Unification of κ-Minkowski and extended Snyder spaces

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ARTICLE INFO

ABSTRACT

In a recent paper, we have studied associative realizations of the noncommutative extended Snyder model, obtained by including the Lorentz generators (tensorial coordinates) and their conjugated momenta. In this paper, we extend this result to also incorporate a covariant realization of the κ-Poincaré spacetime. We obtain the coproduct, the associative star product and the twist in a Weyl-ordered realization, to first order in the noncommutativity parameters. This could help the construction of a quantum field theory based on this geometry.

1. Introduction

In the last decades, noncommutative geometry has become an important topic both in mathematics and in theoretical physics, where it is considered a promising candidate for describing the structure of spacetime at the Planck scale. The mathematical framework most suitable for physical applications is based on the formalism of Hopf algebra. This structure is simultaneously an associative algebra and a coassociative coalgebra, satisfying some compatibility conditions [1], which is apt to describe the quantization of spacetime and the deformation of the momentum composition laws. Its physical applications are based on the observation that most theories of quantum gravity predict a granular structure of spacetime at small scales, with the appearance of a minimal length, that renders the classical geometric description inadequate [2].

Among the most well-known models of this kind are the Snyder space [3,4] and the κ-Poincaré model [5] with its associated κ-Minkowski spacetime [6]. Both of them are based on a deformation of the commutation relations of the position operators $\hat{x}_i$: for Snyder space one has[1]

$$[\hat{x}_i, \hat{x}_j] = i\beta \hat{M}_{ij}, \tag{1}$$

where β is a constant with dimension length square, while the operators $\hat{M}_{ij}$ are the generators of Lorentz transformations. For κ-Minkowski geometry, in a covariant setting,

$$[\hat{x}_i, \hat{x}_j] = i(a_i \hat{x}_j - a_j \hat{x}_i), \tag{2}$$

with a_i a fixed vector with dimension of length. The length scale is usually identified with the Planck length.

While in κ-Minkowski the algebra of the position operators is closed, in the Snyder case a closed algebra is obtained only if one includes the Lorentz generators with standard commutation relations as elements of the defining coordinate algebra. As a consequence, one cannot construct a proper Hopf algebra using a representation of the $\hat{M}_{ij}$ in terms of the position and momentum operators, and in fact such attempt gives rise to a noncoassociative bialgebra [4].

A way to obtain a coassociative Hopf algebra for the Snyder model was proposed in [7] and discussed in detail in [8], and is based on the introduction of new degrees of freedom given by antisymmetric tensors, identified with the Lorentz generators $\hat{M}_{ij}$. In this way, the

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[1] In this paper, Latin indices run from 0 to $N - 1$, Greek indices from 0 to N.

Snyder algebra in N dimensions becomes isomorphic to $so(1, N)$, and has been called extended Snyder algebra, to distinguish it from the standard representation based solely on the N position variables [4,7].

In this paper, using a result from [9], we extend the formalism developed in ref. [8] to include also the κ-Minkowski deformation and then achieve an unification of the extended Snyder model with the κ-Poincaré model. This will be obtained by deforming $so(1, N)$ to an algebra which leaves invariant a suitable metric g and will be denoted $so(1, N; g)$. We then consider a Weyl realization of this algebra in terms of an extended Heisenberg algebra [8] and compute its coproduct, star product and twist.

We shall only consider the Weyl realization of the κ-Poincaré Snyder algebra. Of course more general realizations can be obtained both for Snyder [4,10] and κ-Poincaré [11], that can easily be implemented in our formalism. We should also remark that a unification of the Snyder and κ-Poincaré models has been previously obtained in the standard formalism [12]. In that case, the resulting algebra is however not coassociative and the star products are not associative.

2. κ-Minkowski Snyder spacetime and $so(1, N; g)$ algebra

In ref. [9] it was proposed a formalism for constructing κ-deformations of orthogonal groups by considering transformations that leave invariant generic metric tensors. We apply this formalism to the extended Snyder model of ref. [8] to get an unification of it with κ-Minkowski spacetime.

Let us consider the Lorentz algebra $so(1, N)$ and the related enveloping algebra with generators $M_{\mu\nu}$,

$$[M_{\mu\nu}, M_{\rho\sigma}] = i\lambda(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}), \tag{3}$$

with λ a real parameter, $\eta = \text{diag}(-1, 1, \ldots, 1)$, and perform a change of basis, defining

$$\tilde{M}_{\mu\nu} = (OMO^T)_{\mu\nu}, \qquad g_{\mu\nu} = (O\eta O^T)_{\mu\nu}, \tag{4}$$

where O is an $(N+1) \times (N+1)$ matrix with transpose O^T. Then,

$$\tilde{M}^T = -\tilde{M}, \qquad g^T = g, \qquad \det g \neq 0, \tag{5}$$

and

$$[\tilde{M}_{\mu\nu}, \tilde{M}_{\rho\sigma}] = i\lambda(g_{\mu\rho}\tilde{M}_{\nu\sigma} - g_{\mu\sigma}\tilde{M}_{\nu\rho} - g_{\nu\rho}\tilde{M}_{\mu\sigma} + g_{\nu\sigma}\tilde{M}_{\mu\rho}). \tag{6}$$

We shall denote this algebra with general metric g as $so(1, N; g)$. In particular, we are interested in matrices $g_{\mu\nu}$ of the form

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & \ldots & 0 & g_0 \\ 0 & 1 & \ldots & 0 & g_1 \\ & & \ldots & & \\ 0 & 0 & \ldots & 1 & g_{N-1} \\ g_0 & g_1 & \ldots & g_{N-1} & g_N \end{pmatrix}, \tag{7}$$

with $\det g = -g_0^2 + \sum_{i=1}^{N-1} g_i^2 - g_N$. If $g_N = 0$, the algebra (6) reduces to the κ-Poincaré algebra, if $g_0 = \ldots, g_{N-1} = 0$, to the Snyder one.

In fact, we can introduce extended coordinates $\hat{x}_{\mu\nu} = M_{\mu\nu}$ and $\hat{X}_{\mu\nu} = \tilde{M}_{\mu\nu}$, so that $\hat{X}_{\mu\nu} = (O\hat{x}O^T)_{\mu\nu}$. The $\hat{x}_{\mu\nu}$ satisfy the standard $so(1, N)$ algebra, while we can split the relations (6) as

$$[\hat{X}_{iN}, \hat{X}_{jN}] = i\lambda(g_{iN}\hat{X}_{jN} - g_{jN}\hat{X}_{iN} + g_{NN}\hat{X}_{ij}),$$
$$[\hat{X}_{ij}, \hat{X}_{kN}] = i\lambda(g_{ik}\hat{X}_{jN} - g_{jk}\hat{X}_{iN} - g_{iN}\hat{X}_{jk} + g_{jN}\hat{X}_{ik}),$$
$$[\hat{X}_{ij}, \hat{X}_{kl}] = i\lambda(g_{ik}\hat{X}_{jl} - g_{il}\hat{X}_{jk} - g_{jk}\hat{X}_{il} + g_{jl}\hat{X}_{ik}). \tag{8}$$

If we now define $\hat{X}_{iN} = \kappa \hat{X}_i$ and

$$g_{ij} = \eta_{ij}, \quad g_{iN} = \kappa a_i, \quad g_{NN} = \kappa^2\beta, \tag{9}$$

we get the extended unified κ-Minkowski Snyder spacetime, with commutation relations

$$[\hat{X}_i, \hat{X}_j] = i\lambda(a_i\hat{X}_j - a_j\hat{X}_i + \beta\hat{X}_{ij}),$$
$$[\hat{X}_{ij}, \hat{X}_k] = i\lambda(\eta_{ik}\hat{X}_j - \eta_{jk}\hat{X}_i - a_i\hat{X}_{jk} + a_j\hat{X}_{ik}),$$
$$[\hat{X}_{ij}, \hat{X}_{kl}] = i\lambda(\eta_{ik}\hat{X}_{jl} - \eta_{il}\hat{X}_{jk} - \eta_{jk}\hat{X}_{il} + \eta_{jl}\hat{X}_{ik}). \tag{10}$$

In this way we obtain an unification of the Snyder and κ-Poincaré spacetimes using the formalism of extended coordinates $\hat{X}_{\mu\nu}$.

Example. As an example, let us consider an $so(1, 2; g)$ algebra. Generalization to higher dimensions is straightforward. In this case,

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & g_0 \\ 0 & 1 & g_1 \\ g_0 & g_1 & g_2 \end{pmatrix}. \tag{11}$$

The matrix O is defined up to an $SO(1,2)$ transformation. In fact, given an $SO(1,2)$ matrix R, one has $O R \eta (O R)^T = O \eta O^T = g$. Assuming $g_2 + g_0^2 - g_1^2 > 0$ and defining $\rho = \sqrt{g_2 + g_0^2 - g_1^2}$, one can write down a particularly simple expression for O, namely

$$O_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -g_0 & g_1 & \rho \end{pmatrix}. \tag{12}$$

Using (12), $\hat{X}_{\mu\nu}$ can then be written in terms of $\hat{x}_{\mu\nu}$ as

$$\hat{X}_{01} = \hat{x}_{01}, \qquad \hat{X}_{02} = \rho\, \hat{x}_{02} + g_1\, \hat{x}_{01}, \qquad \hat{X}_{12} = \rho\, \hat{x}_{12} + g_0\, \hat{x}_{01}. \tag{13}$$

3. Weyl representation of $so(1, N; g)$ with general metric

Consider the generalized Heisenberg algebra

$$[x_{\mu\nu}, x_{\rho\sigma}] = [p^{\mu\nu}, p^{\rho\sigma}] = 0, \qquad [x_{\mu\nu}, p^{\rho\sigma}] = i(\delta_\mu{}^\rho \delta_\nu{}^\sigma - \delta_\mu{}^\sigma \delta_\nu{}^\rho), \tag{14}$$

and define $X_{\mu\nu} = (O\, x\, O^T)_{\mu\nu}$ and $P^{\mu\nu} = (O^\ddagger p\, O^{-1})^{\mu\nu}$, where $O^\ddagger = (O^{-1})^T$. These variables still satisfy the commutation relations (14), but their indices are raised and lowered by means of the metric $g_{\mu\nu}$.

We want now to find a representation of the $\hat{X}_{\mu\nu}$ defined in sect. 1 in terms of the Heisenberg algebra generated by $X_{\mu\nu}$ and $P_{\mu\nu}$. The $\hat{X}_{\mu\nu}$ satisfy the $so(1, N; g)$ algebra (6), that we write as

$$[\hat{X}_{\mu\nu}, \hat{X}_{\rho\sigma}] = i\lambda C_{\mu\nu,\rho\sigma}{}^{\alpha\beta} \hat{X}_{\alpha\beta}, \tag{15}$$

where the structure constants are given by

$$C_{\mu\nu,\rho\sigma}{}^{\alpha\beta} = \frac{1}{2}\Big[-g_{\nu\rho}(\delta_\mu{}^\alpha \delta_\sigma{}^\beta - \delta_\mu{}^\beta \delta_\sigma{}^\alpha) + g_{\mu\sigma}(\delta_\rho{}^\alpha \delta_\nu{}^\beta - \delta_\rho{}^\beta \delta_\nu{}^\alpha) - (\mu \leftrightarrow \nu)\Big]. \tag{16}$$

The symmetry properties $C_{\mu\nu,\rho\sigma}{}^{\alpha\beta} = -C_{\nu\mu,\rho\sigma}{}^{\alpha\beta} = -C_{\mu\nu,\sigma\rho}{}^{\alpha\beta} = -C_{\mu\nu,\rho\sigma}{}^{\beta\alpha} = -C_{\rho\sigma,\mu\nu}{}^{\alpha\beta}$ hold.

In general, if the operators $\hat{X}_{\mu\nu}$ generate a Lie algebra with structure constants $C_{\mu\nu,\rho\sigma}{}^{\alpha\beta}$, the universal realization of $\hat{X}_{\mu\nu}$ in terms of the Heisenberg algebra (14), corresponding to Weyl-symmetric ordering, is given by [8,13,14]

$$\hat{X}_{\mu\nu} = X_{\alpha\beta} \left[\frac{\lambda\, \mathcal{C}}{1 - e^{-\lambda\mathcal{C}}} \right]_{\mu\nu}{}^{\alpha\beta}, \tag{17}$$

where $\mathcal{C}_{\mu\nu}{}^{\alpha\beta} = -\frac{1}{2} C_{\mu\nu\rho\sigma}{}^{\alpha\beta} P^{\rho\sigma}$. This realization enjoys the property

$$e^{ik^{\mu\nu}\hat{X}_{\mu\nu}} \rhd 1 = e^{ik^{\mu\nu} X_{\mu\nu}}, \qquad k^{\mu\nu} \in \mathbf{R}, \tag{18}$$

where the action $\rhd$ is defined as

$$X_{\mu\nu} \rhd f(X_{\alpha\beta}) = X_{\mu\nu} f(X_{\alpha\beta}), \qquad P^{\mu\nu} \rhd f(X_{\alpha\beta}) = -i\frac{\partial f(X_{\alpha\beta})}{\partial X_{\mu\nu}} = [P^{\mu\nu}, f(X_{\alpha\beta})]. \tag{19}$$

From (17) it follows that the Weyl realization of $\hat{X}_{\mu\nu}$ in terms of the generalized Heisenberg algebra generated by $X_{\mu\nu}$ and $P^{\mu\nu}$ reads

$$\hat{X}_{\mu\nu} = X_{\mu\nu} + \frac{\lambda}{2} X_{\alpha\beta}\, \mathcal{C}_{\mu\nu}{}^{\alpha\beta} + \frac{\lambda^2}{12} X_{\alpha\beta} \left(\mathcal{C}^2 \right)_{\mu\nu}{}^{\alpha\beta} + \mathcal{O}(\lambda^4), \tag{20}$$

where

$$\mathcal{C}_{\mu\nu}{}^{\alpha\beta} = \frac{1}{2}\Big(\delta_\mu{}^\alpha P_\nu{}^\beta + \delta_\nu{}^\beta P_\mu{}^\alpha - (\alpha \leftrightarrow \beta) \Big),$$

$$\left(\mathcal{C}^2 \right)_{\mu\nu}{}^{\alpha\beta} = \frac{1}{2}\Big(2P_\mu{}^\alpha P_\nu{}^\beta + \delta_\nu{}^\beta P_{\mu\rho} P^{\rho\alpha} + \delta_\mu{}^\alpha P_{\nu\rho} P^{\rho\beta} - (\alpha \leftrightarrow \beta) \Big), \tag{21}$$

and the indices are lowered by means of the metric $g_{\mu\nu}$.

Inserting $\mathcal{C}$ in (20), we find up to order λ^2,

$$\hat{X}_{\mu\nu} = X_{\mu\nu} + \frac{\lambda}{2}\left(X_{\mu\alpha} P_\nu{}^\alpha - X_{\nu\alpha} P_\mu{}^\alpha \right) - \frac{\lambda^2}{12}(X_{\mu\alpha} P_{\nu\beta} P^{\alpha\beta} - X_{\nu\alpha} P_{\mu\beta} P^{\alpha\beta} - 2X_{\alpha\beta} P_\mu{}^\alpha P_\nu{}^\beta), \tag{22}$$

and

$$[\hat{X}_{\mu\nu}, P^{\rho\sigma}] = i(\delta_\mu{}^\rho \delta_\nu{}^\sigma - \delta_\mu{}^\sigma \delta_\nu{}^\rho) + \frac{i\lambda}{2}(\delta_\mu{}^\rho P_\nu{}^\sigma - \delta_\nu{}^\rho P_\mu{}^\sigma + \delta_\nu{}^\sigma P_\mu{}^\rho - \delta_\mu{}^\sigma P_\nu{}^\rho)$$

$$- \frac{i\lambda^2}{12}(\delta_\mu{}^\rho P_{\nu\alpha} P^{\sigma\alpha} - \delta_\mu{}^\sigma P_{\nu\alpha} P^{\rho\alpha} - \delta_\nu{}^\rho P_{\mu\alpha} P^{\sigma\alpha} + \delta_\nu{}^\sigma P_{\mu\alpha} P^{\rho\alpha} + 2P_\mu{}^\rho P_\nu{}^\sigma - 2P_\nu{}^\rho P_\mu{}^\sigma). \tag{23}$$

Using the decomposition of $\hat{X}_{\mu\nu}$ introduced in sect. 1 and defining $X_i = \frac{1}{\kappa} X_{iN}$ and $P_i = \kappa P_{iN}$, we can then write

$$\hat{X}_i = X_i + \frac{\lambda}{2} \left(X_k P_i{}^k - \beta X_{ik} P^k - a_i X_k P^k + a_k X_i P^k + a_j X_{ik} P^{jk} \right) + \mathcal{O}(\lambda^2),$$

$$\hat{X}_{ij} = X_{ij} + \frac{\lambda}{2} \left(X_i P_j + X_{ik} P_j{}^k + a_i X_{jk} P^k - (i \leftrightarrow j) \right) + \mathcal{O}(\lambda^2). \tag{24}$$

4. Coproduct and star product in Weyl realization

Formulae for coproduct and deformed addition of momenta can be deduced from those of ref. [8], with the difference that now the sums are performed with the curved metric (7) instead of the flat metric. In particular, defining

$$e^{\frac{i}{2} k^{\mu\nu} \hat{X}_{\mu\nu}} e^{\frac{i}{2} q^{\rho\sigma} \hat{X}_{\rho\sigma}} = e^{\frac{i}{2} (k^{\mu\nu} \oplus q^{\mu\nu}) \hat{X}_{\mu\nu}} \equiv e^{\frac{i}{2} \mathcal{D}^{\mu\nu}(k,q) \hat{X}_{\mu\nu}}, \tag{25}$$

one has

$$\mathcal{D}^{\mu\nu}(k^{\alpha\beta}, q^{\alpha\beta}) = k^{\mu\nu} + q^{\mu\nu} - \frac{\lambda}{2} \left(k^{\mu\alpha} q^\nu{}_\alpha - k^{\nu\alpha} q^\mu{}_\alpha \right) + \mathcal{O}(\lambda^2). \tag{26}$$

The coproduct $\Delta P^{\mu\nu}$ is then

$$\Delta P^{\mu\nu} = \mathcal{D}^{\mu\nu}(P^{\mu\nu} \otimes 1, 1 \otimes P^{\mu\nu}) = \Delta_0 P^{\mu\nu} - \frac{\lambda}{2} \left(P^{\mu\alpha} \otimes P^\nu{}_\alpha - P^{\nu\alpha} \otimes P^\mu{}_\alpha \right) + \mathcal{O}(\lambda^2), \tag{27}$$

where $\Delta_0 P^{\mu\nu} = P^{\mu\nu} \otimes 1 + 1 \otimes P^{\mu\nu}$. The coproduct (27) is coassociative. Generally, Lie-deformed quantum Minkowski spaces admit both Hopf algebra and Hopf algebroid structures [15] and coproduct of momenta are coassociative [14].

The functions $\mathcal{D}^{\mu\nu}(q, k)$ satisfy the symmetry properties

$$\mathcal{D}^{\mu\nu}(q, k) \big|_\lambda = \mathcal{D}^{\mu\nu}(k, q) \big|_{-\lambda}. \tag{28}$$

Moreover,

$$e^{\frac{i}{2} k^{\mu\nu} X_{\mu\nu}} \star e^{\frac{i}{2} q^{\rho\sigma} X_{\rho\sigma}} = e^{\frac{i}{2} \mathcal{D}^{\mu\nu}(k,q) X_{\mu\nu}}. \tag{29}$$

This star product is associative. $\mathcal{D}^{\mu\nu}$ can be decomposed as

$$\mathcal{D}^i(k, q) = k^i + q^i - \frac{\lambda}{2} \left(k_k q^{ik} - k^{ik} q_k + a_k (k^k q^i - k^i q^k) \right) + \mathcal{O}(\lambda^2)$$

$$\mathcal{D}^{ij}(k, q) = k^{ij} + q^{ij} - \frac{\lambda}{2} \left(k^{ik} q^j{}_k + \beta k^i q^j + a_k (k^{ik} q^j + k^i q^{jk}) - (i \leftrightarrow j) \right) + \mathcal{O}(\lambda^2), \tag{30}$$

with symmetry properties

$$\mathcal{D}^i(q, k) \big|_\lambda = \mathcal{D}^i(k, q) \big|_{-\lambda}, \qquad \mathcal{D}^{ij}(q, k) \big|_\lambda = \mathcal{D}^{ij}(k, q) \big|_{-\lambda}. \tag{31}$$

5. The twist for the Weyl realization

In this section, we construct the twist operator at first order in λ, using the results of [8]. The twist is defined as a bilinear operator such that $\Delta h = \mathcal{F} \Delta_0 h \mathcal{F}^{-1}$ for each $h \in so(1, N)$.

The twist in a Hopf algebroid sense can be computed by means of the formula [16]

$$\mathcal{F}^{-1} \equiv e^F = e^{-\frac{i}{2} P^{\mu\nu} \otimes X_{\mu\nu}} e^{\frac{i}{2} P^{\rho\sigma} \otimes \hat{X}_{\rho\sigma}}. \tag{32}$$

Using the CBH formula one gets

$$F = \frac{i}{2} P^{\mu\nu} \otimes (\hat{X}_{\mu\nu} - X_{\mu\nu}) + \dots. \tag{33}$$

Substituting (20) in (33), one obtains

$$F = \frac{i\lambda}{2} P^{\alpha\gamma} \otimes X_{\alpha\beta} P_\gamma{}^\beta + \mathcal{O}(\lambda^2). \tag{34}$$

It is easy to check that

$$\mathcal{F} \Delta_0 P^{\mu\nu} \mathcal{F}^{-1} = \Delta P^{\mu\nu}, \tag{35}$$

with $\Delta P^{\mu\nu}$ given in (27). In particular,

$$\Delta P^i = \Delta_0 P^i - \frac{\lambda}{2} \left(P^k \otimes P^i{}_k - P^{ik} \otimes P_k + a_k (P^k \otimes P^i - P^i \otimes P^k) \right) + \mathcal{O}(\lambda^2),$$

$$\Delta P^{ij} = \Delta_0 P^{ij} - \frac{\lambda}{2} \left(P^{ik} \otimes P^j{}_k + \beta P^i \otimes P^j + a_k (P^{ik} \otimes P^j + P^i \otimes P^{jk}) \right.$$

$$\left. - (i \leftrightarrow j) \right) + \mathcal{O}(\lambda^2), \tag{36}$$

where

$$\Delta_0 P^i = P^i \otimes 1 + 1 \otimes P^i, \qquad \Delta_0 P^{ij} = P^{ij} \otimes 1 + 1 \otimes P^{ij}. \tag{37}$$

6. Final remarks

Using the formalism introduced in ref. [9], we have been able to unify the extended Snyder model with the κ-Minkowski spacetime. In contrast with the unified model proposed in ref. [12], the resulting coalgebra is coassociative, but this is obtained at the price of introducing new tensorial variables, whose physical interpretation is not evident, although they should be considered as internal degrees of freedom.

Our result could be useful for the construction of dispersion relations [17] and an associative field theory on a general κ-deformed Snyder spacetime.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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