

## On the perturbative approach to the penguin-induced $B \rightarrow \pi\phi$ decay

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### Abstract

Using a modified perturbative approach that includes the Sudakov resummation and transverse degrees of freedom we analyze the penguin-induced  $B^- \rightarrow \pi^- \phi$  decay. The perturbative method enables us to include nonfactorizable contributions and to control virtual momenta appearing in the process. The calculation supports the results obtained in the standard BSW factorization approach, illustrating the electroweak penguin dominance and the branching ratio of order  $\mathcal{O}(10^{-8})$ . However, the estimated prediction of 16% for CP asymmetry is much larger than that obtained in the factorization approach.

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## 1. Introduction

Nowadays, experimental facilities offer a possibility of searching for CP asymmetries in penguin-induced nonleptonic decays, very promising decays to detect direct CP violation. Such decays have small branching ratios (BR), but satisfy both requirements for CP-violating asymmetry, owing to the fact that penguins are loop diagrams with different quark generations contributing with different weak CP-phases from the CKM matrix and that final-state strong interaction phases emerge from the absorptive part of penguin amplitudes.

Our aim is to investigate the penguin-induced  $B^- \rightarrow \pi^- \phi$  decay in the modified perturbative approach.

We present a complete calculation of factorizable and nonfactorizable contributions in the  $B^- \rightarrow \pi^- \phi$  decay up to order  $\mathcal{O}(\alpha_s \alpha_{em})$ . Nonfactorizable contributions from the QCD-penguin operators are examined and their role in the EW-penguin dominated processes, such as  $B^- \rightarrow \pi^- \phi$ , is assigned. Direct CP-asymmetry violation is calculated and all results are compared with those obtained in the standard BSW factorization approach.

## 2. Perturbative model

The nonleptonic  $B^- \rightarrow \pi^- \phi$  decay is governed by the weak decay of the heavy b-quark,  $b \rightarrow ds\bar{s}$ . The light antiquark of the  $B$  meson is the spectator in the decay, being only slightly accelerated by the

exchange of a hard gluon to form a pion in the final state.

We use the NLO weak Hamiltonian in which the renormalization-scheme dependence of the Wilson coefficients is explicitly canceled by the inclusion of the one-loop QED matrix elements of the tree-level operators  $\mathcal{O}_{1,2}^{(q)}$ . The final expression for the matrix element is then

$$\begin{aligned} \langle \pi^- \phi | \mathcal{H}_{eff}(\Delta = -1) | B^- \rangle &= \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \\ &\times \left( \sum_{k=3}^{10} \bar{c}_k(\mu) \langle \mathcal{O}_k \rangle^{\text{tree}} + \frac{\alpha_{em}(\mu)}{9\pi} (3\bar{c}_1(\mu) + \bar{c}_2(\mu)) \times \right. \\ &\left. \left\langle \left( \frac{10}{9} - \Delta G(m_q^2, q^2, \mu^2) \right) (\mathcal{O}_7 + \mathcal{O}_9) \right\rangle^{\text{tree}} \right) \quad (1) \end{aligned}$$

and here  $\langle \mathcal{O}_k \rangle^{\text{tree}} \simeq \langle \pi^- \phi | \mathcal{O}_k | B^- \rangle$ .

The absorptive part of the matrix element needed for a nonvanishing CP asymmetry resides in the  $\Delta G(m_q, q^2, \mu)$  function for  $q^2 \geq 4m_q$ . In the perturbative calculation, the process-dependent virtual momentum  $q^2$  is determined by the momentum distribution in the decay and it is controlled by the momentum distributions in the process.

The nontrivial part is the calculation of the matrix elements of the four-quark operators  $\langle \mathcal{O}_k \rangle^{\text{tree}}$  at the tree level, which we are going to perform in the modified perturbative approach [1].

The matrix element factorizes into the convolution of distribution amplitudes of hadrons involved

into the decay (hadron wave functions) and the hard scattering amplitude of valence partons. The hard scattering amplitude can be straightforwardly calculated perturbatively, taking into account all possible exchanges of a hard gluon between valence partons in a given  $\alpha_s$ -order of the calculation [2].

On the other hand, the hadronic wave functions represent the most speculative part of the perturbative approach. They are of nonperturbative origin and should be a universal, process-independent quantity. However, there are several models of wave functions for each of the particles and it is not simple to rule some of them out.

We make a selection among the wave functions by comparing the results for the  $B \rightarrow \pi$  transition form factor obtained using nonperturbative methods (lattice, QCD sum rules) with those estimated in our modified perturbative approach.

The selected wave functions for the B-meson, the pion, and the  $\phi$ -meson are, respectively,

$$\Phi_B(x) \propto \sqrt{x(1-x)} \exp\left(-\frac{M_B^2}{2\omega^2}x^2\right), \quad (2)$$

$$\Phi_\pi^{CZ}(x, \mu_1) \propto 6x(1-x) \times \left(1 + (5(1-2x)^2 - 1) \left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right)^{50/81}\right) \quad (3)$$

$$\Phi_\phi(x) \propto 6x(1-x). \quad (4)$$

Having selected these wave functions, we are now in a position to give some predictions. For details, the reader is referred to [2].

### 3. Numerical results and conclusions

The results are presented in Tables I and II. The results in column II are obtained by the calculation, in which the  $\mu$  scale-setting ambiguity of Wilson coefficients is moderated by applying the three-scale factorization theorem [3]. The theorem keeps trace of all three scales characterizing the nonleptonic weak decay: the W-boson mass  $M_W$ , the typical scale  $t$  of the process, and the hadronic scale  $\sim \Lambda_{QCD}$ , and proves for the leading-order weak Hamiltonian that Wilson coefficients should be taken at the scale  $t$ . The matrix elements of the operators  $\mathcal{O}_k$  and Wilson coefficients are then both calculated at the same scale, contrary to the standard calculation with the Wilson coefficients  $\bar{c}_k(\mu = m_b)$  represented in column I.

Our results for the branching ratio appear to be in agreement with previous calculations performed in the BSW factorization approach [4], predicting the branching ratio to be of order  $\mathcal{O}(10^{-8})$ ,

**Table 1.** Branching ratios calculated for different penguin contributions, by taking only the factorizable parts or the complete expression into account (see text). The CKM parameters used are  $\bar{\rho} = 0.16$  and  $\bar{\eta} = 0.33$ .

Penguin contributions	BR/ $10^{-10}$		
	BSW	I	II
QCD <sup>fact</sup>	0.20	0.14	1.06
QCD <sup>all</sup>	-	2.51	0.73
(QCD + QED) <sup>fact</sup>	34	38	89
(QCD + QED) <sup>all</sup>	-	44	85

**Table 2.** CP asymmetries calculated for the QCD and QED penguin contributions together, by taking only the factorizable parts or the complete expression into account.

Penguin contributions	$a_{CP}/10^{-2}$		
	BSW	I	II
(QCD + QED) <sup>fact</sup>	-1.9	14.6	15.4
(QCD + QED) <sup>all</sup>	-	16.1	16.3

dominated by the EW penguins. On the other hand, the predicted CP asymmetry differs a lot from that estimated in the BSW factorization approach, being as large as 16% and having an opposite sign for the preferred values of the CKM parameters  $\bar{\rho} = 0.16$  and  $\bar{\eta} = 0.33$ . The large CP asymmetry estimated in the perturbative approach is the result of large on-shell effects of the virtual propagators involved in the calculation [2].

Besides, if the Wilson coefficients are considered to be functions of the scale (results in column II), the same one which appears in the hadronic matrix elements, then the nonfactorizable QCD penguin contributions appear to be negligible, as is the case with the obviously very small nonfactorizable contributions of the EW penguin operators. Furthermore, estimations based on this assumption have produced the branching ratios about a factor of two larger than those calculated with the conventional Wilson coefficients.

### References

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