LO. The author of [40] also chose to neglect NLO-QCD contributions, to both the dimension-three and -six contributions, both of which we find to be significant: the available NLO contributions improve the fit in all cases, as well as reducing dependence of results on the mass scheme.

In addition, the author of [40] enhanced wavefunctions of charmed baryons by an arbitrary coefficient $y = 1.75$, presumably with the intention of bringing lifetimes of antitriplet baryons into agreement with experiment. After then obtaining a large lifetime for Ω_c^0, and a negative semileptonic decay rate $\Gamma^{\mathrm{SL}}(\Omega_c^0)$, a second arbitrary factor, designed to suppress large and negative $\Gamma_{7,\mathrm{int}^+}$ contributions, was introduced. Our results show, however, that the predicted lifetime of Ω_c^0 is compatible with the most recent experimental value without any need for such arbitrary factors. This is true whether or not the Darwin and α_s contributions are accounted for in the decay width, and may therefore be traced to our different parametrization of the P_1 matrix element (cf. discussion after eq. (5.16)) as

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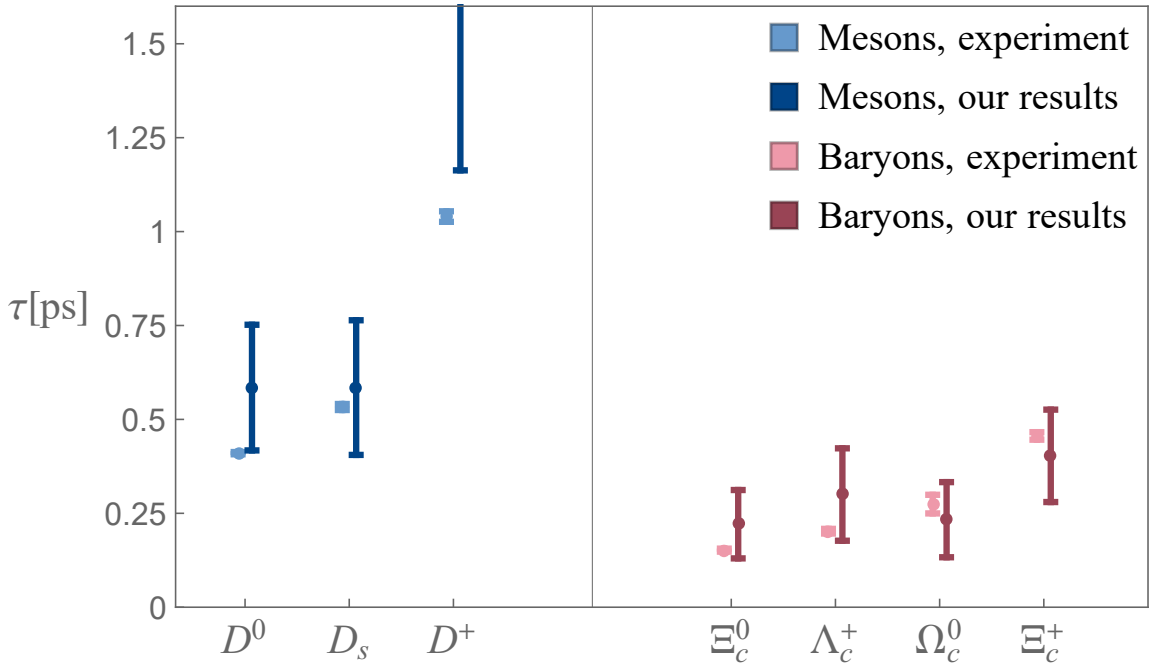

Figure 6. Hierarchy of lifetimes, in ps, of charmed mesons (left, in blue) and singly charmed baryons (right, in red). Experimental values, including the latest LHCb results for Ω_c^0 and Ξ_c^0 [3, 21], are on the left of each pair of values; our predictions, using the kinetic scheme, are on the right.

compared with [32, 40, 44].[19] That we have been able to accommodate the new Ω_c^0 lifetime without needing to introduce any such arbitrary factors lends support to our approach. We therefore do not find evidence for the claim advanced by [146, 147] that the HQE fails to apply for Ω_c^0. Further consideration of the dimension-seven matrix elements, and beyond to higher-order terms in the $1/m_c$ expansion, will be necessary in order to settle the question.

6 Conclusions and outlook

In this work, we have predicted the lifetimes of singly charmed hadrons, with the main results presented in tables 10 and 11 for mesons, and table 14 for baryons. In particular, the results for baryons include the most complete set of contributions to date, and in this sense supersede previous theoretical predictions in [13, 40]. A summary of our predictions, illustrating the lifetime hierarchies among charmed mesons and singly charmed baryons, is exhibited in figure 6.

[19] In [40] it was argued that the factor $(m_{\Lambda_c^+}^2 - m_{[ud]}^2)/m_c^2 - 1$, where $m_{[ud]}$ is the effective mass of the light diquark pair in an antisymmetric spin state, is of order m_q/m_c, which was used to justify the choice of parametrization of the P_1 matrix element made in [40]. However, for the given value of $m_{[ud]}$ [148] we find $((m_{\Lambda_c^+}^2 - m_{[ud]}^2)/m_c^2 - 1) \sim 1.1$, which is no longer of order m_q/m_c. It is perhaps relevant that the first analysis of the dimension-seven corrections [32, 44], on which the approach in [40] was based, focused on the b sector. In inclusive b hadron decays, a potential overestimate of the size of the matrix elements $P_{1,2,3}$ is more tolerable in view of the $1/m_b^3$ suppression of all four-quark contributions.

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While we agree with the recent results for charm meson observables in [39], we generally disagree with the results and conclusions of the analysis of inclusive singly charmed hadron decays from [40, 146, 147]. This arises from several considerations. Firstly, the analysis therein focused on the LO contributions (with the exception of the, at the time unavailable, Darwin contribution, which is itself sizable), but it is apparent from our results (cf. the tables in appendix D) that the NLO contributions to inclusive charm decays are large, and therefore cannot be neglected. Secondly, the analysis in [40] did not present uncertainty estimates. In our analysis, however, supported by [39], the uncertainties of the results are large, so that no conclusions should be drawn merely by focusing on the central values obtained for a given choice of input parameters. Thirdly, the study in [40] made use of arbitrary parameters, in baryon lifetime predictions particularly, in order to compensate for missing contributions and with an eye on reproducing the experimental results. In light of the previous two points, this cannot be justified. Furthermore, we disagree with the claim made in [40, 146, 147] that the HQE is not applicable to Ω_c^0. Although the applicability of the HQE in charm decays remains unclear, there is no indication in our results for such a strong statement only in inclusive Ω_c^0 decays.

A further reason that neglecting NLO contributions, as was done in [40], is insufficient to predict inclusive charm decays reliably is that these contributions significantly stabilize results between different mass schemes. By contrast, there is a large dependence on the choice of charm mass scheme at LO, as the tables in appendix D show. However, the pole mass scheme exhibits huge α_s corrections, with signs that the picture may be even worse at higher orders in the α_s expansion. It is reasonable to expect a similar pattern in the full NLO results, in particular the dimension-seven spectator contributions, supporting the long-standing argument that the pole mass should not be used for reliable predictions in inclusive charm decays. It will be important to examine up to at least NNLO contributions, consistently across both nonleptonic and semileptonic channels, in order to gain a more complete picture. This includes the missing NLO contributions to the dimension-five and -six two-quark contributions. Taken together, these could help to understand how best to address the issues surrounding the convergence of the α_s expansion, most prominently visible in the leading dimension-three term to semileptonic decays [73].

Comparing our results to experimental data, we see some signs of slight tension. For mesons, the decay width of D^+ can be driven to negative values by a sizable Pauli interference contribution, and we find the ratio $\tau(D_s)/\tau(D^0)$ to be closer to unity than in experiment; both observations were also made in [39]. For baryons, while our predictions for the lifetimes are *individually* compatible with experimental measurements, some of the ratios, particularly that of $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$, are not. Such tension can be largely attributed to our central value of the Λ_c^+ lifetime being an overestimate compared with experiment. Nevertheless, the picture elsewhere, with notably the Ω_c^0 lifetime prediction being compatible with the new LHCb results [1, 3], suggests that the application of the HQE to inclusive charm decays remains plausible. In view of the large hadronic uncertainties, chiefly arising from spectator contributions, it is difficult to reach any firm conclusions on this point. Alternative approaches to arranging the HQE for charm, such as those presented in [37, 38], are also worth considering for the future, with in particular [38] likely to be relevant in addressing the issue of a slowly-converging series of four-quark operators.

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Since spectator contributions are present in semileptonic decays, one could be tempted to try to extract the main ingredient of these contributions, the hadron wavefunctions, from precisely-measured semileptonic decay rates. Unfortunately, due to large uncertainties present in other matrix elements, this seems not to be possible at the moment, but the situation can improve once the non-spectator matrix elements and their higher-order contributions are known with larger precision. Lattice computations of all relevant matrix elements, both for baryons and mesons, with the latter complementing the HQET estimates in [83], could serve to address this. On the other hand, without any control over the uncertainties of such matrix elements, merely going to higher orders in the $1/m_c$ expansion is unlikely to improve the theoretical predictions meaningfully.

Finally, one should not forget that in charmed hadron decays there is a possibility of quark-hadron duality violation, one of the crucial elements in the calculation of the inclusive widths within the HQE, which goes beyond the operator product expansion. This could bring non-negligible contributions $\sim e^{-m_c/\mu_{\rm hadr}}$ that could be in particular notable in semileptonic decays, as discussed in [149–151] for example, and recently analysed in the context of heavy meson decays in [152]. More generally, it could be that the charm quark mass is not heavy enough for duality to set in. In any case, the concept of duality is worthy of further investigation.

We urge for a new, independent measurement of the Ω_c^0 lifetime, to be compared with the surprisingly large LHCb result,[20] which radically changed the long-standing lifetime hierarchy of singly charmed baryons. Measurements of semileptonic branching fractions of other baryons, apart from Λ_c^+, would likewise be more than welcome.

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[20] There is a dedicated group at Belle II working on an independent determination of $\tau\left(\Omega_c^0\right)$, and it will be interesting to see their results soon [153].

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Parameter	Numerical value
G_F	$1.1663787(6) \cdot 10^{-5}$ GeV^{-2}
$\overline{m}_s(2\,\text{GeV})$	0.093 GeV
$\overline{m}_c(\overline{m}_c)$ [104, 105]	1.280(13) GeV
$\overline{m}_b(\overline{m}_b)$ [104, 105]	4.198(12) GeV
$\alpha_s(m_Z)$	0.1180(7)
m_Z	91.1876 GeV
m_μ	0.105658 GeV

Table 15. Values of input parameters used in the numerical analysis. Uncertainties in the final digit(s), which are neglected in this study, are given in brackets. The value of $\overline{m}_c(\overline{m}_c)$ is the average by the Flavour Lattice Averaging Group (FLAG) [104, 105] of the lattice QCD results in [93–97]. The value of $\overline{m}_b(\overline{m}_b)$ is the FLAG average of the lattice QCD results in [154–156].

	$D^\pm$	D^0	D_s
m_M	1.86966(5)	1.86484(5)	1.96835(7)
f_M	0.2120(7)	0.2120(7)	0.2499(5)

Table 16. Masses and decay constants of D mesons in GeV, from the latest PDG [21] and FLAG [104, 157, 158] values. Uncertainties in the final digit(s), which are neglected in this study as they are dominated by other effects, are given in brackets.

	Λ_c^+	Ξ_c^+	Ξ_c^0	Ω_c^0
m_H	2.28646(14)	2.46771(23)	2.47044(28)	2.6952(17)

Table 17. Masses of singly charmed baryons, in GeV, taken from PDG [21]. Uncertainties in the final digit(s) are neglected in this study, but are given in brackets.

A Numerical inputs

In this section we collect the numerical inputs used in determining the lifetime values. Values of the inputs are taken from PDG [21] except where stated.

Table 15 contains the input values of quark (in the $\overline{\text{MS}}$ scheme) and lepton masses, alongside input parameters relevant for α_s running. Tables 16 and 17 contain the masses of mesons and baryons respectively, alongside the meson decay constants. Tables 18 and 19 contain resonance masses useful for computing μ_G^2 (see section 3.1) and μ_π^2 (see section 3.2).

The magnitudes of the CKM parameters are [21]

$$|V_{\text{CKM}}| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97401 & 0.22650 & 0.00361 \\ 0.22636 & 0.97320 & 0.04053 \\ 0.00854 & 0.03978 & 0.999172 \end{pmatrix}, \tag{A.1}$$

where we have neglected the corresponding uncertainties, which are negligible relative to other uncertainties in this paper.

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	D_0^*	$D_\pm^*$	D_s^*	B^*	B_s^*
m_{M^*}	2.00685(5)	2.01026(5)	2.1122(4)	5.32470(21)	$5.4154^{+0.0018}_{-0.0015}$

Table 18. Masses of the excited mesons in GeV, relevant for spectroscopic determinations, taken from PDG values [21]. Uncertainties in the final digit(s) are neglected in this study, but are given in brackets, except for B_s^* which currently has asymmetric uncertainty.

	Ω_c^*	Σ_c	Σ_c^*	Ξ_c'	Ξ_c^*
m_H	2.7659(20)	2.4529(4)	2.5175(23)	2.5782(5)	2.64616(25)

Table 19. Masses, in GeV, of (excited) baryon states relevant for determining the baryon wavefunctions in section 5.1 using the representation in eq. (5.14) [141], taken from PDG [21]. Uncertainties in the final digit(s) are neglected in this study, but are given in brackets.

Scale	$\alpha_s(\mu)$ (two-loop)	$\alpha_s(\mu)$ (five-loop)
$\mu = 1.28\,\text{GeV}$	0.371	0.385
$\mu = 1.50\,\text{GeV}$	0.340	0.349
$\mu = 3.0\,\text{GeV}$	0.251	0.253

Table 20. Numerical values of $\alpha_s(\mu)$ evaluated with two-loop and five-loop running. For our numerical evaluations we use the values obtained with five-loop running. We used the initial value of the strong coupling constant $\alpha_s^{(5)}(m_Z = 91.1876\,\text{GeV}) = 0.1180$. See the text for further details.

Scale	$C_1^{\text{LO}}(\mu)$	$C_2^{\text{LO}}(\mu)$	$C_1^{\text{NLO}}(\mu)$	$C_2^{\text{NLO}}(\mu)$
$\mu = 1.28\,\text{GeV}$	-0.52	1.27	-0.40	1.20
$\mu = 1.50\,\text{GeV}$	-0.47	1.24	-0.36	1.18
$\mu = 3.0\,\text{GeV}$	-0.32	1.15	-0.24	1.10

Table 21. Numerical values of the Wilson coefficients $C_{1,2}$ at the leading order (LO) and next-to-leading-order (NLO) evaluated with the five-loop running of $\alpha_s(\mu)$.

A.1 α_s and Wilson coefficients

For the evaluation of the two-loop running of the strong coupling constant α_s we use the function `AsRunDec` from the version 3.1 of the software package `RunDec` [106, 107]. This function automatically performs the flavour decoupling across the b- and c quark thresholds at the default values $\mu_c = 1.5\,\text{GeV}$ and $\mu_b = 4.8\,\text{GeV}$. We used $\alpha_s(M_Z) = 0.1180$. For easier future comparisons, we list the values of $\alpha_s(\mu)$ for few reference values of the scale in table 20. For our evaluations we used the five-loop values. The coefficients of the α_s expansion of eq. (2.14) depend on the Wilson coefficients $C_{1,2}$. In the case of a specific contribution for which the expansion in α_s is known beyond the leading order, we use the $C_{1,2}^{\text{NLO}}(\mu)$-values for the evaluation of the leading coefficient $\mathcal{C}_n^{(0)}$, and the $C_{1,2}^{\text{LO}}(\mu)$-values for evaluation of the next-to-leading coefficient $\mathcal{C}_n^{(1)}$. However, in the case that only the leading order coefficient $\mathcal{C}_n^{(0)}$ is known, we employ the LO results $C_{1,2}^{\text{LO}}(\mu)$. We illustrate the running of $C_{1,2}$ for few reference renormalization scale points in table 21.

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B Analytic forms for leading order coefficients in the HQE

Here we compile analytic expressions for the coefficients of the contributions to the inclusive decay width (2.13), up to leading order.

B.1 Non-spectator contributions

Recall from (2.22) that the LO coefficients c_i have the general form

$$c_n^{(0)} = N_C C_1^2 \mathcal{K}_{n,11}^{(0)} + 2C_1 C_2 \mathcal{K}_{n,12}^{(0)} + N_C C_2^2 \mathcal{K}_{n,22}^{(0)} + \mathcal{K}_{n,\mathrm{SL}}^{(0)}\,, \tag{B.1}$$

where the $\mathcal{K}_{n,ij}$ can then be written in terms of phase space functions. For $c_{3,\pi,G}$, the relevant functions appearing in (2.24) and (2.25) are $I_{0,1,2}(x,y,z)$, where, explicitly [17, 18, 40, 56–59],

$$\begin{aligned}
&I_0\left(0,0,0\right) = 1\,,\\
&I_0\left(x,0,0\right) = I_0\left(0,x,0\right) = I_0\left(0,0,x\right) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x\,,\\
&I_0\left(x,x,0\right) = \sqrt{1-4x}\left(1 - 14x - 2x^2 - 12x^3\right) + 24x^2\left(1-x^2\right)\ln\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\,,\\
&I_0\left(x,y,0\right) = \sqrt{\lambda}\left(1 - 7\left(x+y\right) - 7\left(x^2+y^2\right) + x^3 + y^3 + xy\left(12 - 7\left(x+y\right)\right)\right)\\
&\qquad\quad + 12x^2\left(1-y^2\right)\ln\frac{1+v_x}{1-v_x} + 12y^2\left(1-x^2\right)\ln\frac{1+v_y}{1-v_y}\,,
\end{aligned} \tag{B.2}$$

where $\lambda \equiv \lambda(1,x,y)$ is the Källén function, and v_x, v_y can be interpreted as the maximal velocities of particles x and y. They are given by

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab+bc+ca)\,, \quad v_x = \frac{\sqrt{\lambda}}{1+x-y}\,, \quad v_y = \frac{\sqrt{\lambda}}{1+y-x}\,. \tag{B.3}$$

In the limit $y = x$, then $v_x = v_y = \sqrt{1-4x}$. The function I_1 is related to I_0 by

$$\begin{aligned}
I_1(x,y,0) &= \frac{1}{2}\left(2 - x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}\right) I_0(x,y,0)\,,\\
I_1(x,x,0) &= \frac{1}{2}\left(2 - x\frac{d}{dx}\right) I_0(x,x,0)\,,\\
I_1(x,0,0) &= \frac{1}{2}\left(2 - x\frac{d}{dx}\right) I_0(x,0,0)\,,
\end{aligned} \tag{B.4}$$

whereas I_2 is new:

$$\begin{aligned}
&I_2\left(x,0,0\right) = \left(1-x\right)^3\,,\\
&I_2\left(x,x,0\right) = \sqrt{1-4x}\left(1 + \frac{1}{2}x + 3x^2\right) - 3x\left(1-2x^2\right)\ln\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\,.
\end{aligned} \tag{B.5}$$

To the best of our knowledge, no explicit form for $I_2(x,y,0)$ is available, but fortunately for charmed hadrons the contribution never arises, so the explicit forms above suffice in computing the decay width.

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For the Darwin term, we give the results of [39], translated to our notation and conventions. Starting again from (2.22), we have

$$\begin{aligned}\mathcal{K}^{(0)}_{\rho,ij} &= \sum_{q_1,q_2=d,s} |V_{cq_1}V_{uq_2}|^2 \mathcal{K}^{(q_1\bar{q}_2)}_{\rho,ij}\,,\\ \mathcal{K}^{(0)}_{\rho,\mathrm{SL}} &= |V_{cs}|^2\left(\mathcal{K}^{(se)}_{\rho}+\mathcal{K}^{(s\mu)}_{\rho}\right)+|V_{cd}|^2\left(\mathcal{K}^{(de)}_{\rho}+\mathcal{K}^{(d\mu)}_{\rho}\right)\,.\end{aligned} \tag{B.6}$$

Note that the Darwin operator mixes with the four-quark operators under operator renormalization [37, 43]. In the results below, this gives rise to terms proportional to $\log \mu_0^2/m_c^2$, where μ_0 is the renormalization scale for the operators. In our results, we will take $\mu_0 = 1.5\,\mathrm{GeV}$ throughout.

The LO coefficients for nonleptonic decays are [39]

$$\mathcal{K}^{(d\bar{d})}_{\rho,11} = \mathcal{K}^{(d\bar{d})}_{\rho,22} = 6+8\ln\frac{\mu_0^2}{m_c^2}\,, \qquad \mathcal{K}^{(d\bar{d})}_{\rho,12} = -\frac{34}{3}\,, \tag{B.7}$$

$$\begin{aligned}\mathcal{K}^{(d\bar{s})}_{\rho,11} &= \mathcal{K}^{(d\bar{s})}_{\rho,22} = \frac{2}{3}(1-x_s)\left[9+11x_s-25x_s^2+5x_s^3-12x_s^2\ln x_s-24\left(1-x_s^2\right)\ln(1-x_s)\right]\\ &\quad +8(1-x_s)\left(1-x_s^2\right)\ln\frac{\mu_0^2}{m_c^2}\,,\\ \mathcal{K}^{(d\bar{s})}_{\rho,12} &= -\frac{2}{3}\left[17-26x_s+18x_s^2-38x_s^3+5x_s^4+12x_s\left(5+2x_s-2x_s^2\right)\ln x_s\right.\\ &\quad \left.+48(1-x_s)\left(1-x_s^2\right)\ln(1-x_s)\right]-16x_s\left(1+x_s-x_s^2\right)\ln\frac{\mu_0^2}{m_c^2}\,,\end{aligned} \tag{B.8}$$

$$\begin{aligned}\mathcal{K}^{(s\bar{d})}_{\rho,11} &= \frac{2}{3}(1-x_s)\left[9+11x_s-25x_s^2+5x_s^3-12x_s^2\ln x_s-24\left(1-x_s^2\right)\ln(1-x_s)\right]\\ &\quad +8(1-x_s)\left(1-x_s^2\right)\ln\frac{\mu_0^2}{m_c^2}\,,\\ \mathcal{K}^{(s\bar{d})}_{\rho,12} &= -\frac{2}{3}\left[17-50x_s+90x_s^2-54x_s^3+5x_s^4+12x_s^2(3-x_s)\ln x_s-24(1-x_s)^3\ln(1-x_s)\right]\\ &\quad +8x_s\left(3-3x_s+x_s^2\right)\ln\frac{\mu_0^2}{m_c^2}\,,\\ \mathcal{K}^{(s\bar{d})}_{\rho,22} &= \frac{2}{3}\left[9-16x_s-12x_s^2+16x_s^3-5x_s^4\right]+8\ln\frac{\mu_0^2}{m_c^2}\,,\end{aligned} \tag{B.9}$$

$$\begin{aligned}\mathcal{K}^{(s\bar{s})}_{\rho,11} &= \frac{2}{3}\left[\sqrt{1-4x_s}\left(9+22x_s-34x_s^2-60x_s^3+24\ln x_s-24\ln(1-4x_s)\right)\right.\\ &\quad \left.+24\left(1-2x_s-x_s^2-2x_s^3-5x_s^4\right)\ln\frac{1+\sqrt{1-4x_s}}{1-\sqrt{1-4x_s}}\right]+8\sqrt{1-4x_s}\ln\frac{\mu_0^2}{m_c^2}\,,\end{aligned}$$

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$$
\begin{aligned}
\mathcal{K}_{\rho,12}^{(s\bar{s})} &= \frac{2}{3}\Big[\sqrt{1-4x_s}\left(-33+46x_s-106x_s^2-60x_s^3+24\ln x_s-24\ln(1-x_s)\right)\\
&\quad +4(1-x_s)^2(4-x_s)+12(1-x_s)^3\ln x_s+12\Big(3-2x_s+4x_s^2-16x_s^3\\
&\quad -10x_s^4\Big)\ln\frac{1+\sqrt{1-4x_s}}{1-\sqrt{1-4x_s}}\Big]-8\left(1-\sqrt{1-4x_s}-3x_s+x_s^2-x_s^3\right)\ln\frac{\mu_0^2}{m_c^2}\,,\\
\mathcal{K}_{\rho,22}^{(s\bar{s})} &= \frac{2}{3}\Bigg[\sqrt{1-4x_s}\left(17+8x_s-22x_s^2-60x_s^3\right)-4\left(2-3x_s+x_s^3\right)\\
&\quad -12\left(1-x_s-2x_s^2+2x_s^3+10x_s^4\right)\ln\frac{1+\sqrt{1-4x_s}}{1-\sqrt{1-4x_s}}-12(1-x_s)\left(1-x_s^2\right)\ln x_s\Bigg]\\
&\quad +8(1-x_s)\left(1-x_s^2\right)\ln\frac{\mu_0^2}{m_c^2}\,.
\end{aligned}
\tag{B.10}
$$

For semileptonic decays, the LO coefficients are

$$
\begin{aligned}
\mathcal{K}_{\rho}^{(de)} &= 6+8\ln\frac{\mu_0^2}{m_c^2}\,,\\
\mathcal{K}_{\rho}^{(d\mu)} &= \frac{2}{3}(1-x_\mu)\left[9+11x_\mu-25x_\mu^2+5x_\mu^3-12x_\mu^2\ln x_\mu-24(1-x_\mu^2)\ln(1-x_\mu)\right]\\
&\quad +8(1-x_\mu)(1-x_\mu^2)\ln\frac{\mu_0^2}{m_c^2}\,,\\
\mathcal{K}_{\rho}^{(se)} &= \frac{2}{3}\left[9-16x_s-12x_s^2+16x_s^3-5x_s^4\right]+8\ln\frac{\mu_0^2}{m_c^2}\,,\\
\mathcal{K}_{\rho}^{(s\mu)} &= \frac{2}{3}\Bigg[\sqrt{1-4x_s}\left(17+8x_s-22x_s^2-60x_s^3\right)-4\left(2-3x_s+x_s^3\right)\\
&\quad -12\left(1-x_s-2x_s^2+2x_s^3+10x_s^4\right)\ln\frac{1+\sqrt{1-4x_s}}{1-\sqrt{1-4x_s}}-12(1-x_s)(1-x_s^2)\ln x_s\Bigg]\\
&\quad +8(1-x_s)(1-x_s^2)\ln\frac{\mu_0^2}{m_c^2}\,,
\end{aligned}
\tag{B.11}
$$

where the approximation $m_\mu \approx m_s$ has been made in the last expression, as the result for the semileptonic decay $c \to s\mu^+\nu_\mu$ is not yet available. The SL contributions can also be obtained from the NL contributions by the replacement rules $N_C \to 1$, $C_1 \to 0$, $C_2 \to 1$, and $x_s \to x_\mu$ as appropriate.

B.2 Spectator contributions

The leading-order expressions for the four-quark operators, the spectator contributions, are provided below. In many cases these have been known for a long time, but we bring the results into a symmetric form and present results for arbitrary masses in the loop, thus unifying previous presentations.

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The leading-order dimension-six results in the meson basis are (eg [28, 39, 40])

$$
\begin{aligned}
\widehat{\Gamma}^q_{6,\mathrm{WA}}(x_1,x_2) &= \frac{\Gamma_0}{2m_M}\frac{16\pi^2\sqrt{\lambda}}{m_Q^3}\Bigg\{\left[\frac{1}{N_C}\left((x_1-x_2)^2+x_1+x_2-2\right)(C_1+N_CC_2)^2\right]\mathcal{O}_1^q \\
&\quad -\left[\frac{2}{N_C}\left(2(x_1-x_2)^2-x_1-x_2-1\right)(C_1+N_CC_2)^2\right]\mathcal{O}_2^q \\
&\quad +2\left[\left((x_1-x_2)^2+x_1+x_2-2\right)C_1^2\right]\mathcal{T}_1^q \\
&\quad -4\left[\left(2(x_1-x_2)^2-x_1-x_2-1\right)C_1^2\right]\mathcal{T}_2^q\Bigg\}, \\
\widehat{\Gamma}^q_{6,\mathrm{PI}}(x_1,x_2) &= \frac{\Gamma_0}{2m_M}\frac{16\pi^2\sqrt{\lambda}}{m_Q^3}\Bigg\{\left[\frac{6(1-x_1-x_2)}{N_C}\left(C_1^2+2N_CC_1C_2+C_2^2\right)\right]\mathcal{O}_1^q \\
&\quad +\left[4N_C(1-x_1-x_2)\left(C_1^2+C_2^2\right)\right]\mathcal{T}_1^q\Bigg\}, \\
\widehat{\Gamma}^q_{6,\mathrm{WE}}(x_1,x_2) &= \widehat{\Gamma}^{q\prime}_{6,\mathrm{WA}}(x_1,x_2)\big|_{C_1\longleftrightarrow C_2}, \\
\widehat{\Gamma}^{q,\mathrm{SL}}_{6,\mathrm{WA}}(x_\ell,0) &= \widehat{\Gamma}^q_{6,\mathrm{WA}}(x_\ell,0)\big|_{C_1\to 0,C_2\to 1,N_C\to 1},
\end{aligned}
\tag{B.12}
$$

where $\lambda \equiv \lambda(1,x_1,x_2)$ is the Källén function, defined in eq. (B.3), and $x_a = m_a^2/m_c^2$, where $a = 1,2,\ell$, and $m_{q_{1,2}}$ and m_ℓ denote the masses of the light quarks and leptons in the loop, see figure 2. Note that the right-hand side of the WE-WA relation also involves appropriate replacement of the light quark flavour. The above expressions are supplemented with the α_s corrections [28, 30, 31, 45], which are here not shown explicitly, but are taken into account in our evaluations of the lifetimes. The results below are given in the HQET basis of operators; the equivalent expressions in the QCD basis can be recovered by replacing $\mathcal{O}_i^q \to O_i^q$ and $\mathcal{T}_i^q \to T_i^q$.

The equivalent expressions for the dimension-seven contribution are [28, 39, 40]

$$
\begin{aligned}
\widehat{\Gamma}^q_{7,\mathrm{WA}} = \frac{\Gamma_0}{2m_M}\frac{16\pi^2}{m_c^4}\sqrt{\lambda}\frac{(C_1+N_CC_2)^2}{N_C}\Bigg\{&\left[(x_1-x_2)^2+x_1+x_2-2\right]\mathcal{R}_1^q \\
&-2\left[1-2(x_1-x_2)^2+x_1+x_2\right]\left(\mathcal{P}_1^q+\mathcal{P}_1^{q\dagger}-\mathcal{R}_2^q\right) \\
&+\frac{2}{\lambda}\left[(x_1+x_2-1)\left((x_1-x_2)^2+x_1+x_2-2\right)+\lambda\left(2(x_1-x_2)^2+x_1+x_2\right)\right]\mathcal{P}_2^q \\
&+\frac{4}{\lambda}\Bigg[(1-x_1-x_2)\left(\lambda+(x_1-x_2)^2+x_1+x_2-2\right) \\
&+\lambda\left(1+2x_1+2x_2-6(x_1-x_2)^2\right)\Bigg]\mathcal{P}_3^q\Bigg\} \\
&+\left\{P_i^q\to S_i^q, \mathcal{R}_i^q\to\mathcal{U}_i^q, \frac{(C_1+N_CC_2)^2}{N_C}\to 2C_1^2\right\},
\end{aligned}
$$

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$$
\begin{aligned}
\widehat{\Gamma}^q_{7,\mathrm{PI}} &= \frac{\Gamma_0}{2m_M}\frac{16\pi^2\sqrt{\lambda}}{m_Q^4}\frac{C_1^2+2N_CC_1C_2+C_2^2}{N_C}\Bigg\{\left[\frac{12\left((1-x_1-x_2)^2+(x_1+x_2)\,\lambda\right)}{\lambda}\right]P_2^q \\
&\quad +6\,(1-x_1-x_2)\,\mathcal{R}_1^q\Bigg\} \\
&\quad +\left\{\mathcal{P}_2^q\to\mathcal{S}_2^q\,,\mathcal{R}_1^q\to\mathcal{U}_1^q\,,\frac{C_1^2+2N_CC_1C_2+C_2^2}{N_C}\to 2\left(C_1^2+C_2^2\right)\right\}\,, \\
\widehat{\Gamma}^q_{7,\mathrm{WE}} &= \widehat{\Gamma}^{q'}_{7,\mathrm{WA}}\big|_{C_1\longleftrightarrow C_2}\,, \\
\widehat{\Gamma}^{q,\mathrm{SL}}_{7,\mathrm{WA}}(x_\ell,0) &= \widehat{\Gamma}^q_{7,\mathrm{WA}}(x_\ell,0)\big|_{C_1\to 0,C_2\to 1,N_C\to 1}\,,
\end{aligned}
\tag{B.13}
$$

with the same notation as in eq. (B.12) and the (x_1, x_2) dependence suppressed. To switch between the HQET and QCD bases, one replaces the operators $\mathcal{P}_i^q \to P_i^q$ and $\mathcal{R}_i^q \to 0$. Note also that, in the HQET basis, additional non-local matrix elements are generated, but since these are reabsorbed by replacing the static decay constant F_M with the physical decay constant f_M, cf. appendix C and [39, 53, 87, 144], we do not explicitly present them here.

The equivalent expressions to eqs. (B.12) and (B.13) for baryons can be derived by applying the change of basis in (2.38), but are provided explicitly for convenience:

$$
\begin{aligned}
\widehat{\Gamma}^q_{6,\mathrm{int}^+}(x_1,x_2) &= \frac{\Gamma_0}{2m_M}\frac{16\pi^2\sqrt{\lambda}}{m_Q^3}\Bigg\{\left[\left((x_1-x_2)^2+x_1+x_2-2\right)\left(2C_1C_2+N_CC_2^2\right)\right]O_1^q \\
&\quad -\left[2\left(2\,(x_1-x_2)^2-x_1-x_2-1\right)\left(2C_1C_2+N_CC_2^2\right)\right]O_2^q \\
&\quad +\left[\left((x_1-x_2)^2+x_1+x_2-2\right)C_1^2\right]\tilde{O}_1^q \\
&\quad -2\left[\left(2\,(x_1-x_2)^2-x_1-x_2-1\right)C_1^2\right]\tilde{O}_2^q\Bigg\}\,, \\
\widehat{\Gamma}^q_{6,\mathrm{exc}}(x_1,x_2) &= \frac{\Gamma_0}{2m_B}\frac{16\pi^2}{m_Q^3}\left(2N_C\sqrt{\lambda}\,(1-x_1-x_2)\right)\left\{[2C_1C_2]\,O_1^q+\left[C_1^2+C_2^2\right]\tilde{O}_1^q\right\}\,, \\
\widehat{\Gamma}^q_{6,\mathrm{int}^-}(x_1,x_2) &= \widehat{\Gamma}^q_{6,\mathrm{int}^+}(x_1,x_2)\,\big|_{C_1\longleftrightarrow C_2}\,, \\
\widehat{\Gamma}^{q,\mathrm{SL}}_{6,\mathrm{int}^+}(x_\ell,0) &= \widehat{\Gamma}^q_{6,\mathrm{int}^+}(x_\ell,0)\,\big|_{C_1\to 0,C_2\to 1,N_C\to 1}\,,
\end{aligned}
\tag{B.14}
$$

and for dimension-seven,

$$
\begin{aligned}
\widehat{\Gamma}^q_{7,\mathrm{int}^+} &= \frac{\Gamma_0}{2m_M}\frac{16\pi^2\sqrt{\lambda}}{m_Q^4}\left(2C_1C_2+N_CC_2^2\right)\Bigg\{2\left[2\,(x_1-x_2)^2-x_1-x_2-1\right]\left(P_1^q+P_1^{q\dagger}\right) \\
&\quad +\frac{2}{\lambda}\left[(x_1+x_2-1)\left((x_1-x_2)^2+x_1+x_2-2\right)+\lambda\left(2\,(x_1-x_2)^2+x_1+x_2\right)\right]P_2^q \\
&\quad +\frac{4}{\lambda}\Big[(1-x_1-x_2)\left(\lambda+(x_1-x_2)^2+x_1+x_2-2\right) \\
&\quad +\lambda\left(1+2x_1+2x_2-6\,(x_1-x_2)^2\right)\Big]P_3^q\Bigg\} \\
&\quad +\left\{P_i^q\to\tilde{P}_i^q,\left(2C_1C_2+N_CC_2^2\right)\to C_1^2\right\}\,,
\end{aligned}
$$

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$$\widehat{\Gamma}^q_{7,\text{exc}} = \frac{\Gamma_0}{2m_B}\frac{16\pi^2}{m_Q^4}\left[\frac{12\left((1-x_1-x_2)^2+(x_1+x_2)\,\lambda\right)}{\sqrt{\lambda}}\right]\left\{[2C_1C_2]\,P_2^q+\left[C_1^2+C_2^2\right]\tilde{P}_2^q\right\},$$
$$\widehat{\Gamma}^q_{7,\text{int}^-} = \widehat{\Gamma}^q_{7,\text{int}^+}\big|_{C_1\longleftrightarrow C_2},$$
$$\widehat{\Gamma}^{q,\text{SL}}_{7,\text{int}^+}(x_\ell,0) = \widehat{\Gamma}^q_{7,\text{int}^+}(x_\ell,0)\big|_{C_1\to 0,C_2\to 1,N_C\to 1}, \tag{B.15}$$

with again the same notation as in eq. (B.12) and the (x_1, x_2) dependence suppressed. Note that results throughout for baryons are presented in the QCD basis, since we do not consider HQET matrix elements for baryons.

C A note on the relation between the HQET and QCD four-quark matrix elements at $O(1/m_Q^3)$

Given that the basis of HQET operators and QCD operators are related, but are structurally different, it might well be asked how free is the choice to use one basis or the other, and what is the difference between each choice. In one sense, the answer is clear, because the two ought to be related up to higher-order corrections in the $1/m_c$ and α_s expansions. However, this is far from trivial: at the very least, it is important to be able to quantify this difference.

To illustrate the point, we consider spectator contributions up to dimension-seven in semileptonic decay width for mesons. Helicity suppression implies that, in the limit $m_{s,\mu} \to 0$, $\hat{\Gamma}_{6+7,\text{SL}} \to 0$, with equality in the VIA. At the operator level, this can be written in the QCD basis as

$$\hat{\Gamma}_{6+7,\text{SL}} = -\frac{G_F^2|V_{cs}|^2m_c^2}{6\pi}\frac{1}{2m_M}\left(O_1 - O_2 + \frac{2}{m_c}(P_1 - P_2) + O\left(m_s^2, m_\ell^2\right)\right), \tag{C.1}$$

where we neglect corrections from m_s and m_μ. Using the QCD parametrizations within the VIA [85]

$$\langle D|O_1|D\rangle = f_D^2m_D^2, \qquad \langle D|O_2|D\rangle = f_D^2m_D^2\left(\frac{m_D}{m_c+m_q}\right)^2,$$
$$\langle D|P_1|D\rangle = -m_qf_D^2m_D^2, \qquad \langle D|P_2|D\rangle = -f_D^2m_D^2\frac{m_D^2-m_c^2-m_q^2}{2m_c}, \tag{C.2}$$

we can see that this is indeed the case: the combination (C.1) vanishes in the limit $m_q \to 0$. Note that the parametrization of O_2 includes the factor $(m_D/(m_c+m_q))^2$, which should be set to one if dimension-seven contributions were excluded. The equivalent expression in the HQE basis is

$$\hat{\Gamma}_{6+7,\text{SL}} = -\frac{G_F^2|V_{cs}|^2m_c^2}{6\pi}\frac{1}{2m_M}\left(\mathcal{O}_1 - \mathcal{O}_2 + \frac{1}{m_c}\Big(2\mathcal{P}_1 - 2\mathcal{P}_2 + \mathcal{R}_1 - \mathcal{R}_2 + \mathcal{M}_{\pi,1} - \mathcal{M}_{\pi,2} + \mathcal{M}_{G,1} - \mathcal{M}_{G,2}\Big) + O\left(m_q^2\right)\right), \tag{C.3}$$

where the $\mathcal{M}$ are non-local operators whose forms are given explicitly in [39, 53, 144] (and see also [87]). Inserting the parametrizations in eqs. (4.1) and (4.5), supplemented by the non-local parametrizations from appendix C in [39], again shows that this cancels, but this

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time order-by-order in the $1/m_c$ expansion rather than mixing across dimensions, as was the case in the QCD basis.

Moreover, applying the relation [39, 87, 144]

$$F_D^2\left(1-\frac{\bar{\Lambda}}{m_c}+\frac{2G_1}{m_c}+\frac{12G_2}{m_c}\right)=f_D^2 m_D+O\left(1/m_c^2\right)\,, \tag{C.4}$$

(where $G_{1,2}$ parametrize nonlocal, $1/m_c$ suppressed, matrix elements defined in appendixC in [39]), which is valid at a fixed scale $\mu=m_c$ and within the VIA, then the two expressions (C.1) and (C.3) can be directly related by making the identification

$$\begin{aligned}
&\langle\mathcal{O}_1\rangle+\frac{1}{m_c}(\langle\mathcal{R}_1\rangle+\langle\mathcal{M}_{\pi,1}\rangle+\langle\mathcal{M}_{G,1}\rangle)\to\langle\mathcal{O}_1\rangle|_{F_D^2\to f_D^2 m_D}\,,\\
&\langle\mathcal{O}_2\rangle+\frac{1}{m_c}(\langle\mathcal{R}_2\rangle+\langle\mathcal{M}_{\pi,2}\rangle+\langle\mathcal{M}_{G,2}\rangle)\to\langle\mathcal{O}_2\rangle|_{F_D^2\to f_D^2 m_D}+\frac{2}{m_c}\langle\mathcal{R}_2\rangle\,.
\end{aligned} \tag{C.5}$$

Finally, we may relate them to the full QCD basis by noting that

$$\langle\mathcal{O}_2\rangle+\frac{2}{m_c}\langle\mathcal{R}_2\rangle=f_D^2 m_D^2\left(1+2\frac{\bar{\Lambda}-m_q}{m_c}\right)\approx f_D^2 m_D^2\left(\frac{m_D}{m_c+m_q}\right)^2=\langle O_2\rangle\,, \tag{C.6}$$

where the second step is valid as usual up to higher-order corrections in the HQE. This justifies the parametrizations employed in 4.1.

This re-absorption effect can be extended, in principle, to include higher-order terms arising from the HQE or the α_s expansion, although in the latter case the discussion is more complicated owing to operator mixing [53].

The discussion does not, however, naturally extend to baryons, not least because there is no factorization of 4-quark matrix elements in baryons and no information about the baryon matrix elements is available beyond QCD. The conjecture of [80], that such a reabsorption of dimension-seven non-local contributions in the f_D decay constant does *not* occur for the triplet (Λ_c^+, Ξ_c^+, Ξ_c^0), but *would* occur in the Ω_c^0, therefore remains to be tested.

D Supplementary tables of results

The following tables provide a detailed breakdown of contributions to meson and baryon decay widths, arranged so that it is possible to see the relative sizes of contributions entering at different orders in the $1/m_Q$ and α_s expansion. We present results in the pole and kinetic schemes, for illustrative purposes; the remaining two mass schemes employing in this paper, the MSR and $\overline{\text{MS}}$ schemes, are not presented explicitly. Specifically:

- Table 22 presents contributions to decay widths of mesons in the pole mass scheme;
- table 23 presents contributions to decay widths of mesons in the kinetic mass scheme;
- table 24 presents contributions to decay widths of baryons in the pole mass scheme;
- table 25 presents contributions to decay widths of baryons in the kinetic mass scheme.

In all cases, only the central values are given, so it should be borne in mind that each contribution therefore comes with, potentially sizeable, uncertainties due to variations of the hadronic parameters or the scale μ. The size of the uncertainties of the relevant contributions can be seen in the tables 10, 11, and 14 in the main text.

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		$\Gamma_{3+5}{}^{\rm NL}$	$\Gamma_{3+5}{}^{\rm SL}$	$\Gamma_\rho^{\rm NL}$	$\Gamma_\rho^{\rm SL}$	$\tilde{\Gamma}_{6,\rm WE}$	$\tilde{\Gamma}_{6,\rm PI}$	$\tilde{\Gamma}_{6,\rm WA}$	$\tilde{\Gamma}_{6,\rm WA}^{\rm SL}$	$\tilde{\Gamma}_{7,\rm WE}$	$\tilde{\Gamma}_{7,\rm PI}$	$\tilde{\Gamma}_{7,\rm WA}$	$\tilde{\Gamma}_{7,\rm WA}^{\rm SL}$	$\Gamma_{\rm tot}^{\rm SL}$	$\Gamma_{\rm tot}$	τ	$\tau_{\rm exp.}$
D^0	LO	1.05	0.30	0.19	0.05	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.35	1.64	0.61	0.41
	NLO	1.31	0.15	0.19	0.05	0.04	-0.01	-0.01	-0.01	0.0	0.0	0.0	0.0	0.19	1.71	0.59	
D^+	LO	1.05	0.30	0.19	0.05	0.0	-1.87	0.0	0.0	0.0	1.14	0.0	0.0	0.35	0.87	1.15	1.04
	NLO	1.31	0.15	0.19	0.05	0.0	-2.90	-0.01	-0.01	0.0	1.14	0.0	0.0	0.19	-0.07	-	
D_s^+	LO	1.06	0.28	0.29	0.08	0.0	-0.13	-0.01	0.0	0.0	0.11	0.0	0.0	0.36	1.66	0.60	0.53
	NLO	1.31	0.13	0.29	0.08	0.0	-0.22	0.0	0.01	0.0	0.11	0.0	0.0	0.22	1.71	0.58	

Table 22. Central values of various contributions to decay widths of charmed mesons, in the *pole* scheme for the charm quark mass, indicating also the leading order (LO) and next-to-leading order (NLO) contributions where available. Decay widths are given in units ps^{-1}, and lifetimes in units of ps. The NLO contributions to Γ_ρ and $\tilde{\Gamma}_7$ are not known; for the corresponding NLO entries we repeat the LO results, for the reader's convenience. The semileptonic (SL) contributions involve both electron and muon channels. Note that, for the D_s^+ meson, the given value of $\tau^{\rm exp}$ results after subtracting the decay width of the channel $D_s \to \tau\nu$, see eq. (2.2). All hadronic matrix elements and other numerical inputs used to obtain these results correspond to those in section 4.1.

		$\Gamma_{3+5}{}^{\rm NL}$	$\Gamma_{3+5}{}^{\rm SL}$	$\Gamma_{\rho}^{\rm NL}$	$\Gamma_{\rho}^{\rm SL}$	$\tilde{\Gamma}_{6,\rm WE}$	$\tilde{\Gamma}_{6,\rm PI}$	$\tilde{\Gamma}_{6,\rm WA}$	$\tilde{\Gamma}_{6,\rm WA}^{\rm SL}$	$\tilde{\Gamma}_{7,\rm WE}$	$\tilde{\Gamma}_{7,\rm PI}$	$\tilde{\Gamma}_{7,\rm WA}$	$\tilde{\Gamma}_{7,\rm WA}^{\rm SL}$	$\Gamma_{\rm tot}^{\rm SL}$	$\Gamma_{\rm tot}$	τ	$\tau_{\rm exp.}$
D^0	LO	0.79	0.20	0.19	0.05	0.04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	1.28	0.78	0.41
	NLO	1.28	0.23	0.19	0.05	0.04	-0.01	-0.01	-0.01	0.0	0.0	0.0	0.0	0.28	1.77	0.56	
D^+	LO	0.79	0.20	0.19	0.05	0.0	-1.66	0.0	0.0	0.0	1.08	0.0	0.0	0.25	0.65	1.54	1.04
	NLO	1.28	0.23	0.19	0.05	0.0	-2.89	-0.01	-0.01	0.0	1.08	0.0	0.0	0.28	-0.07	-	
D_s^+	LO	0.79	0.19	0.28	0.08	0.0	-0.12	-0.01	0.0	0.0	0.10	0.0	0.0	0.27	1.31	0.76	0.53
	NLO	1.29	0.22	0.28	0.08	0.0	-0.22	0.0	0.01	0.0	0.10	0.0	0.0	0.30	1.77	0.56	

Table 23. Central values of various contributions to decay widths of charmed mesons, in the *kinetic* scheme for the charm quark mass, indicating also the leading order (LO) and next-to-leading order (NLO) contributions where available. Decay widths are given in units ps^{-1}, and lifetimes in units ps. See the caption of table 22 for further details.

		$\Gamma_{3+5}{}^{\rm NL}$	$\Gamma_{3+5}{}^{\rm SL}$	$\Gamma_{\rho}^{\rm NL}$	$\Gamma_{\rho}^{\rm SL}$	$\tilde{\Gamma}_{6,\rm exc}$	$\tilde{\Gamma}_{6,\rm int^-}$	$\tilde{\Gamma}_{6,\rm int^+}$	$\tilde{\Gamma}^{\rm SL}_{6,\rm int^+}$	$\tilde{\Gamma}_{7,\rm exc}$	$\tilde{\Gamma}_{7,\rm int^-}$	$\tilde{\Gamma}_{7,\rm int^+}$	$\tilde{\Gamma}^{\rm SL}_{7,\rm int^+}$	$\Gamma^{\rm SL}_{\rm tot}$	$\Gamma_{\rm tot}$	τ	$\tau_{\rm exp.}$
Λ_c^+	LO	0.90	0.43	0.15	0.04	1.04	−0.38	0.03	0.02	0.46	0.11	−0.01	−0.01	0.48	2.78	3.59	
	NLO	1.15	0.32	0.15	0.04	1.28	−0.25	0.03	0.01	0.46	0.11	−0.01	−0.01	0.37	3.29	3.04	2.02
Ξ_c^+	LO	0.89	0.42	0.18	0.05	0.08	−0.39	0.81	0.50	0.04	0.12	−0.29	−0.18	0.80	2.22	4.51	
	NLO	1.14	0.32	0.18	0.05	0.09	−0.26	0.79	0.37	0.04	0.12	−0.29	−0.18	0.55	2.36	4.25	4.56
Ξ_c^0	LO	0.89	0.42	0.18	0.05	1.15	0	0.84	0.52	0.52	0	−0.30	−0.19	0.81	4.10	2.45	
	NLO	1.14	0.32	0.18	0.05	1.42	0	0.82	0.38	0.52	0	−0.30	−0.19	0.56	4.35	2.31	1.52
Ω_c^0	LO	0.98	0.34	0.23	0.06	0.48	0	2.85	1.77	0.22	0	−1.50	−0.94	1.23	4.49	2.23	
	NLO	1.23	0.20	0.23	0.06	0.59	0	2.54	1.21	0.22	0	−1.50	−0.94	0.54	3.86	2.59	2.74

Table 24. Central values of various contributions to decay widths of baryons, in the *pole* scheme for the charm quark mass, indicating also the leading order (LO) and next-to-leading order (NLO) contributions where available. Decay widths are given in units $\rm ps^{-1}$, and lifetimes in units 10^{-13}s. See the caption of table 22 for further details.

		$\Gamma_{3+5}{}^{\rm NL}$	$\Gamma_{3+5}{}^{\rm SL}$	$\Gamma_{\rho}^{\rm NL}$	$\Gamma_{\rho}^{\rm SL}$	$\tilde{\Gamma}_{6,\rm exc}$	$\tilde{\Gamma}_{6,\rm int^-}$	$\tilde{\Gamma}_{6,\rm int^+}$	$\tilde{\Gamma}^{\rm SL}_{6,\rm int^+}$	$\tilde{\Gamma}_{7,\rm exc}$	$\tilde{\Gamma}_{7,\rm int^-}$	$\tilde{\Gamma}_{7,\rm int^+}$	$\tilde{\Gamma}^{\rm SL}_{7,\rm int^+}$	$\Gamma^{\rm SL}_{\rm tot}$	$\Gamma_{\rm tot}$	τ	$\tau_{\rm exp.}$
Λ_c^+	LO	0.65	0.31	0.15	0.04	0.92	−0.34	0.03	0.02	0.44	0.11	−0.01	−0.01	0.37	2.31	4.32	
	NLO	1.15	0.38	0.15	0.04	1.32	−0.29	0.03	0.02	0.44	0.11	−0.01	−0.01	0.43	3.33	3.00	2.02
Ξ_c^+	LO	0.65	0.31	0.18	0.05	0.07	−0.35	0.72	0.45	0.04	0.11	−0.28	−0.17	0.64	1.77	5.65	
	NLO	1.15	0.37	0.18	0.05	0.10	−0.30	0.83	0.40	0.04	0.11	−0.28	−0.17	0.65	2.48	4.03	4.56
Ξ_c^0	LO	0.65	0.31	0.18	0.05	1.02	0	0.75	0.47	0.49	0	−0.29	−0.18	0.65	3.45	2.90	
	NLO	1.15	0.37	0.18	0.05	1.47	0	0.86	0.42	0.49	0	−0.29	−0.18	0.66	4.53	2.21	1.52
Ω_c^0	LO	0.72	0.24	0.22	0.06	0.43	0	2.53	1.57	0.21	0	−1.41	−0.88	0.99	3.69	2.71	
	NLO	1.22	0.29	0.22	0.06	0.61	0	2.66	1.32	0.21	0	−1.41	−0.88	0.78	4.30	2.33	2.74

Table 25. Central values of various contributions to decay widths of baryons, in the *kinetic* scheme for the charm quark mass, indicating also the leading order (LO) and next-to-leading order (NLO) contributions where available. Decay widths are given in units $\rm ps^{-1}$, and lifetimes in units 10^{-13}s. See the caption of table 22 for further details.

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