IRB-TH-4/99

July, 1999.

Enhancement of preasymptotic effects in inclusive beauty decays

B. Guberina[*], B. Melić[†], H. Štefančić[‡]

Theoretical Physics Division, Rudjer Bošković Institute,
P.O.Box 1016, HR-10001 Zagreb, Croatia

Abstract

We extend Voloshin's recent analysis of charmed and beauty hyperon decays based on $SU(3)$ symmetry and heavy quark effective theory, by introducing a rather moderate model-dependence, in order to obtain more predictive power, e.g. the values of lifetimes of the (Λ_b, Ξ_b) hyperon triplet and the lifetime of Ω_b. In this way we obtain an improvement of the ratio $\tau(\Lambda_b)/\tau(B_d^0) \sim 0.9$ and the hierarchy of lifetimes $\tau(\Lambda_b) \simeq \tau(\Xi_b^0) < \tau(\Xi_b^-) < \tau(\Omega_b)$ with lifetimes of Ξ_b^- and Ω_b exceeding the lifetime of Λ_b by 22% and 35%, respectively.

PACS: 14.20.Mr, 13.20.He, 12.39.Hg, 12.39.Jh

Keywords: beauty baryons, lifetimes, inclusive decays, four-quark operators

[*]guberina@thphys.irb.hr

[†]melic@thphys.irb.hr

[‡]shrvoje@thphys.irb.hr

Weak decays of beauty mesons and baryons are believed to be a nice playground where a variety of phenomena should be well described and understood in the framework of the operator product expansion (OPE) and heavy quark effective theory (HQET) [1, 2]. The essential underlying idea in both theories is the expansion in inverse powers of heavy-quark mass – the mass of the beauty quark, $m_b \sim O(5\,GeV)$, is considered to be heavy compared with the typical hadron scale of $0.5 - 1\,GeV$. This is to be compared with the case of charmed mesons and baryons, where the mass of the charmed quark, $m_c \sim 1.3\,GeV$, is hardly an ideal expansion parameter.

The rate of the beauty-hadron decay is given by

$$\begin{aligned}\Gamma(H_b \to f) &= \frac{G_{\mathrm{F}}^2 m_b^5}{192\pi^3}|V|^2 \frac{1}{2M_{H_b}}\{c_3^f \langle H_b|O_3|H_b\rangle + c_5^f \frac{\langle H_b|O_5|H_b\rangle}{m_b^2} \\ &+ \sum_i c_6^f \frac{\langle H_b|O_6^i|H_b\rangle}{m_b^3} + O(1/m_b^4) + ...\}\,,\end{aligned} \tag{1}$$

where c_j^f are the Wilson coefficients and

$$\frac{1}{m_b^{D-3}}\langle H_b|O_D|H_b\rangle \tag{2}$$

are matrix elements of the D-dimensional operators which appear in the OPE multiplied by the appropriate power of inverse quark mass. The sum in (1) starts with $D = 3$, i.e. with $O_3 = \overline{b}b$ giving

$$\frac{1}{2M_{H_b}}\langle H_b|O_3|H_b\rangle = 1 + O(1/m_b^2)\,. \tag{3}$$

Clearly, in the asymptotic limit $m_b \to \infty$, one recovers the parton model result – as long as m_b is large enough, one expects all corrections to stay moderate. Furthermore, it is obvious from (3) that there are no $1/m_b$ corrections – a consequence of the nonexistence of independent operators of dimension four.

The experimental situation is as follows: the lifetimes of beauty hadrons follow the simple theoretical $m_b \to \infty$ prediction within $5 - 10\%$:

$$\tau(B^+) = \tau(B_d^0) = \tau(B_s^0) = \tau(\Lambda_b)\,, \tag{4}$$

except for the lifetime of Λ_b, which appears to be by $15-25\%$ smaller than predicted in (4). More precisely [3],

$$\frac{\tau(B^+)}{\tau(B_d^0)} = 1.07 \pm 0.03\,, \tag{5}$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d^0)} = 0.81 \pm 0.05\,. \tag{6}$$

The lifetimes of b hadrons are

$$\begin{aligned} \tau(B_d^0) &= (1.54 \pm 0.03)\, ps\,, \\ \tau(B^+) &= (1.65 \pm 0.03)\, ps\,, \\ \tau(\Xi_b\; mixture) &= (1.39\; {}^{+0.34}_{-0.28}\;)\, ps \\ \tau(\Lambda_b) &= (1.24 \pm 0.08)\, ps\,. \end{aligned} \tag{7}$$

Theoretical estimates [4] predict the ratio (5) to be $1 + 0.05(f_B/200MeV)^2$, in accordance with experiment, but the ratio (6) is predicted to be in the range

$$\frac{\tau(\Lambda_b)}{\tau(B_d^0)} \sim 0.95 - 0.98\,, \tag{8}$$

which seems to be an overestimate.

It appears, however, that the ratio $\tau(\Lambda_b)/\tau(B_d^0)$ is not easy to lower down to the experimental value, the reason being that the $1/m_b$ expansion converges rapidly. For example, keeping only operators with $D = 3$ and $D = 5$, one obtains 0.98 for the ratio (6). Thus it seems difficult to accommodate this ratio with the same mass m_b entering the decay rates of both Λ_b and B_d^0. In fact, strangely enough, putting the physical hadron masses instead of m_b would give $\tau(\Lambda_b)/\tau(B_d^0) = 0.73$, up to the $O(1/m_b^2)$, in good agreement with experiment. However, this nice Ansatz, proposed in [5] completely spoils the OPE and contradicts other OPE predictions confirmed by experiments.

Therefore, the only hope to obtain the ratio (6) in the framework of the OPE and HQET is to look for the possible larger contributions coming from the operators with dimension $D = 6$ or higher. These operators are known to play an important role in charmed-meson decays, in which, owing to the Pauli interference effect [6, 7, 8], there is a dilation of the lifetime of the D^+ meson. In charmed-baryon decays, their role is even more pronounced: they give the dominant contribution leading to the well-established lifetime hierarchy which was successfully predicted prior to experiment [9, 10]. Unfortunately, the calculation of the matrix elements of the operators with dimension $D = 6$ requires the use of quark models and is, therefore, strongly model dependent.

Recently, Voloshin [11] proposed the way to avoid the use of phenomenological models. He showed that using $SU(3)$ symmetry and HQET it was possible to relate the measured lifetimes of charmed hyperons to the differences in semileptonic decay rates, the differences in the Cabibbo suppressed decay rates of charmed hyperons and the splitting of the total decay rates of b hyperons, without invoking the quark model results for the matrix elements [12, 13]. He confirmed the predicted difference in the semileptonic decay rates between the Ξ_c and Λ_c by a factor 2 to 3, and the enhancement of the semileptonic branching ratio for Λ_c^+ coming from the Cabibbo suppressed decay rate. When applied to beauty decays, Voloshin's approach leads to a difference of 14% in the lifetime of Ξ_b^- with respect to the lifetime of Λ_b.

In this paper we extend Voloshin's analysis introducing a rather moderate model dependence, in order to obtain more predictive power, e.g. the values for the lifetimes of the (Λ_b, Ξ_b) hyperon triplet and the lifetime of Ω_b. Basically, we express the decay rates in terms of the nonrelativistic (NR) wave function at the origin $\Psi(0)$, the value of which we determine using Voloshin's method.

Our starting point is the expression (1). It is argued [14] that the beauty mass which enters the expression (1) is a running mass $m_b(\mu)$. In the limit $\mu \to 0$, one obtains the *pole* mass which is very often used in calculations. It would be perfectly legitimate to use the pole mass in pure perturbative theory (with no nonperturbative contribution). However, the use of the pole mass is very problematic when nonperturbative corrections are calculated, because of the renormalon singularities resulting in an irreducibile uncertainty of $O(\Lambda_{QCD}/m_b)$ that is larger than the nonperturbative corrections we are calculating. Shielding $m_b(\mu)$ against renormalon ambiguities by choosing $\mu > 0.5\,GeV$, one avoids problems with the pole mass. In fact, a natural choice for the scale μ is $m_b/5 \sim 1\,GeV$, as argued in [15]. Such a relatively low scale makes the

$\overline{MS}$ mass inadequate for treating the decays. A natural definition of the running mass would be that with the linear dependence on μ:

$$\frac{d\,m(\mu)}{d\,\mu} = -c_m \frac{\alpha_s(\mu)}{\pi} + \dots . \tag{9}$$

The recent value for $m_b(\mu)$ at $\mu \sim 1\,GeV$ and for $c_m = 16/9$ is given by [16, 17]

$$m_b(\mu = 1\,GeV) = (4.59 \pm 0.08)\,GeV\,, \tag{10}$$

which is slightly lower than the usual values. In this calculation we use $m_b(\mu = 1\,GeV)$ in the range $4.6\,GeV < m_b(1\,GeV) < 4.8\,GeV$.

Next, we turn to the calculation of the matrix elements of the O_6^i (four-quark) operators. We follow the approach given by Voloshin [11] based on HQET and flavor $SU(3)$ symmetry. A suitable parameter to express these matrix elements is *the effective decay constant* F_B^{eff} which is an analogue of the static decay constant used to evaluate four-quark matrix elements in decays of heavy baryons [2, 13, 18].

We use the following two differences of decay rates: For $\Delta_b^1 = \Gamma(\Lambda_b) - \Gamma(\Xi_b^0)$, we have

$$\Delta_b^1 = \frac{G_F^2 m_b^2}{4\pi} \mid V_{cb} \mid^2 (c^2 - s^2)(\sqrt{1-4z} - (1-z)^2(1+z))[C_5(m_b)x + C_6(m_b)y], \tag{11}$$

where $z = m_c^2/m_b^2$ and c and s stand for $\cos\theta_c$ and $\sin\theta_c$, respectively (θ_c is the Cabibbo angle). For $\Delta_b^2 = \Gamma(\Xi_b^-) - \Gamma(\Lambda_b)$, we have

$$\Delta_b^2 = \frac{G_F^2 m_b^2}{4\pi} \mid V_{cb} \mid^2 [l_1 x + l_2 y], \tag{12}$$

where l_1 and l_2 are the abbreviations for the following expressions:

$$l_1 = (1-z)^2 C_1(m_b) - [c^2\sqrt{1-4z} + s^2(1-z)^2(1+z)]C_5(m_b), \tag{13}$$

$$l_2 = (1-z)^2 C_2(m_b) - [c^2\sqrt{1-4z} + s^2(1-z)^2(1+z)]C_6(m_b). \tag{14}$$

In the equations displayed above, C_i stand for special combinations of Wilson coefficients described in [11], while x and y denote combinations of heavy-baryon matrix elements introduced first in the same reference:

$$x = \Big\langle \frac{1}{2}(\overline{b}\Gamma^{\mu}b)[(\overline{u}\Gamma_{\mu}u) - (\overline{s}\Gamma_{\mu}s)]\Big\rangle_{\Xi_b^- - \Lambda_b} = \Big\langle \frac{1}{2}(\overline{b}\Gamma^{\mu}b)[(\overline{s}\Gamma_{\mu}s) - (\overline{d}\Gamma_{\mu}d)]\Big\rangle_{\Lambda_b - \Xi_b^0}, \tag{15}$$

$$y = \Big\langle \frac{1}{2}(\overline{b^i}\Gamma^{\mu}b^j)[(\overline{u^j}\Gamma_{\mu}u^i) - (\overline{s^j}\Gamma_{\mu}s^i)]\Big\rangle_{\Xi_b^- - \Lambda_b} = \Big\langle \frac{1}{2}(\overline{b^i}\Gamma^{\mu}b^j)[(\overline{s^j}\Gamma_{\mu}s^i) - (\overline{d^j}\Gamma_{\mu}d^i)]\Big\rangle_{\Lambda_b - \Xi_b^0}, \tag{16}$$

Similar relations are valid (through HQET and $SU(3)$ symmetry) for the respective members of the charmed hyperon triplet [11].

The procedure of extraction of the effective parameter F_B^{eff} is based on equating expressions obtained in two approaches. In the first approach, for the matrix elements x and y we use the $SU(3)$ hypothesis which basically comprises using values of matrix elements extracted from experimental data on charmed baryons for calculations in the beauty-baryon sector. This approach is based on the assumptions of $SU(3)$ and heavy-quark symmetry. In the second approach, x and y are calculated using the nonrelativistic quark model, already frequently employed for similar calculations [2, 13, 18]. Within this model, for x and y we have

$$x = -y = - \mid \Psi_{\Lambda_b}(0) \mid^2 \tag{17}$$

Equation (17) clearly shows that the valence approximation is used in the calculation of the matrix elements. The connection between the wave function squared, $\mid \Psi_{\Lambda_b}(0) \mid^2$, and F_B^{eff} is given by the relation [19, 20]

$$\mid \Psi_{\Lambda_b}(0) \mid^2 = T(F_B^{eff})^2, \tag{18}$$

where

$$T = 4\frac{M(\Sigma_b^0) - M(\Lambda_b^0)}{M^2(B^*) - M^2(B)} m_u^* (\frac{1}{12} M(B)\kappa(\mu)^{-\frac{4}{9}}). \tag{19}$$

Here $\mu \sim 1\,GeV$ is a typical hadronic scale of hybrid renormalization κ. The decay rate differences obtained in the first and in the second approach are denoted by $\Delta^i_{b,SU(3)}$ and $\Delta^i_{b,model}$, respectively $(i = 1, 2)$.

The effective parameter $F_{B,i}^{eff}$ is now extracted from the equation

$$\Delta^i_{b,SU(3)} = \Delta^i_{b,model}. \tag{20}$$

The expressions obtained for $F_{Beff,i}$, $i = 1, 2$, are

$$F_{B,1}^{eff} = \sqrt{\frac{C_5(m_b)x + C_6(m_b)y}{T(C_6(m_b) - C_5(m_b))}}, \tag{21}$$

$$F_{B,2}^{eff} = \sqrt{\frac{l_1 x + l_2 y}{T(l_2 - l_1)}}. \tag{22}$$

The final numerical value is calculated taking into consideration the errors of the expressions (21) and (22) and combining all numerical values appropriately. For $m_c = 1.25\,GeV$ and $m_b = 4.7\,GeV$, we obtain

$$F_B^{eff} = (0.441 \pm 0.026)\,GeV\,. \tag{23}$$

The parameter F_B^{eff} shows a slight mass dependence which was incorporated in numerical calculations.

Next, the numerical value displayed in (23) is used in (18) to obtain the values of the O_6^i matrix elements. As all the matrix elements in the expression (1) for the decay rate are now available, we can calculate the lifetimes of beauty baryons accordingly. The lifetimes of B mesons are calculated in a "standard" way using the B-meson decay constant f_B.

Since absolute results for lifetimes are not so reliable owing to ambiguities in quark mass, we shall express our results mainly in the form of lifetime ratios.

Our results for the ratio $r_{\Lambda B} = \tau(\Lambda_b)/\tau(B_d^0)$ are shown in Fig.1. The Wilson coefficients in (1) have been calculated at one loop using $\Lambda_{QCD} = 300\,MeV$. Other relevant numerical parameters used throughout the paper are [1, 2] $\mu_\pi^2 = 0.5\,GeV^2$, $\mu_G^2(\Omega_b) = 0.156\,GeV^2$, $\mu_G^2(\Lambda_b, \Xi_b) = 0$. The effect of introducing F_B^{eff}, which we have calculated, is to bring the ratio $r_{\Lambda B}$ from 0.96 to 0.9 – still two standard deviations from experiment. It is clear from Fig.1 that variation in m_b has almost no effect. Also, the variation of μ_π^2 does change $r_{\Lambda B}$ at the permile level. If the experimental ratio (6) persists, then there might be the problem in b decays.

An interesting effect in this approach noticed by Voloshin is the enhancement of the lifetime of Ξ_b^- compared with the lifetime of Λ_b. Using the "standard" B-meson decay constant $f_B \sim$

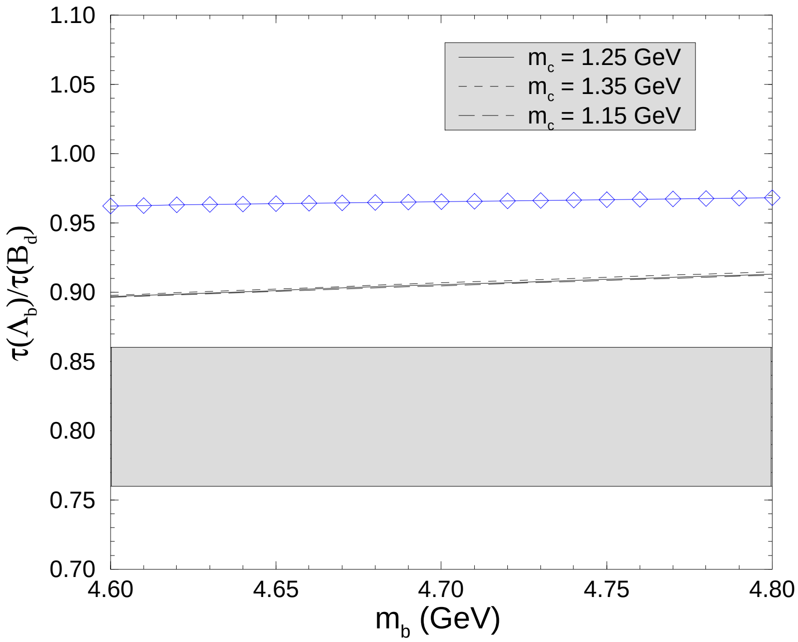

Figure 1: The shaded area represents the experimental value of the ratio $r_{\Lambda B}$ within one standard deviation. The line with diamonds represents the calculated value of $r_{\Lambda B}$ for the "standard" value $f_B = 160\,MeV$. The values of $r_{\Lambda B}$ using F_B^{eff} are calculated for three different values of mass m_c and are represented by lines without symbols. The significant shift from the "standard" value result is visible, but the deviation from the experimental band is still substantial.

$160\,MeV$, instead of F_B^{eff}, one obtains for the ratio $\tau(\Xi_b^-)/\tau(\Lambda_b) \sim 1.03$. Our calculation gives, Fig.2,

$$\frac{\tau(\Xi_b^-)}{\tau(\Lambda_b)} \simeq 1.22\,, \tag{24}$$

i.e. a relative enhancement of the $\tau(\Xi_b^-)/\tau(\Lambda_b)$ ratio of the order 20%, which is in fair agreement with the preliminary experimental results (7) [3]. The main reason for this enhancement is the large (positive) exchange contribution to $\Gamma(\Lambda_b)$ versus the large negative Pauli interference contribution in $\Gamma(\Xi_b^-)$.

Such an enhancement is even more pronounced in the lifetime of Ω_b, giving the ratio

$$\frac{\tau(\Omega_b)}{\tau(\Lambda_b)} \simeq 1.35\,, \tag{25}$$

i.e. a relative enhancement of the ratio $\tau(\Omega_b)/\tau(\Lambda_b)$ of the order 30%, Fig.3. This result is a

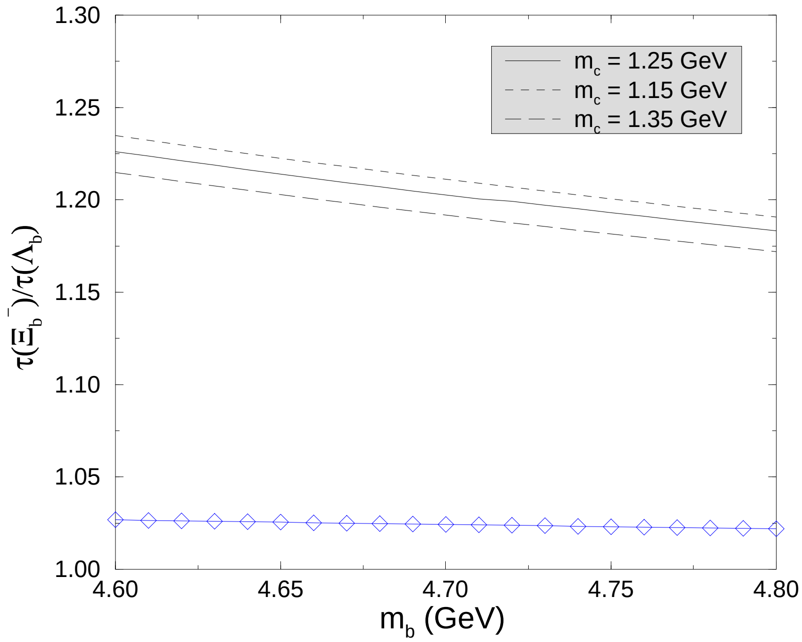

Figure 2: The line with diamonds represents the value of the ratio $\tau(\Xi_b^-)/\tau(\Lambda_b)$ for the value $f_B = 160\,MeV$. Values of the same ratio calculated using F_B^{eff} are given for three m_c masses and are represented by lines without symbols. The difference of $\sim 20\%$ is clearly visible.

consequence of the even stronger (compared with Ξ_b^-) negative Pauli interference contribution in $\Gamma(\Omega_b)$. The results (24) and (25) should both serve as crucial predictions to be checked in experiment. The calculation gives $\tau(\Xi_b^0)$ approximately the same as $\tau(\Lambda_b)$, which together with (24) and (25) leads to the following lifetime hierarchy:

$$\tau(\Lambda_b) \simeq \tau(\Xi_b^0) < \tau(\Xi_b^-) < \tau(\Omega_b)\,. \tag{26}$$

As long as the absolute value of the Λ_b lifetime is concerned, the effect of F_B^{eff} is to lower the theoretical value of $\tau(\Lambda_b)$ by $\sim 10\%$, giving

$$\tau(\Lambda_b) \sim 2.0\,ps\,, \tag{27}$$

which is too high compared with the measured value $(1.24 \pm 0.08)\,ps$. To obtain better agreement with experiment, one needs larger m_b. If, for example, we use the pole masses $m_b^{pole} = 5.1\,GeV$, $m_c^{pole} = 1.6\,GeV$, the result is $\tau(\Lambda_b)^{pole} = 1.6\,ps$ – still too high vis-à-vis experiment. However,

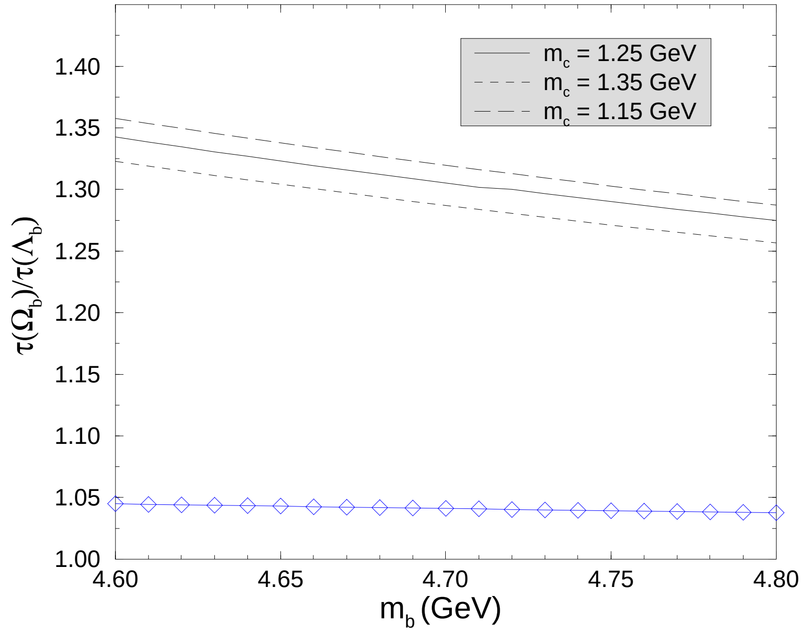

Figure 3: The value of the ratio $\tau(\Omega_b)/\tau(\Lambda_b)$ obtained using $f_B = 160\,MeV$ is represented by the line with diamonds. Values of the same ratio for F_B^{eff}, represented by lines without symbols, are calculated for three different m_c masses. Calculations using f_B and F_B^{eff} differ by $\sim 30\%$.

playing with a large pole mass is merely an introduction of an additional parameter – a consistent treatment requires having the running mass $m_b(\mu)$ in the expansion and its value could hardly reach more than $4.7\,GeV$.

Much the same situation appears in the calculation of B-meson lifetimes, which is not affected by our approach. Typically, one obtains $\tau(B) \sim 2 - 2.5\,ps$ for $m_b = 4.6\,GeV$ and $\tau(B) \sim 1.75 - 2.2\,ps$ for $m_b = 4.7\,GeV$, the range of values for τs coming from the variation of m_c, $1.15\,GeV < m_c < 1.35\,GeV$. Comparing with the results for the calculated value of $\tau(\Lambda_b)$, one sees that it is easier to have $\tau(B_d^0)$ near to the experimental value. This may suggest that the problem with too large a theoretical value of $r_{\Lambda B}$ lies in the theoretical overestimate of $\tau(\Lambda_b)$.

To conclude, we point out the following. The calculations presented in this paper rely upon HQET and flavor $SU(3)$ symmetry and are therefore reliable up to violations of these symmetries. Still, we expect the effects of these violations to be smaller than the main effect of our approach.

The procedure applied above significantly increases the contribution of four-quark operators and numerical results show a significant, albeit still unsufficient shift towards experimental values, especially in the case of the $r_{\Lambda B}$ ratio. In our approach, to reach the experimental value of $r_{\Lambda B}$, would require F_B^{eff} to have the value $0.72\,GeV$, which can be hardly achieved. The discrepancy, still remaining after increasing the preasymptotic effects coming from four-quark operators, indicates that there should be other, yet unknown, sources of enhancement of preasymptotic effects and that these effects should also produce significant contributions. Also, there remains the possibility of violation of some of the underlying concepts, such as quark-hadron duality, but a consistent treatment of these problems is still out of the reach of the present theory. In our approach, the large contributions of the operators of $D = 6$ also suggest a much wider spread of lifetimes in the sector of beauty baryons. The extent of this spread is to be verified by future experiments. At the end, we state that a systematic application of the OPE, HQET and a moderately model-dependent procedure of enhancement of preasymptotic effects improves the $r_{\Lambda B}$ ratio significantly, although it cannot resolve the problem completely. We consider this problem along with the problem of absolute lifetimes of beauty hadrons to be one of the most important issues that heavy-quark physics should address in the future.

Acknowledgements

This work was supported by the Ministry of Science and Technology of the Republic of Croatia under the contract Nr. 00980102.

References

[1] M. Neubert, "B Decays and the Heavy-Quark Expansion", preprint CERN-TH/97-24 and hep-ph/9702375, to appear in "Heavy Flavors II", eds. A.J. Buras and M. Lindner.

[2] B. Blok, M. Shifman: "Lifetimes of Charmed Hadrons Revised - Facts and Fancy." in Proc. of the Workshop on the Tau-Charm Factory, p.247, Marbella, Spain, 1993, eds. J. Kirkby and R. Kirkby (Editions Frontiers, Gif-sur-Yvette, 1994), TPI-MINN-93/55-T, UMN-TH-1227/93.

[3] C. Caso et al, Eur. Phys. J. C3 (1998) 1 and 1999 off-year partial update for the 2000 edition available on PDG WWW pages (URL:http://pdg.lbl.gov/).

[4] I. Bigi, B. Blok, M. Shifman, N. Uraltsev, A. Vainshtein, preprint CERN-TH.7132/94, TPI-MINN-94/1-T, UMN-TH-1234/94, UND-HEP-94-BIG01, TECHNION-PH-94-1 and hep-ph/9401298. To appear in the second edition of the book "B Decays", S. Stone (ed), World Scientific, Singapore.

[5] G. Altarelli, G Martinelli, S. Petrarca, F. Rapuano, Phys. Lett. B 382 (1996) 409.

[6] B. Guberina, S. Nussinov, R.D. Peccei, R. Rückl, Phys. Lett. B 89 (1979) 111.

[7] N. Bilić, B. Guberina, J. Trampetić, Nucl. Phys. B 248 (1984) 261.

[8] M. Shifman, M.B. Voloshin, Yad. Fiz. 41 (1985) 463 [Sov. J. Nucl. Phys. 41 (1985) 120].

[9] M. Shifman, M.B. Voloshin, ZhETF 91 (1986) 1180 [JETP 64 (1986) 698].

[10] B. Guberina, R. Rückl, J. Trampetić, Z. Phys. C 33 (1986) 297.

[11] M.B. Voloshin, preprint TPI-MINN-99/07-T, UMN-TH-1742-99 and hep-ph/9901445.

[12] M.B. Voloshin, Phys. Lett. B 385 (1996) 369.

[13] B. Guberina, B. Melić, Eur. Phys. J. C 2 (1998) 697.

[14] N. Uraltsev, "Heavy Flavour Physics: A Probe of Nature's Grand Design", Proc. Intern. School of Physics "Enrico Fermi", Course CXXXVII, Varenna, 1997, I. Bigi and L. Moroni, Eds. (IOS Press, Amsterdam, 1998) p. 329.

[15] I. Bigi, M. Shifman, N. Uraltsev, A. Vainshtein, Phys. Rev. D 56 (1997) 4017.

[16] M. Beneke, A. Signer, preprint CERN-TH/99-163, DTP/99/60 and hep-ph/9906475.

[17] I. Bigi, Contribution to the Informal Workshop on the Derivation of $\mid V_{cb} \mid$ and $\mid V_{ub} \mid$, CERN, 1999, preprint hep-ph/9907270.

[18] B. Guberina, B. Melić, H. Štefančić, Eur. Phys. J. C 9 (1999) 213.

[19] A. De Rújula, H. Georgi, S.L. Glashow, Phys. Rev. D 12 (1975) 147.

[20] J.L. Cortes, J. Sanchez Guillen, Phys. Rev. D 24 (1981) 2982.