Controlling systemic risk: Network structures that minimize it and node properties to calculate it

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Evaluation of systemic risk in networks of financial institutions in general requires information of interinstitution financial exposures. In the framework of the DebtRank algorithm, we introduce an approximate method of systemic risk evaluation which requires only node properties, such as total assets and liabilities, as inputs. We demonstrate that this approximation captures a large portion of systemic risk measured by DebtRank. Furthermore, using Monte Carlo simulations, we investigate network structures that can amplify systemic risk. Indeed, while no topology in general sense is *a priori* more stable if the market is liquid (i.e., the price of transaction creation is small) [T. Roukny *et al.*, Sci. Rep. **3**, 2759 (2013)], a larger complexity is detrimental for the overall stability [M. Bardoscia *et al.*, Nat. Commun. **8**, 14416 (2017)]. Here we find that the measure of scalar assortativity correlates well with level of systemic risk. In particular, network structures with high systemic risk are scalar assortative, meaning that risky banks are mostly exposed to other risky banks. Network structures with low systemic risk are scalar disassortative, with interactions of risky banks with stable banks.

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I. INTRODUCTION

In the past, the stability of the banking sector was mostly analyzed considering measures of the individual banks. Only recently, especially after the 2008 crisis, this approach has changed. The negative consequences of an interconnected economy and also of a interconnected financial sector became more clear. For this reason, scientists and policy makers concerned with systemic risk (a risk that a large number of institutions will simultaneously get in serious financial problems) started to recognize that there was a serious lack of knowledge on the mechanisms through which the interconnectedness affects the financial stability. This triggered a series of new analyses that included network effects and propagation of unfavorable financial conditions (technically known as financial distress). As a result, a series of new models and approaches were proposed and incorporated into systemic risk measures [1–5]. The most recent research of systemic risk measures extend the definition of financial system to multilayer networks, by the inclusion of different assets and valuations, as well as types and maturities of loans, etc. [6–8]. Applications of these models include central bank regulation [3], individual assessment of systemic risk [9], and simulations of different policies, such as bank taxation [10,11].

However, in order to compute these network risk measures, we need both a detailed knowledge of the interconnection network of institutions (for example, the investments between all pairs of banks [12]) and the knowledge of the dynamics of the evolution of this system. Unfortunately, these pieces of information about the network are known only by the regulating authorities (and just in a few cases); for that reason several methods of reconstructing the graph from partial information have been proposed [13–17], with methods and checks aimed at determining the best possible reconstruction [18].

Here we present a complementary approach to that of a reconstruction, by showing that a series of risk measures (including network effects) can be understood to a great extent by analyzing the properties of single banks. Indeed, here we show that in the calculation of systemic risk measures the presence of network can be taken into account by inspection of local (single banks and pairs of banks) properties. First, single bank measures as the interbank leverage (ratio of total investments of a bank into other banks over this banks equity) is enough to understand the first steps of stress propagation (that account for a large part of total stress propagation). Second, the investments between pairs of highly leveraged banks are also increasing stress propagation. We find that investment networks with high systemic risk are highly *scalar assortative* with respect to single bank risk, while networks with lowest systemic risk are *scalar disassortative* [19]. Scalar assortative networks are networks in which every node has associated scalar value and in which starting from the node with relatively large scalar value and randomly choosing its neighbor, it is more probable to observe large scalar value on its neighbor than would be expected by chance. Thus, the correlations among nodes with a scalar property can be described with the scalar assortativity measure $r = [\sum_{xy} xy(e_{xy} - a_x b_y)]/\sigma_a\sigma_b$ [19]. Here, x and y are some scalar measures associated with vertices which are not necessarily network related but can represent the total bank assets or the bank's credit rating and similar. Clearly, degree assortativity, which is more commonly understood as assortativity, is just a special case of scalar assortativity in which scalar variables are the degrees of the network. Furthermore, e_{xy} is the fraction of links from a vertex with scalar x to a vertex with scalar y, and $a_x = \sum_y e_{xy}$ and $b_y = \sum_x e_{xy}$.

We use data taken from the Italian electronic broker market e-MID (Market for Interbank Deposits) run by e-MID S.p.A. "Società Interbancaria per l'Automazione" (SIA), Milan. The Italian electronic broker Market for Interbank Deposit (e-MID) covers the entire overnight deposit market in Italy. The information about the parties involved in a transaction allows us to perform risk propagation on real networks as well as providing a basis from which we create artificial networks. As mentioned before, there are a number of papers which study the risk propagation with a network-based quantity, namely, the DebtRank [3], both for direct application to stress tests [20] and to realize a plausible scenario to understand systemic risk [21]. Here we follow the Debtrank approach presented in [12] since it simplifies the DebtRank method allowing to use simple linear algebra, while still preserving the conclusions obtained in other variants of DebtRank.

The paper is organized as follows. First, we reintroduce the DebtRank algorithm as proposed in [12]. Second, we analyze the amplification mechanism of the method and rewrite the algorithm in such a way that (a) single node, (b) neighborhood (local), and (c) global contributions to the DebtRank are clearly separated. Third, we propose a Monte Carlo network creation algorithm to test which network configurations are extremal (maximal or minimal) with respect to the DebtRank. Fourth, we present a simple illustrative example, which is followed by empirical results computed from the real data and analytically solvable examples. We finish with analysis of finite size and varying distributions effects on our results presented in previous sections.

II. BACKGROUND: PROPAGATING SHOCKS WITH DEBTRANK

Assume N banks, each with equity E_i. For every bank i, we additionally know how much it invested in total into other banks. We call this the interbank assets A_i of bank i in the interbank market. Additionally, we know the debts of each bank i to all other banks, called liabilities L_i. Initially (time $t = 0$) we assume no distress, and $\sum_i A_i(0) = \sum_i L_i(0)$. We have to stress that these are not the total assets and liabilities but only the portion which is network exposed in interbank network. For $t = 1$, we assume external distress on the banks $h(1)$. According to this distress, the assets A_i reduce their value, as the distressed banks are more likely to bankrupt and therefore not to pay back their debt. On the other hand, liabilities do not get reduced. Here we want to understand network effects of the positive feedback between reduced equity and asset value. For this we follow the DebtRank scenario. More precisely, we are interested in small everyday shocks, where no bank loses all its equity.

To compute the equity losses, let us assume for the moment we know not only the total amount A_i of assets of bank i, but also in which banks j they invested, denoted with the asset matrix $A_{ij}(0)$. We have definition of single bank i assets $A_i(0)$,

$$A_i(0) = \sum_j A_{ij}(0), \tag{1}$$

and single bank j liabilities $L_j(0)$,

$$L_j(0) = \sum_i A_{ij}(0). \tag{2}$$

Further we define the matrix Λ with elements $\Lambda_{ij} = A_{ij}(0)/E_i(0)$, and the distress parameter h_i describing the relative loss of equity of bank i, $h_i(t) = 1 - E_i(t)/E_i(0)$. The financial distress of a financial institution i, h_i, measures the reduction in its market value due to the reduction of the market value of its assets such as loans or investments in equity of other financial institutions. As a simple example, consider a bank i lending to bank j at a given interest rate r. If the financial situation of the bank j deteriorates, any other bank would then give to the bank j a similar loan but with a larger interest rate $r' > r$. In this way the market value of the existing loan (which is an asset of bank i) is reduced. According to [12] we have

$$h_i(t) = h_i(1) + \sum_j \Lambda_{ij} h_j(1) + \sum_j (\Lambda^2)_{ij} h_j(1) + \cdots + \sum_j (\Lambda^{t-1})_{ij} h_j(1). \tag{3}$$

We study a homogeneously distributed initial distress affecting each institution in the same way by reducing its equity before distress $E_i(0)$ at the start by $E_i(1) = (1 - \psi)E_i(0)$. Here ψ gives the proportion of equity lost due to the initial shock

$$h_i(t)/\psi = 1 + \sum_j \Lambda_{ij} + \sum_j (\Lambda^2)_{ij} + \cdots + \sum_j (\Lambda^{t-1})_{ij}. \tag{4}$$

In the remainder of the text we are primarily interested in the total relative systemic equity loss

$$H(t) = \sum_i h_i(t) E_i(0) \Big/ \sum_j E_j(0) \tag{5}$$

and especially in its asymptotic value $\lim_{t\to\infty} H(t) \equiv H^\infty$.

III. AMPLIFICATION OF A SMALL SHOCK HITTING ALL BANKS

For a general vector of initial distress $h_i(1)$, the total relative systemic equity loss can be expressed as

$$H^\infty = \frac{\sum_i E_i(0) h_i(1)}{\sum_k E_k(0)} + \frac{\sum_i L_i(0) h_i(1)}{\sum_k E_k(0)} + \frac{1}{\sum_k E_k(0)} \sum_{jl} \frac{L_j(0) A_{jl}(0) h_l(1)}{E_j(0)} + O(A^2). \tag{6}$$

As described above, we are interested in a small shock ψ hitting all banks equally, which roughly corresponds to shocks at the macroeconomic level. Although this is necessarily an approximation, it allows us to obtain even more detailed analytical insight into the total relative systemic equity loss using the data on individual banks (node specific data). Here we introduce a suitable variable: the macroeconomic multiplier $\Psi = H^\infty/\psi$. This variable is used to describe how external shocks are amplified in the banking system by relating the total (relative to systemic effects) equity loss to the portion of equity lost initially. We can now divide Eq. (5) by external shock parameter ψ and put in Eq. (4). After some rewriting using the definitions of bank assets A_i in Eq. (1) and liabilities L_j in Eq. (2), the following form is obtained:

$$\Psi = 1 + \frac{\sum_i A_i(0)}{\sum_k E_k(0)} + \frac{\sum_i A_i(0)L_i(0)/E_i(0)}{\sum_k E_k(0)} + \frac{\sum_{ij} A_{ij}(0)L_i(0)A_j(0)/(E_i(0)E_j(0))}{\sum_k E_k(0)} + \Psi^{(\mathrm{res})} \quad (7)$$

$$\equiv 1 + \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)} + \Psi^{(\mathrm{res})}. \quad (8)$$

Notice that the terms up to $\Psi^{(2)}$ only depend on the asset and liability sums A_i and L_i. The term $\Psi^{(3)}$ is the lowest order term including the investment matrix $A_{ij}(0)$. Now we define a risk matrix

$$R_{ij}^{(3)} = L_i(0)A_j(0)/(E_i(0)E_j(0)) \times (1 - \delta_{ij}), \quad (9)$$

whose meaning is to relate the relative exposure of bank i liabilities-equity ratio given by L_i/E_i to the relative assets-equity ratio A_j/E_j of the counterparty. $\Psi^{(3)}$ can be written in a more compact way by introducing the auxiliary dimensionless quantities:

$$\alpha_{ij} = A_{ij}(0)\Big/\sum_k E_k(0), \quad a_i = A_i(0)\Big/\sum_k E_k(0),$$

$$l_i = L_i(0)\Big/\sum_k E_k(0), \quad e_i = E_i(0)\Big/\sum_k E_k(0). \quad (10)$$

Now the simplified equation for contribution of counterparty banks to the systemic risk of the bank i is

$$\Psi^{(3)} = \sum_{ij} \alpha_{ij} R_{ij}^{(3)}. \quad (11)$$

In the end, the rest of the financial network contributes to distress through $\Psi^{(\mathrm{res})}$ which can be written as

$$\Psi^{(\mathrm{res})} = \sum_{t=4}^{\infty} \frac{\sum_{ij} E_i(0)(\Lambda^t)_{ij}}{\sum_k E_k(0)} = \sum_{t=4}^{\infty} \sum_{ij} e_i(\Lambda^t)_{ij}, \quad (12)$$

where again $\Lambda_{ij} = A_{ij}(0)/E_i(0) = \alpha_{ij}/e_i$. If the eigenvalue of the matrix Λ_{ij} with the largest absolute value (in the following called λ) has the absolute value considerably smaller than one, we can expect the residual term to be a minor correction in Ψ. If, on the other hand, $\lambda \geqslant 1$, the equity loss accelerates infinitely and at least one bank bankrupts.

Finally, for $t \to \infty$ the relation (3) can be written at the matrix level as

$$h(1) = (I - \Lambda)h^\infty. \quad (13)$$

From the condition that all elements of h^∞ are below 1, corresponding to no bankruptcies in the system, it is possible to obtain conditions on initial distress. This result directly reflects the fact that, owing to the network structure encoded in Λ, the stability of the entire financial network has different sensitivity on the same level of initial distress at various nodes. This information may be of practical importance to financial regulators. In particular, if some $h_i(1)$ is outside allowed range obtained by (13), i.e., some element of h^∞ raises above 1 with change in $h_i(1)$, regulators should consider intervention, possibly in the form of restructuring the financial network. The practical calculation of $h(1)$ using analytical methods might be prohibitively complicated even for networks of moderate size. A more convenient approach is based on simulations. One can randomly select each component of h^∞ in the interval of values corresponding to no bankruptcy ($0 \leqslant h_i^\infty < 1$) and calculate $h(1)$ using (13). With a sufficiently large number of such calculations one can obtain estimates of no-bankruptcy intervals for all components of $h(1)$. This analysis is left for future work. For the effect of relative shock on overall DebtRank one can consult Ref. [12].

IV. MINIMAL AND MAXIMAL SHOCK AMPLIFICATION Ψ

For understanding the bounds of systemic risk in measures of the shock multiplier Ψ, let us minimize or maximize $\Psi(\alpha_{ij})$ by varying α_{ij}, given single bank properties $A_i(0)$, $L_i(0)$, and $E_i(0)$. For this purpose, we use a stochastic optimization process. For variables α_{ij} we have the following constraints, which can in practice be obtained from the bank financial reports:

$$\sum_j \alpha_{ij} = a_i, \quad \sum_i \alpha_{ij} = l_j, \quad \alpha_{ij} \geqslant 0, \quad \alpha_{ii} = 0. \quad (14)$$

To find network configurations with extremal values of risk, we first need to define an optimization process which in particular does not depend on the choice of the risk or on some other property of financial network. In particular, we want to study a more general nonlinear function $F(\alpha_{ij})$, which we want to maximize. In our case, this function is the macroeconomic multiplier Ψ computed in Eq. (7). In other settings it is possible to use the same method for different ways of valuation of economic multiplier or some other risk related function. In any case, to the matrix α_{ij} fulfilling all constraints given in Eq. (14), we can add to it a matrix D, defined in the following way:

$$D(i_1, j_1, i_2, j_2)_{ij} = d\delta_{i,i_1}\delta_{j,j_1} + d\delta_{i,i_2}\delta_{j,j_2} - d\delta_{i,i_1}\delta_{j,j_2} - d\delta_{i,i_2}\delta_{j,j_1}, \quad (15)$$

which clearly preserves the sums in rows and columns of the matrix α and in which d is a parameter, chosen to be the smallest value of α in the given realization of the network. Therefore, d is a parameter that changes with every given network we study. Take, for example, a system with five banks and an initial valid matrix with elements α_{ij}. Then, one possible update D to this matrix is

$$D = \begin{bmatrix} 0 & d & 0 & -d & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -d & 0 & d & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (16)$$

We could start the process with α_{ij} taken from the data but in principle we start with $\tilde{\alpha}_{ij} = a_i l_j / \sum_k a_k$, as a guess for $\tilde{\alpha}_{ij}$, but which by construction is a complete, and we find that the rate of convergence to extremal solution is better. We successively subtract matrices of the form $D(i_1, i_1, i_2, i_2)$, until only one diagonal element is left, and further $D(i_1, i_1, i_2, j_2 \neq i_2)$ to eliminate the last diagonal element as well. For finding matrices α_{ij} with extremal $F(\alpha_{ij})$, we propose updates $\alpha \rightarrow \alpha + D(i_1, j_1, i_2, j_2)$, with D involving only off-diagonal elements. These matrices are drawn with uniform probability, i.e., every choice of indices i_1, i_2, j_1, j_2 is equally likely. If for the updated matrix it would hold $\alpha_{ij} \geqslant 0$, we accept updates with standard simulated annealing probability

$$\min\{1, \exp\{\beta[F(\alpha + D) - F(\alpha)]\}\}. \tag{17}$$

For $F = \Psi$, Ψ is maximized, while for $F = -\Psi$ it is minimized. The positive parameter β regulates how likely updates away from the optimization goal are accepted. For large β, such updates are accepted very unlikely. Small β can be used to escape local extrema (often combined with an increasing parameter β over time, to approach the global extreme in the end of the optimization procedure). In order to force additional constraints for the investment matrix α_{ij}, we add further terms

$$F = \pm\Psi - \beta_k \bar{k}(\alpha_{ij}) - \beta_{\text{asym}} \frac{\sum_{ij} \alpha_{ij}\alpha_{ji}}{\sum_{ij} \alpha_{ij}^2}. \tag{18}$$

For $\beta_k > 0$, α_{ij} is more sparse after optimization. The average degree is calculated as $\bar{k} = \bar{k}_{\text{in}} = \bar{k}_{\text{out}} = \sum_{ij} \Theta(\alpha_{ij})/N$, with theta function $\Theta(x > 0) = 1$ and $\Theta(0) = 0$. With $\beta_{\text{asym}} > 0$, the investment matrix is forced to be asymmetric. This has the following meaning: If for a pair of banks i, j it holds $\alpha_{ij}\alpha_{ji} > 0$, bank i invests into bank j, while at the same time bank j invests into bank i. In the e-MID data there are a number of closed loops of length 2, but for the purposes of this paper we chose to suppress them. The reason for this choice is that in overnight markets one can easily clear the debt between two parties, and we choose the DebtRank version presented in [12] which does not provide stop in iterations of the DebtRank algorithm. Short loops therefore iterate shock propagation between two banks *ad infinitum* and the correct way to alleviate this problem is to "clear" them into a one directional edge whose weight is the difference between the values of two reciprocal edges. The optimal choices of optimization parameters depend on the properties of networks we are studying and are given for each of the studied networks in the following sections. When this netting procedure is performed, it should be kept in mind that it is only a simplifying approximation since liability of bank i to bank j may have a different duration from the liability of bank j to bank i.

A. Illustrative example

For illustration, let us first discuss an artificial example of a network of interbank liabilities. We use a small network with $N = 30$ banks, equities from a Pareto distribution with exponent three, and interbank leverages $0.32 < A_i/E_i < 0.96$ from a uniform distribution. As it is easiest to illustrate and understand the case with $A_i = L_i$, we start with this case. For optimization, we use parameters $\beta = 10^6$, $\beta_k = 0.1$, and $\beta_{\text{asym}} = 2.0$. We sum up the first 50 terms of Ψ for assessing update trials, and once a sweep we calculate Ψ using the first 200 terms, with results plotted on the upper panel of Fig. 1(a). The final optimized networks have average degree $\bar{k} = 2.0$ (minimization) and $\bar{k} = 2.6$ (maximization). Largest eigenvalues are $\lambda = 0.67$ (minimization) and $\lambda = 0.83$ (maximization). Both connection matrices are strictly asymmetric at the end of optimization. On the lower panel of Fig. 1(a), we see a scalar assortativity measure with respect to interbank leverage A_i/E_i. Therefore, in this case, interbank leverage A_i/E_i is a scalar property $A_i/E_i = x = y$ in the equation for scalar assortativity introduced before. To compute scalar assortativity, we used an implementation provided with graph tool [22], where the variance is obtained with the jackknife method. For small systemic risk, highly leveraged banks should both lend from and borrow to banks with small leverage. This is the case for the network with minimized Ψ shown in Fig. 1(b). If highly leveraged banks lend among each other, systemic risk is high, as can be seen in Fig. 1(c).

We find that the scalar assortativity r of interbank leverage A_i/E_i is closely connected to the risk of an interbank network. In the following, we will see that simpler, purely topological network properties are less indicative for the interbank network risk. The global clustering coefficient is defined as the fraction of closed triplets among all triplets in a network, where the link direction is ignored. An opened triplet happens when three nodes are connected by only two links leaving the third possible connection open, while in a closed triplet all links between the three nodes are present. The global clustering coefficient for the interbank network with minimized risk of 0.023 is in the range 0.065 ± 0.053 expected for random networks with 30 nodes and average degree of $\bar{k} = 2.0$. We created 1000 random networks and calculated the global clustering coefficients of each, defining the range as two standard deviations around the mean value. For the interbank network with maximized risk, the global clustering coefficient of 0.213 is larger than for random networks with a range 0.085 ± 0.048 for average degree of $\bar{k} = 2.6$. We understand that this could be a consequence of the scalar assortativity of interbank leverage. In Fig. 1(c), it can be seen that the interactions of similar banks with respect to their individual leverage increase the number of triangles. However, triangles between banks with heterogeneous interbank leverage A_i/E_i would also increase the global clustering coefficient, but the shock amplification would stay at smaller values. Therefore, the global clustering coefficient alone is not a good indicator for interbank network risk. The degree assortativity is calculated as the scalar assortativity r used so far, but in this case by using the degree k_i of the nodes instead of the interbank leverage A_i/E_i. We find the degree assortativity to be -0.49 for the interbank network with minimized risk and -0.23 for maximization. Both cases are in the disassortative regime, meaning that banks with large degree tend to interact with small degree banks. We see that degree assortativity does not give a clear hint on the interbank network risk. The in- and out-degree distributions of the networks with minimized as well as with maximized risk are compatible with the binomial distribution (or limiting case of Poisson distribution) of simple random networks. As a simple test we generate 1000 random networks and calculate the degree distribution for each of them. This allows us to calculate

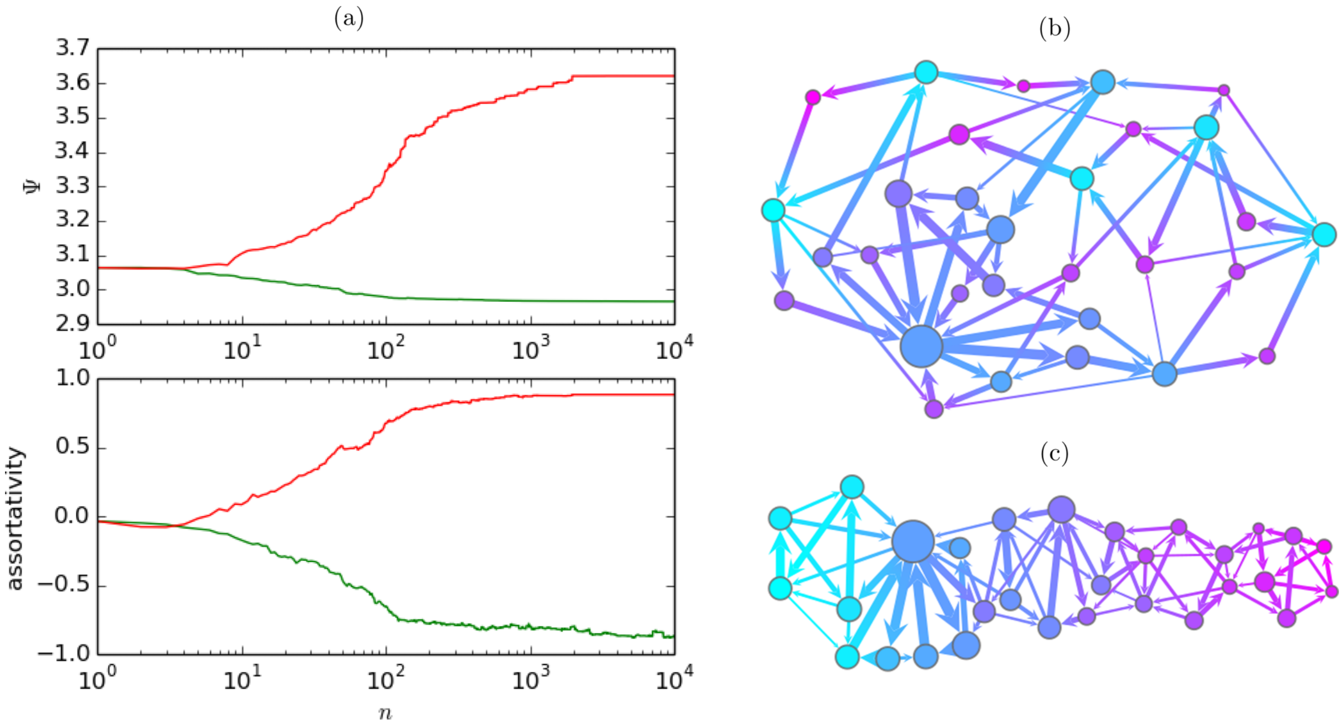

FIG. 1. (a) In the upper panel, the shock multiplier Ψ is shown during maximization (red line) and minimization (green line), where n denotes sweeps (with N^2 update trials). This is for an artificial example with $N = 30$ banks, equities from a Pareto distribution with exponent three, and interbank leverages $0.32 < A_i/E_i = L_i/E_i < 0.96$ from a uniform distribution. In the lower panel, a scalar assortativity measure with respect to interbank leverage is shown for the same optimization runs. Minimal systemic risk is connected to disassortative networks [see also (b)], while maximal systemic risk is connected to assortative networks [see also (c)]. The networks in (b) and (c) encode the total assets A_i of a bank i as node size, and the interbank leverage A_i/E_i as node color from pink (light shade - low values) to cyan (darker shade - high values).

regimes of two standard deviations for the probability of each degree, and the degree distributions of optimized interbank networks are compatible with these regimes for all degrees. We find that the interbank network risk has no effect on the degree distribution. Finally, the density of the network can be chosen from a wide range by varying the parameter β_k in the optimization, both for minimized and maximized risk. Therefore, the average degree alone is a poor indicator for the network risk. We repeat the optimization with $\beta_k = 0$ which means that the density of the optimized networks is not forced to be small. The shock amplification is similar as before with $\Psi = 2.97$ for minimization and $\Psi = 3.86$ for maximization. The average degree is $\bar{k} = 12.4$ for the minimized risk and $\bar{k} = 12.7$ for the maximized risk. However, most of the links have small weights below 10^{-3} with 78% in the network with minimized risk and 81% for the maximized risk. Such small weights have a minor share below 10% in the sparse optimized networks discussed before.

B. Empirical results

We use an interbank liability data set for the European market involving Italian banks in the year 1999. For a shock in the night before the last trading day in July, Friday July 30. 1999, we consider all outstanding liabilities with lifetime at least the next five trading days. These contracts, thus, have to be repaid earliest the upcoming Friday after one week. This choice is to guarantee that shock propagation due to devaluation of contracts has time to take place, a point that is in question for overnight obligations. Credits with shorter duration bring with them a different kind of risk into the interbank market: In times of crisis, banks which are no longer trustworthy for the others will have problems to renew short-lasting contracts and to borrow money. However, this different source of risk is not well represented with DebtRank modeling because it is not related to the devaluation of existing contracts for lenders, but rather to the worsened market conditions for borrowers who are trying to arrange new contracts. As short-lasting contracts contribute a large part to all obligations, in this case about 61% of all obligations are held in contracts lasting up to one week, it would be very interesting to address this additional aspect in future studies. This goes beyond the scope of this study and we ignore contracts with a duration shorter than two weeks. Possible contract durations are thus starting from two weeks, up to one year. We construct the network of all 218 banks involved, and reduce it to the largest strongly connected component, including $N = 53$ banks. As the data set is anonymized, we have to reconstruct the equity of the banks. We choose $E_i = \max(A_i, L_i) \times 1.25 \times \xi_i$ with ξ_i from a normal distribution with mean one and standard deviation 0.2. The resulting network can be seen in Fig. 2(a). Total assets A_i of a bank i shown as node size, and the interbank L_i/E_i as edge color at the edge source, A_j/E_j at edge target, node color from pink (light shade - low values) to cyan (darker shade - high values). We found

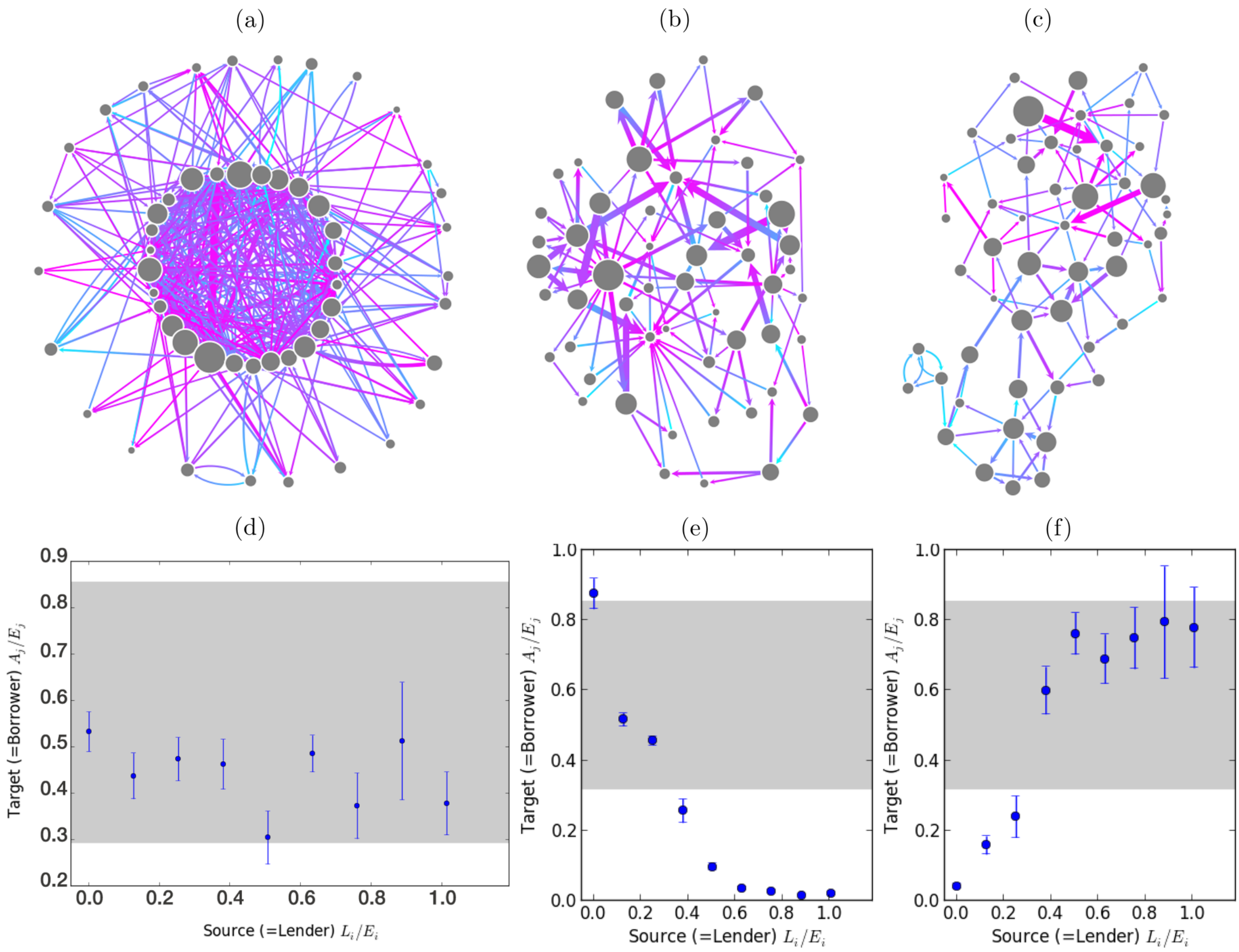

FIG. 2. (a) Liability network with $N = 53$ banks in the Italian market in 1999. (d) For this network, source and target properties are uncorrelated. (e) After minimizing systemic risk, the network becomes disassortative, with anticorrelations among different scalar properties for source and target node. For $A_i = L_i$, these correlations can be described with the simpler measure of scalar disassortativity as shown in Fig. 1(a) on the bottom. The gray band indicates the area between first and third quartiles for A_i/E_i, so half of the values A_i/E_i around the median lie within the gray area. We see that for small systemic risk, a bank i with high L_i/E_i should lend to a bank j with low A_j/E_j. Interpretation: A bank i with high L_i/E_i has high impact on its lenders, while a bank j with low A_j/E_j has only small exposure to its borrowers, thus shocks are dampened. (b) The network with minimal risk. Total assets A_i of a bank i shown as node size, and the interbank L_i/E_i as edge color at the edge source, A_j/E_j at edge target, node color from pink (light shade - low values) to cyan (darker shade - high values). (c) Network with maximal risk. (f) Maximal systemic risk is connected to assortativity.

a shock amplifier $\Psi = 1.90$ for this network. In Fig. 2(d) we analyze for this network correlations between lenders liabilities divided by equity (source L_i/E_i) and borrowers leverage (target A_i/E_i). The average A_i/E_i of all target nodes is plotted which are reached from source nodes with values L_i/E_i from a certain interval. We see that there are no significant correlations between lenders' liabilities divided by equities and borrowers' leverage. For $A_i = L_i$, these correlations simplify and can be described with the scalar assortativity as shown in Fig. 1(a) on the bottom.

The network consists of 763 edges among the 53 banks, therefore, the average degree is 14.4. Assets A_i and liabilities L_i are mildly correlated with a Pearson correlation of 0.11. In total, 36 of the directed edges have a counterpart in the opposite direction, so some loops of length two are present. The largest A_i is 875 million euros, the largest L_i is 1 132 million euros. All assets sum up to 7.04 billion euros, so do the liabilities. Using 100 different samples of equities E_i we found $\langle\Psi\rangle = 1.87$ with standard deviation 0.06.

As for the illustrative example, we minimize and maximize the shock amplifier Ψ with final sweep $n = 10^4$, $\beta = 10^6$, $\beta_k = 0.1$, and $\beta_{\text{asym}} = 2.0$. We sum up the first 50 terms of Ψ for assessing update trials. After the final sweep we calculate Ψ using the first 200 terms, with results $\Psi = 1.80$ (minimization) and $\Psi = 2.25$ (maximization). The final optimized networks both have average degree $\bar{k} = 2.0$. The connection matrix minimizing shock amplification is strictly asymmetric at the end of optimization, while for maximization we see a small number of loops of length two. Results are shown in Fig. 2. With Figs. 2(b) and 2(e) we see that an investment matrix with minimized Ψ has a more subtle kind of scalar disassortativity, as compared to the illustrative example with $A_i/E_i = L_i/E_i$. Here, systemic risk is minimized, when banks with high L_i/E_i lend to banks with low A_i/E_i. With Figs. 2(c) and 2(f), we see that the network of the maximized systemic risk is assortative. It is important to stress that this structure is very different from the typical core-periphery structure usually observed in financial networks [23–26]. It is also important to stress that risk minimization in principle reduces the number of edges in the network, therefore reducing the risk diversification of single financial institution. The apparent paradox was previously addressed in [27]. We also have to stress that scalar assortativity is very different from network assortativity. Previous analysis of cascades in complex networks [28,29] showed that cascades are (analogous to systemic risk spreading) inhibited by *network assortative* structures, while this analysis shows that systemic risk is amplified with *scalar assortativity*. These results are not opposed to each other but are complementary to each other.

C. Analytically solvable examples

There is another strong indicator why correlations between source L_i/E_i and target A_j/E_j are dominating in the optimization: For constant $C = L_i/E_i$ or constant $C = A_i/E_i$ (e.g., no positive or negative correlations possible), Ψ is constant, independent of the investment matrix A_{ij}. Let us first show this for $C = A_i/E_i$. The terms up to $\Psi^{(2)}$ are anyhow independent of A_{ij}. For higher terms we can write $\Psi^{(3)} + \Psi^{(\mathrm{res})} = \sum_{t=3}^{\infty}\sum_{ij} e_i(\Lambda^t)_{ij}$. We can define a stochastic matrix with elements $S_{ij} = A_{ij}/E_iC$, as $A_i/E_i = \sum_j A_{ij}/E_i = C$. With $\sum_j S_{ij} = 1$ and $\Lambda_{ij} = S_{ij}C$, we have

$$\Psi^{(3)} + \Psi^{(\mathrm{res})} = \sum_{t=3}^{\infty}\sum_{ij} e_i C^t (S^t)_{ij} = \sum_{t=3}^{\infty} C^t \sum_i e_i = \sum_{t=3}^{\infty} C^t. \tag{19}$$

Here we use properties of stochastic matrices $\sum_j (S^2)_{ij} = \sum_{jk} S_{ik}S_{kj} = 1$, etc. For liability sums being constant $C = L_i/E_i$, we can define a stochastic matrix $S_{ij} = A_{ij}/E_jC$, here with $\sum_i S_{ij} = 1$. We have $\Lambda_{ij} = S_{ij}CE_j/E_i$, and $\sum_{ijkl} E_i\Lambda_{ij}\Lambda_{jk}\Lambda_{kl} = C^3\sum_{ijkl} S_{ij}S_{jk}S_{kl}E_l = C^3\sum_l E_l$, with the same result $\Psi^{(3)} = C^3$ as for constant leverage. The same holds for higher terms. With this finding, other more subtle properties of the investment matrix, as second neighbor correlations, can only play a limited role. Further, we found an approximation for banks with interbank leverage from a sharply peaked distribution ($\max_i|A_i/E_i - \langle A_j/E_j\rangle_j| \ll \langle A_i/E_i\rangle_i$). In this case, the macroscopic shock amplification is mostly independent of the investment network and a simple function of the average leverage $\Psi \approx \sum_{t=0}^{\infty}(\langle A_i/E_i\rangle_i)^t = 1/(1 - \langle A_i/E_i\rangle_i)$, with geometric sum only for $\langle A_i/E_i\rangle_i < 1$.

Let us now discuss a case, where the optimization of Ψ can be performed explicitly. We have $N = n_1 + n_2$ banks with identical equity $E_i = E$, $e_i = 1/N$. With this choice, we have $\Lambda_{ij} = A_{ij}/E$. The first n_1 banks have $A_i/E = L_i/E = c_1$, while the last n_2 banks are less leveraged with $A_i/E = L_i/E = c_2 < c_1$. Illustrated for $n_1 = 2$ and $n_2 = 3$, let us introduce the following parametrized matrix:

$$\Lambda = \begin{bmatrix} \frac{c_1}{n_1} & \frac{c_1}{n_1} & 0 & 0 & 0 \\ \frac{c_1}{n_1} & \frac{c_1}{n_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{c_2}{n_2} & \frac{c_2}{n_2} & \frac{c_2}{n_2} \\ 0 & 0 & \frac{c_2}{n_2} & \frac{c_2}{n_2} & \frac{c_2}{n_2} \\ 0 & 0 & \frac{c_2}{n_2} & \frac{c_2}{n_2} & \frac{c_2}{n_2} \end{bmatrix} + \kappa \begin{bmatrix} -\frac{n_2}{n_1} & -\frac{n_2}{n_1} & 1 & 1 & 1 \\ -\frac{n_2}{n_1} & -\frac{n_2}{n_1} & 1 & 1 & 1 \\ 1 & 1 & -\frac{n_1}{n_2} & -\frac{n_1}{n_2} & -\frac{n_1}{n_2} \\ 1 & 1 & -\frac{n_1}{n_2} & -\frac{n_1}{n_2} & -\frac{n_1}{n_2} \\ 1 & 1 & -\frac{n_1}{n_2} & -\frac{n_1}{n_2} & -\frac{n_1}{n_2} \end{bmatrix} = \Lambda_a + \kappa\Delta. \tag{20}$$

The matrix Λ_{a} is maximally assortative, as only banks of the same type interact. For a simpler notation, we allow for self-links. The diagonal elements can easily be emptied into links among banks of the same type. This keeps Ψ unchanged. With $\Lambda_{ij} \geqslant 0$, we have $0 \leqslant \kappa \leqslant \min(c_1/n_2, c_2/n_1)$. For the maximal value of κ, the two bank types interact as much as the constraints allow. Therefore, this is the maximally disassortative case. The change in Ψ for an infinitesimal increase of disassortativity, going from Λ to $\Lambda + d\kappa\Delta$, is

$$d\Psi = \sum_{t=3}^{\infty}\sum_{p=2}^{t-1}\sum_{ij}(\Lambda^p\Delta\Lambda^{t-p})_{ij}d\kappa/N, \tag{21}$$

$$\sum_{ij}(\Lambda^p\Delta\Lambda^{t-p})_{ij} = -f(\Lambda^p)f(\Lambda^{t-p}) \quad \text{with}$$

$$f(\Lambda^p) = n_1(\Lambda^p)_{11} + (n_2 - n_1)(\Lambda^p)_{1N} - n_2(\Lambda^p)_{NN}. \tag{22}$$

We neglect higher order terms in $d\kappa$ and use the fact that $\sum_i \Delta_{ij} = 0$, such that this matrix only occurs in between matrices Λ. With showing that $\sum_{ij}(\Lambda^p\Delta\Lambda^{t-p})_{ij} \leqslant 0$ for all $0 < p < t$, we show that the most assortative connection matrix implies largest shock propagation, while the most disassortative matrix implies smallest shock propagation. We show that $f(\Lambda^p) = f(\Lambda)C_p$ with C_p positive. Using $(\Lambda^p)_{1N} = n_1\Lambda_{11}(\Lambda^{p-1})_{1N} + n_2\Lambda_{1N}(\Lambda^{p-1})_{NN}$ and analog expressions for $(\Lambda^p)_{11}$ and $(\Lambda^p)_{NN}$, we can write $f(\Lambda^p) = n_1(\Lambda^{p-1})_{11}f(\Lambda) + n_2\Lambda_{NN}f(\Lambda^{p-1})$. This is a positive multiple of $f(\Lambda)$, if this holds for $f(\Lambda^{p-1})$. With the condition being trivially fulfilled for $f(\Lambda^1)$, we can use induction to prove it for any p.

For $\Lambda = \Lambda_{\mathrm{ass}}$, the simple closed form result $\Psi = \sum_{t=0}^{\infty} c_1^t + c_2^t$ holds. The dominating term c_1^t grows or shrinks exponentially with t. The minimized Ψ is a lengthy polynomial in c_1, c_2, n_1, and n_2 which cannot be easily reduced into a closed form expression. With an ansatz $v = (1, 1, \dots, a, a, \dots)$ for the eigenvector with largest eigenvalue λ, we find

$$\lambda = \frac{c_1 - \kappa n_2 + c_2 - \kappa n_1}{2} + \left\{\frac{[c_1 - \kappa n_2 - (c_2 - \kappa n_1)]^2}{4} + \kappa^2 n_1 n_2\right\}^{1/2}. \tag{23}$$

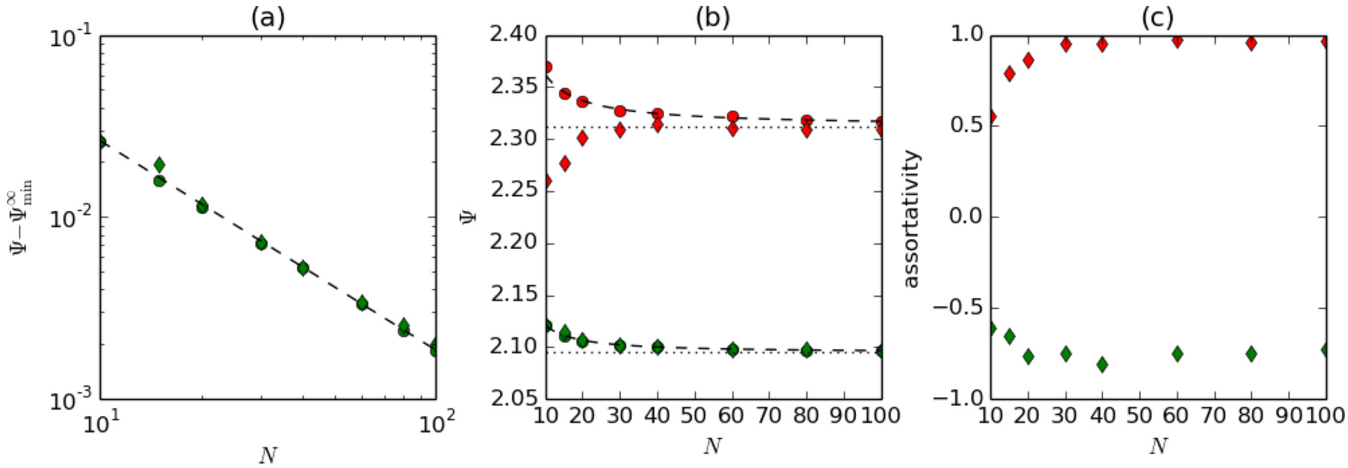

FIG. 3. Finite size effects for unrestricted optimization (circles) and restricted optimization, where small degree and asymmetric investment matrix is forced (diamonds). (a) With finite size scaling we find that for large N, Ψ after minimization approaches $\Psi^{\infty}_{\min} \approx 2.0947 \pm 5 \times 10^{-4}$, with a finite size deviation about $\propto N^{-1.15}$. Numerical results are shown with circles (unrestricted) and diamonds (restricted optimization). The dashed line indicates a power law with exponent -1.15. (b) Results of (a) are repeated with linear scale (green symbols and lower dashed line), and compared to results of maximization (red symbols and upper dashed line indicating results of a finite size scaling). The dotted lines indicate $\Psi^{\infty}_{\min}$ and $\Psi^{\infty}_{\max} \approx 2.315 \pm 0.01$. (c) Assortativity after restricted optimization for minimization (green diamonds) and maximization (red diamonds). Results indicate that, independent of the network size, least risky networks are strongly disassortative, while most risky networks are strongly assortative with respect to leverage.

Assume many healthy banks and a few highly leveraged banks: $n_1 = 5$, $c_1 = 2$, $n_2 = 50$, $c_2 = 0.5$. For Λ_a we have $\lambda = c_1 = 2$ with the first n_1 banks going bankrupt. For largest possible κ, we have $\lambda = 0.8$. Here all banks survive a small macroeconomic shock. In this latter case, the first n_1 banks do not lend among each other, and the healthy banks dedicate a share of $\frac{2}{5}$ for interactions with the first n_1 banks and remain a share of $\frac{3}{5}$ for interactions among each other.

D. Finite size effects and varying distributions of single bank properties

To discuss the finite size effects with varying network size N, we choose $E_i = 1$ for all banks, and $A_i/E_i = L_i/E_i = 0.2 + 0.6i/(N-1)$. In this way, the single bank properties for networks of different sizes are similar, and there is no need to average over many realizations of them. We optimize for $n_{\max} = 5 \times 10^3$ sweeps with increasing parameter $\beta = 10 \times N^2 \times 100^{n/n_{\max}}$. The algorithmic cost per optimization sweep scales with N^4, as for every microscopic update trial, matrix multiplications have to be performed (scaling with N^2), and there are N^2 microscopic update trials in a sweep. With a choice of small values for leverage, we can use only the first 13 terms in Ψ for assessing update trials. Final Ψ is calculated with 103 terms. Unrestricted optimization is performed with $\beta_k = \beta_{\text{asym}} = 0$, results with restrictions are found using $\beta_k = 0.1$ and $\beta_{\text{asym}} = 2.0$. In Fig. 3(a) we see a finite-size scaling for unrestricted (green circles) and restricted (green diamonds) minimization. We found an asymptotic result $\Psi^{\infty}_{\min} \approx 2.0947 \pm 5 \times 10^{-4}$, and $\Psi - \Psi^{\infty}_{\min} \propto N^{-1.15}$. We also performed a finite size scaling for results of unrestricted maximization of Ψ (not shown). This has less convincing results, indicating that local maxima are a problem. We found $\Psi^{\infty}_{\max} \approx 2.315 \pm 0.01$. In Fig. 3(b) we see results for minimization (green) and maximization (red). For small networks, restricted maximization results (red diamonds) are far below the unrestricted case (red circles). However, deviations are small for larger networks. In Fig. 3(c) we see that networks with maximized Ψ are strongly assortative, while networks with minimized Ψ are strongly disassortative.

We already discussed how correlations among single bank properties A_i/E_i and L_i/E_i affect results. Let us now discuss the outcome with rescaling $A_i/E_i \to c \times A_i/E_i$ and $L_i/E_i \to c \times L_i/E_i$. We choose $E_i = 1$ for all banks, and $A_i/E_i = L_i/E_i = c \times (0.2 + 0.6i/(N-1))$, with $N = 30$ banks. In Fig. 4 we see results for $c = 1$ (solid lines) and $c = 2$ (dashed lines). We optimize for $n_{\max} = 5 \times 10^3$ sweeps with increasing parameter $\beta = 10 \times N^2 \times 100^{n/n_{\max}}$. As the losses grow exponentially for $c = 2$, we only use the first six terms in Ψ for assessing update trials. On the left of the figure, we see the largest eigenvalue λ of the stress propagation matrix Λ during optimization. λ is larger than one for $c = 2$ (dashed lines). This means that even a very small initial shock causes an exponentially growing stress propagation, finally causing at least one bankrupt bank. On the right of the figure, we see that monitoring assortativity while optimization indicates a similar behavior, even if stress propagation changes from dampened (solid lines) to exponentially growing (dashed lines).

V. SUMMARY AND OUTLOOK

We saw that in the framework of DebtRank, most risky investment networks are highly assortative with respect to lenders' liabilities divided by equity (source L_i/E_i) and borrowers' leverage (target A_i/E_i). We tested this for artificial samples of single bank properties, finding that the effect is robust regarding to correlations among single bank properties, network size, and, finally, it is also a common feature of dampened or exponentially growing stress propagation;

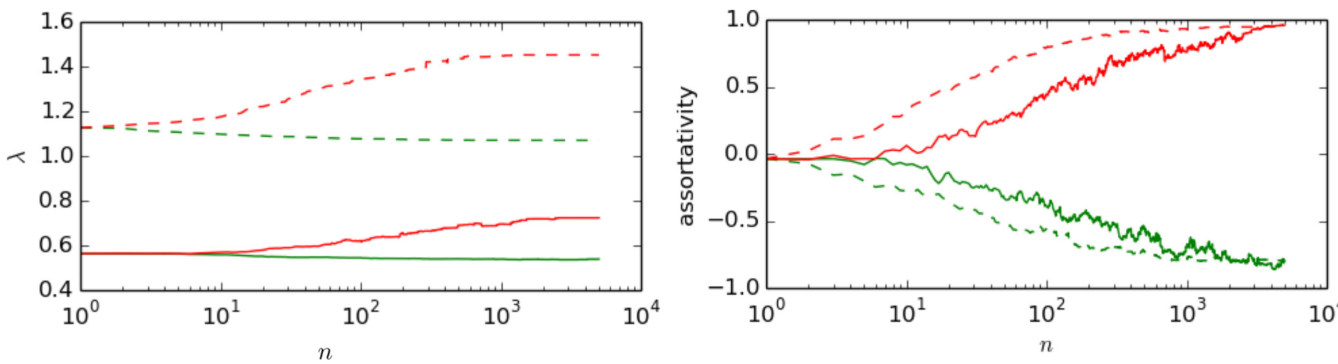

FIG. 4. Rescaling single bank leverage $A_i/E_i \rightarrow c \times A_i/E_i$, stress propagation can switch from dampened to exponentially growing. We see this for an example case with $c = 2$. While the largest eigenvalue λ of the stress propagation matrix Λ increases from below one (solid lines, minimization green, maximization red) to above one (dashed lines), the optimization procedure has a similar outcome with respect to assortativity in both cases.

we found this behavior also in empirical data. Finally, we performed the optimization analytically for a network with two types of banks.

The two main results of this paper are as follows: (i) shock propagation in financial networks can be approximately computed from single node properties only and (ii) this shock propagation can be minimized by making financial networks disassortative. Aside from an obvious advantage in (i) that the computation using single node properties only is simpler and faster, other advantages and potential applications of these results are possible. For example, the possibility to (approximately) estimate shock amplification in interbank networks from single bank properties brings additional advantage for financial regulators. Namely, single bank properties necessary for such estimation, such as their total assets A_i and liabilities L_i are cumulative quantities and, as such, they change more slowly than changes in the structure of the interbank networks. In particular, on a daily basis, we do not expect total assets or liabilities of the bank to change significantly. However, it is reasonable to expect that at the same daily timescale any bank in the network would engage in lending to or borrowing from many new banks, or changing the amount of lending or borrowing for other banks that the said bank is already connected to. Thus, in the regime where the approximation of shock propagation is reliable using only first terms that depend on single bank properties, these estimates are also expected to remain reliable for as long as these single bank properties do not change significantly, and much longer than the typical scale on which the interbank network changes.

The association of scalar disassortative network structures with lower systemic risk gives to regulators more "degrees of freedom" in resolving situations where vulnerability of a small number of banks threatens the entire network. Namely, there are many network structures with high disassortativity and it is easier for regulators to find or realize one of them if realistic legal, liquidity, or even political constraints exist.

An interesting parallel with physical systems also arises from this analysis. Namely, if we classify leverage into discrete categories, then we can possibly map them to spin systems like the Potts model. If this analogy holds, one could associate low risk structures with a variant of antiferro-Potts model, while networks which exhibit more risk could possibly be associated with the ferro variant of the Potts model. Whether this analogy holds is beyond the scope of this paper, but if the mapping of systemic risk model to such a well studied statistical physics model would be obtained, a community of scientists that study systemic risk could greatly benefit from the accumulated knowledge.

Finally, the approach of estimating shock propagation in interbank networks from single bank properties only provides a novel possibility for public oversight of financial system stability. As banks publish public financial statements in regular intervals, and these statements contain data on total borrowing from or lending to other banks in the financial system, it is in principle possible for anyone to compute the lower bound on the systemic risk for various scenarios of initial financial distress. In this way, monitoring systemic risk in the financial system would no longer be limited to regulatory authorities.

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