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Revealing neutrino nature and CPT violation with decoherence effects

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Abstract We study decoherence effects on mixing among three generations of neutrinos. We show that in presence of a non-diagonal dissipation matrix, both Dirac and Majorana neutrinos can violate the CPT symmetry and the oscillation formulae depend on the parametrization of the mixing matrix. We reveal the CP violation in the transitions preserving the flavor, for a certain form of the dissipator. In particular, for such dissipators, the CP violation affects all the transitions in the case of Majorana neutrinos, unlike Dirac neutrinos which still preserve the CP symmetry in one of the transitions flavor preserving. The precise form of the dissipation matrix is not known a-priori as it depends on the nature of the phenomenon that originates it. However, our theoretical results show that decoherence effects, if exist for neutrinos, could allow to reveal the neutrino nature and to test fundamental symmetries.

1 Introduction

Nowadays the concept of neutrino mixing/oscillation represent one of the main missing ingredient in the Standard Model of particles, indeed its experimental verifications [1–6] stimulated lots of new investigations aimed to extend the standard theory by including a non-zero mass for neutrinos. One of the most important open issues, at both theoretical and experimental levels, is to determine the values of neutrino masses and to understand their real nature, i.e. whether they are Dirac or Majorana particles.

The most known and studied physical effect which could shed some light on neutrino nature is the neutrinoless double beta decay for which several experiments have been proposed [7], but so far no results have been obtained. Recently, to discriminate between Dirac and Majorana neutrinos also other scenarios have been proposed in which the fundamental physical quantity is not the decay rate of a process but, for instance, the Leggett–Garg K_3 quantity [8] and the geometric phase for neutrinos [9]. Moreover, it is also well known that the neutrino oscillation formulae in the presence of decoherence can depend on the Majorana phase [10,11]. This feature was used by the authors in Ref. [12] in the case of two flavors neutrinos to explicitly show how an off-diagonal dissipator can distinguish between the two kind of neutrinos and that one of the physical implications is the violation of CPT symmetry.

According to the CPT theorem, the Hamiltonian of a Lorentz invariant local quantum field theory is invariant under a simultaneous transformation of charge conjugation C, parity inversion P and time reversal T, so that CPT turns out to be an exact fundamental symmetry [13]. However, such a theorem is based on the crucial assumption that any kind of decoherence and dissipation effects are negligible.

The phenomena of dissipation and decoherence could be consequences of the interaction between neutrinos and the surrounding environment, or space–time fluctuations induced by quantum gravity effects. Many efforts have been already made in order the study dissipation and its origin in neutrino oscillations [10,11,14–25].

Here, we extend the study performed in [12] to the case of three flavors neutrinos and we reveal new features due to the presence of Dirac and Majorana phases in the mixing matrix. We consider diagonal and off-diagonal dissipators and we analyze the time evolution of the density matrix for neutrinos. We show that for an off-diagonal dissipator, in the

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three flavor mixing case, CPT symmetry can be broken both for Dirac and Majorana neutrinos because of the presence of different phases in the mixing matrix. This result is different with respect to that obtained in the case of two flavor mixing for which CPT symmetry is violated only by Majorana neutrinos [12]. Another characteristic behavior of the mixing among three families here revealed is that, for a simple off-diagonal dissipator, Majorana neutrinos can violate CP symmetry in all the flavor preserving neutrino transitions because of the presence of three phases (the Dirac phase and the two Majorana phases) in the mixing matrix. On the contrary, Dirac neutrinos can break CP symmetry only in two of the three flavor preserving transitions.

Moreover, we show that the oscillation formulae for Majorana neutrinos depend on the parametrization of the Majorana mixing matrix. Indeed, in the absence of decoherence, we know that the mixing matrix contain three phases: one for Dirac and two for Majorana, and the latter are not observable in neutrino oscillations. However, we show that for some choice of the dissipation matrix the Majorana phases turn out to be observable quantities as the oscillation formulae explicitly depend on them. As a consequence different parametrizations of the mixing matrix will give different results for the oscillation formulae.

An important point to keep in mind throughout the paper is that a-priori both the form of the dissipator and the real nature of neutrinos (Dirac or Majorana) are unknown. However, these two ingredients are generally independent of each other. The precise form of the dissipator depends on the nature of the environment and, in principle, it can be fixed with other measurements/experiments which are not related with neutrino physics. Indeed, in our theoretical setup we will assume the existence of a precise dissipator and consequently analyse the behavior of neutrinos in the corresponding environment, which already provides a suitable framework to discriminate (at least theoretically) between Dirac and Majorana neutrino. Therefore, if the decoherence affects neutrino evolution, the oscillation formulae could be able to reveal the neutrino nature and, if the neutrinos are Majorana fermions, one could also determine the right parametrization of the Majorana mixing matrix.

In this paper we only consider neutrino oscillations in vacuum in which case the violation of CP and CPT symmetries due to the decoherence are not affected by other phenomena. In fact, for neutrinos travelling, for example, through Earth, the MSW effect [26–28] already introduces an additional degree of CPT violation [29]. Therefore, one has to be careful to identify the right contribution responsible for violations purely induced by decoherence. Since we are mainly interested in highlighting the difference between Dirac and Majorana neutrinos, for simplicity we compare them in the vacuum. It is worthwhile mentioning that matter effects might be negligible or even vanishing if we assume

neutrino propagation in empty space where the source of decoherence could be attributed, for example, to spacetime fluctuations [30–36]. However, in a forthcoming paper we will extend our treatment in the presence of matter.

The work is organized as follows. In Sect. 2 we briefly review the concepts of Dirac and Majorana neutrinos and introduce the mathematical tools of the density matrix needed to compute all the oscillation formulae for three flavors neutrinos in presence of decoherence. In Sect. 3, we consider a diagonal dissipator and show that in this case the oscillation formulae are independent of the neutrino nature. In Sect. 4, we show the effects of an off-diagonal dissipator on the oscillation formulae and on the violation of CP and CPT symmetries. Moreover, we show the dependence of these quantities on the representation of the Majorana mixing matrix. In Sect. 5, we make a more quantitative but still theoretical comparison between Dirac and Majorana neutrinos and leave for future works a more realistic phenomenological analysis including matter effects. In Sect. 6 we summarize the contents of this paper by emphasizing the relevance of the main results, and draw our conclusions.

2 Neutrino mixing and decoherence

The main distinction between Dirac and Majorana neutrinos relies on the fact that Dirac Lagrangian is invariant under the global transformation of $U(1)$ so that all the associated charges (like electric, leptonic, etc.) turn out to be conserved, while Majorana Lagrangian breaks the $U(1)$ symmetry. A process in which the lepton number is violated and therefore would be allowed only for Majorana neutrinos and not for Dirac is the neutrinoless double beta decay.

The breaking of the $U(1)$ global symmetry has also consequences on the form of the mixing matrix [37] which contains a different number of physical phases for the two kind of neutrinos. Indeed, in the general case of the mixing with n Dirac fields, there exist $N_D = \frac{(n-1)(n-2)}{2}$ physical phases, while for n Majorana fields, one has additional $N_M = \frac{n(n-1)}{2}$ phases. The $n-1$ extra phases are called Majorana phases and their detection would allow to identify the nature of neutrinos.

Let us recall that the Lagrangian density for Dirac neutrinos in flavor basis is given by

$$\mathcal{L}(x) = \bar{\Psi}_f(x)\left(i\slashed{\partial} - M\right)\Psi_f(x), \tag{1}$$

where $\Psi_f^T = (\nu_e, \nu_\mu, \nu_\tau)$ and $M^\dagger = M$ is the mixed mass term. The mixing relations are [37–39]:

$$\Psi_f(x) = \mathcal{U}_D \Psi_m(x)$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \Psi_m(x), \tag{2}$$

where $\mathcal{U}_D$ is the Dirac mixing matrix, δ is the Dirac phase, $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$, with θ_{ij} being the mixing angles between the fields with definite masses ν_i, ν_j with $i, j = e, \mu, \tau$, and $\Psi_m^T = (\nu_1, \nu_2, \nu_3)$. Eq. (1) is diagonalized by using Eq. (2), so that we obtain the Lagrangian for free Dirac fermions with masses m_1, m_2 and m_3 :

$$\mathcal{L}(x) = \bar{\Psi}_m(x) \left(i \not{\partial} - M_d \right) \Psi_m(x), \tag{3}$$

where $M_d = \text{diag}(m_1, m_2, m_3)$.

For Majorana neutrinos, different parametrizations of the mixing matrix $\mathcal{U}_M$, exist. In fact, when decoherence is negligible, and even in the case of a diagonal dissipator, all the transition probabilities turn out to be invariant under the rephasing $\mathcal{U}_{\alpha k} \rightarrow e^{i\phi_k} \mathcal{U}_{\alpha k}$ ($\alpha = e, \mu$; $k = 1, 2$). This means that the Majorana phases ϕ_i do not affect the oscillation formulae which are the same as for Dirac neutrinos [40,41]. For instance, one can write

$$\mathcal{U}_M = \mathcal{U}_D \cdot \text{diag} \left(1, e^{i\phi_1}, e^{i\phi_2} \right), \tag{4}$$

where ϕ_1 and ϕ_2 are the two Majorana phases. Another possible parametrization is the following:

$$\mathcal{U}_M = \left(1, e^{-i\phi_1}, e^{-i\phi_2} \right) \cdot \mathcal{U}_D \cdot \text{diag} \left(1, e^{i\phi_1}, e^{i\phi_2} \right)$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\phi_1} & s_{13}e^{i(\phi_2-\delta)} \\ -s_{12}c_{23}e^{-\phi_1} - c_{12}s_{23}s_{13}e^{i(\delta-\phi_1)} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i(\phi_2-\phi_1)} \\ s_{12}s_{23}e^{-i\phi_2} - c_{12}c_{23}s_{13}e^{i(\delta-\phi_2)} & -c_{12}s_{23}e^{i(\phi_1-\phi_2)} - s_{12}c_{23}s_{13}e^{i(\delta+\phi_1-\phi_2)} & c_{23}c_{13} \end{pmatrix}, \tag{5}$$

and other choices leading to the same oscillation formulae are presented in Ref. [42].

This fact is no longer true when there are off-diagonal elements in dissipation matrix and also in the case of diagonal dissipator with $\gamma_1 \neq \gamma_2$, or $\gamma_4 \neq \gamma_5$, or $\gamma_6 \neq \gamma_7$. Indeed, one can obtain oscillation formulae for Majorana neutrinos depending on the phases ϕ_i, and on the parametrization of the mixing matrix, as shown in Ref. [12] for two flavor mixing and non-diagonal dissipator. In the following we will consider the case of three flavor neutrino mixing and reveal new aspects of neutrino oscillations which are absent in the case of mixing between two neutrinos. In the rest of the paper, we mainly focus on the matrix given in (5), which will be very useful to highlight the main features in presence of decoherence.

By treating the neutrino as an open quantum system, we analyze the physical implications of decoherence in flavor mixing. In particular, we study the time evolution of the density matrix corresponding to the neutrino state in the flavor basis and compute several transition probabilities for both diagonal and non-diagonal dissipation matrix.

The state evolution of neutrinos seen as an open system, can be described by the Lindblad–Kossakowski master equation [43,44]:

$$\frac{\partial \rho(t)}{\partial t} = -i \left[H, \rho(t) \right] + D[\rho(t)], \tag{6}$$

where $H = H^\dagger$ is the total Hamiltonian of the system and $D[\rho(t)]$ is the dissipator defined as

$$D[\rho(t)] = \frac{1}{2} \sum_{i,j=0}^{N^2-1} a_{ij} \left(\left[F_i \rho(t), F_j^\dagger \right] + \left[F_i, \rho(t) F_j^\dagger \right] \right), \tag{7}$$

with a_{ij} Kossakowski coefficients whose form is related to the characteristics of the environment [10,11]. The operators F_i, with $i = 1, \ldots, N^2 - 1$, satisfy the relations $\text{Tr}(F_i) = 0$ and $\text{Tr}\left(F_i^\dagger F_j \right) = \delta_{ij}$, and in the case of three flavor neutrinos they are the Gell-Mann matrices λ_i which satisfy the following properties:

$$\lambda_i^\dagger = \lambda_i, \quad [\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k,$$
$$f_{ijk} = -\frac{i}{4} \text{Tr} \left(\lambda_i [\lambda_j, \lambda_k] \right). \tag{8}$$

Here the non-vanishing f_{ijk} are given by $f^{123} = 1$, $f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}$, $f^{458} = f^{678} = \frac{\sqrt{3}}{2}$.

Let us now expand Eqs. (6) and (7) in the basis of $SU(3)$:

$$\dot{\rho}_\mu(t) = f_{ij\mu} H_i \rho_j(t) + D_{\mu\nu} \rho_\nu(t), \tag{9}$$

where $\rho_\mu = \text{Tr}\left(\rho \lambda_\mu \right)$, with $\mu = 0, \ldots, 8$. Given the mass differences $\Delta m_{21}^2 = m_2^2 - m_1^2$ and $\Delta m_{31}^2 = m_3^2 - m_1^2$, the Hamiltonian reads

$$H = \frac{1}{2E} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix}, \tag{10}$$

where $\Delta_{21} = \frac{1}{2E} \Delta m_{21}^2$ and $\Delta_{31} = \frac{1}{2E} \Delta m_{31}^2$. One can show that the only non-vanishing components H_μ are

$$H_0 = \Delta_{21} + \Delta_{31}, \quad H_3 = -\Delta_{21},$$
$$H_8 = \frac{1}{\sqrt{3}} \left(\Delta_{21} - 2\Delta_{31} \right). \tag{11}$$

The dissipator in Eq. (9) is given by

$$
D_{\mu\nu} = - \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_1 & \alpha_1 & \beta_1 & \delta_1 & \chi_1 & \xi_1 & \zeta_1 & \eta_1 \\
0 & \alpha_1 & \gamma_2 & \alpha_2 & \beta_2 & \delta_2 & \chi_2 & \xi_2 & \zeta_2 \\
0 & \beta_1 & \alpha_2 & \gamma_3 & \alpha_3 & \beta_3 & \delta_3 & \chi_3 & \xi_3 \\
0 & \delta_1 & \beta_2 & \alpha_3 & \gamma_4 & \alpha_4 & \beta_4 & \delta_4 & \chi_4 \\
0 & \chi_1 & \delta_2 & \beta_3 & \alpha_4 & \gamma_5 & \alpha_5 & \beta_5 & \delta_5 \\
0 & \xi_1 & \chi_2 & \delta_3 & \beta_4 & \alpha_5 & \gamma_6 & \alpha_6 & \beta_6 \\
0 & \zeta_1 & \xi_2 & \chi_3 & \delta_4 & \beta_5 & \alpha_6 & \gamma_7 & \alpha_7 \\
0 & \eta_1 & \zeta_2 & \xi_3 & \chi_4 & \delta_5 & \beta_6 & \alpha_7 & \gamma_8
\end{pmatrix}. \tag{12}
$$

where we considered the probability conservation which implies $D_{\mu 0} = D_{0\nu} = 0$. All the elements in the matrix (12) are real and the ones on the diagonal are positive in order to satisfy the relation $\mathrm{Tr}\,(\rho(t)) = 1$. Hence, from Eq. (9) it is now clear that we have nine equations among which the $\mu = 0$ component is trivial. Indeed, since $f_{ij0} = 0$ and $D_{0\nu} = 0$ we obtain $\dot{\rho}_0(t) = 0 \Rightarrow \rho_0(t) = 1$.

The density matrix written in terms of the components ρ_μ in the basis λ_μ reads

$$
\rho(t) = \frac{1}{3}\rho_0(t)\lambda_0 + \frac{1}{2}\sum_{i=1}^{8}\rho_i(t)\lambda_i
$$

$$
= \frac{1}{2}\begin{pmatrix}
\frac{2}{3}\rho_0 + \rho_3 + \frac{\rho_8}{\sqrt{3}} & \rho_1 - i\rho_2 & \rho_4 - i\rho_5 \\
\rho_1 + i\rho_2 & \frac{2}{3}\rho_0 - \rho_3 + \frac{\rho_8}{\sqrt{3}} & \rho_6 - i\rho_7 \\
\rho_4 + i\rho_5 & \rho_6 + i\rho_7 & \frac{2}{3}\rho_0 - \frac{2}{\sqrt{3}}\rho_8
\end{pmatrix}. \tag{13}
$$

With this expression of the density matrix, the neutrino oscillation formulae reads

$$
P_{\nu_a \to \nu_b} = \frac{1}{3} + \frac{1}{2}\sum_{i=1}^{8}\rho_{a,i}(t)\rho_{b,i}(0). \tag{14}
$$

Generally, the dissipation matrix can depend on whether we consider neutrino or anti-neutrino. In fact, if the nuclear interactions are also sources of decoherence, then, it exists the distinction between the dissipation matrices of particles and antiparticles. In this paper we analyse neutrino oscillations in vacuum where the neutrino weak and strong interactions are negligible, so that decoherence effects do not discriminate between neutrino and anti-neutrino and we can assume the dissipator to be the same in both cases. The difference between neutrinos and antineutrinos is expressed only by the opposite sign of the Dirac phase and of the Majorana phases for particles and antiparticles. Moreover, throughout our study we assume that the elements of the dissipation matrix are given a-priori and we do not question what kind of model or experiment can determine their specific values. Indeed, in our theoretical setup we assume the existence of a precise dissipator and consequently analyse the behavior of

neutrinos in the corresponding environment, and this procedure already provides a suitable framework to discriminate between Dirac and Majorana neutrinos. To further motivate our analysis let us also mention that our assumption can be relevant at the phenomenological level when considering decoherence effects induced by spacetime fluctuations which are predicted by several models of quantum gravity [30–36]; see also Ref. [45] where a specific form of dissipator was derived in the context of decoherence induced by a quantum gravitational field. In this case the only relevant interaction is gravity which is universal and cannot distinguish between particles and anti-particles.

Notice that, the CP symmetry violation is defined as $\Delta CP_{ab} \equiv P_{\nu_a \to \nu_b} - P_{\bar{\nu}_a \to \bar{\nu}_b} \neq 0$ and the T violation is given by $\Delta T_{ab} \equiv P_{\nu_a \to \nu_b} - P_{\nu_b \to \nu_a} \neq 0$. The CPT symmetry is violated when $\Delta CP \neq \Delta T$.

3 Diagonal dissipator

We now study decoherence effects considering the mixing matrix (5). We analyze both cases of zero and non-zero Majorana phases. We start by solving the set of equations (9) in the simpler case of a diagonal dissipator:

$$
D_{\mu\nu} = -\mathrm{diag}\,(0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8). \tag{15}
$$

Then, the system of differential equations (9) becomes

$$
\begin{aligned}
\dot{\rho}_0(t) &= 0, \\
\dot{\rho}_1(t) &= \Delta_{21}\rho_2(t) - \gamma_1\rho_1(t), \\
\dot{\rho}_2(t) &= -\Delta_{21}\rho_1(t) - \gamma_2\rho_2(t), \\
\dot{\rho}_3(t) &= -\gamma_3\rho_3(t), \\
\dot{\rho}_4(t) &= \Delta_{31}\rho_5(t) - \gamma_4\rho_4(t), \\
\dot{\rho}_5(t) &= -\Delta_{31}\rho_4(t) - \gamma_5\rho_5(t), \\
\dot{\rho}_6(t) &= \Delta_{32}\rho_7(t) - \gamma_6\rho_6(t), \\
\dot{\rho}_7(t) &= -\Delta_{32}\rho_6(t) - \gamma_7\rho_7(t), \\
\dot{\rho}_8(t) &= -\gamma_8\rho_8(t),
\end{aligned} \tag{16}
$$

where $\Delta_{32} = \Delta_{31} - \Delta_{21} = \frac{\Delta m_{32}^2}{2E}$.

We consider now the diagonal dissipator Eq. (15) with the conditions: $\gamma_1 = \gamma_2 = \gamma_{12}$, $\gamma_4 = \gamma_5 = \gamma_{45}$, $\gamma_6 = \gamma_7 = \gamma_{67}$. This choice is consistent with that of Ref. [46]. The system of equations can be solved as follows:

$$
\begin{aligned}
\rho_0(t) &= 1, \\
\rho_1(t) &= e^{-\gamma_{12}t}\left[\rho_1(0)\cos(\Delta_{21}t) + \rho_2(0)\sin(\Delta_{21}t)\right], \\
\rho_2(t) &= e^{-\gamma_{12}t}\left[\rho_2(0)\cos(\Delta_{21}t) - \rho_1(0)\sin(\Delta_{21}t)\right], \\
\rho_3(t) &= e^{-\gamma_3 t}\rho_3(0), \\
\rho_4(t) &= e^{-\gamma_{45}t}\left[\rho_4(0)\cos(\Delta_{31}t) + \rho_5(0)\sin(\Delta_{31}t)\right], \\
\rho_5(t) &= e^{-\gamma_{45}t}\left[\rho_5(0)\cos(\Delta_{31}t) - \rho_4(0)\sin(\Delta_{31}t)\right],
\end{aligned}
$$

$$\rho_6(t) = e^{-\gamma_{67}t} \left[\rho_6(0)\cos(\Delta_{32}t) + \rho_7(0)\sin(\Delta_{32}t) \right],$$
$$\rho_7(t) = e^{-\gamma_{67}t} \left[\rho_7(0)\cos(\Delta_{32}t) - \rho_6(0)\sin(\Delta_{32}t) \right],$$
$$\rho_8(t) = e^{-\gamma_8 t} \rho_8(0). \tag{17}$$

The initial conditions $\rho_i(0)$ can be found by employing the following relations: $\rho_a(0) = |\nu_a\rangle \langle \nu_a|$, $a = e, \mu, \tau$. For electronic neutrino we have

$$\rho_{e,0}(0) = 1,$$
$$\rho_{e,1}(0) = \sin(2\theta_{12})\cos^2\theta_{13}\cos\phi_1,$$
$$\rho_{e,2}(0) = \sin(2\theta_{12})\cos^2\theta_{13}\sin\phi_1,$$
$$\rho_{e,3}(0) = \cos^2\theta_{13}\left(2\cos^2\theta_{12} - 1\right),$$
$$\rho_{e,4}(0) = \sin(2\theta_{13})\cos\theta_{12}\cos(\phi_2 - \delta),$$
$$\rho_{e,5}(0) = \sin(2\theta_{13})\cos\theta_{12}\sin(\phi_2 - \delta),$$
$$\rho_{e,6}(0) = \sin(2\theta_{13})\sin\theta_{12}\cos(\phi_2 - \phi_1 - \delta),$$
$$\rho_{e,7}(0) = \sin(2\theta_{13})\sin\theta_{12}\sin(\phi_2 - \phi_1 - \delta),$$
$$\rho_{e,8}(0) = \sqrt{3}\left(\frac{1}{3} - \sin^2\theta_{13}\right). \tag{18}$$

For muon neutrino we obtain

$$\rho_{\mu,0}(0) = 1,$$
$$\begin{aligned}\rho_{\mu,1}(0) = &-\sin(2\theta_{12})\cos^2\theta_{23}\cos\phi_1 \\ &-\sin(2\theta_{23})\sin\theta_{13}\cos^2\theta_{12}\cos(\delta - \phi_1) \\ &+\sin(2\theta_{23})\sin^2\theta_{12}\sin\theta_{13}\cos(\delta + \phi_1) \\ &+\sin(2\theta_{12})\sin^2\theta_{23}\sin^2\theta_{13}\cos\phi_1,\end{aligned}$$
$$\begin{aligned}\rho_{\mu,2}(0) = &-\sin(2\theta_{12})\cos^2\theta_{23}\sin\phi_1 \\ &+\sin(2\theta_{23})\sin\theta_{13}\cos^2\theta_{12}\sin(\delta - \phi_1) \\ &+\sin(2\theta_{23})\sin^2\theta_{12}\sin\theta_{13}\sin(\delta + \phi_1) \\ &+\sin(2\theta_{12})\sin^2\theta_{23}\sin^2\theta_{13}\sin\phi_1,\end{aligned}$$
$$\begin{aligned}\rho_{\mu,3}(0) = &-1 + \sin^2\theta_{23}\cos^2\theta_{13} + 2\sin^2\theta_{12}\cos^2\theta_{23} \\ &+2\cos^2\theta_{12}\sin^2\theta_{23}\sin^2\theta_{13} \\ &+\sin(2\theta_{12})\sin(2\theta_{23})\sin\theta_{13}\cos\delta,\end{aligned}$$
$$\begin{aligned}\rho_{\mu,4}(0) = &-\sin(2\theta_{23})\sin\theta_{12}\cos\theta_{13}\cos\phi_2 \\ &-\sin(2\theta_{13})\cos\theta_{12}\sin^2\theta_{23}\cos(\phi_2 - \delta),\end{aligned}$$
$$\begin{aligned}\rho_{\mu,5}(0) = &-\sin(2\theta_{23})\sin\theta_{12}\cos\theta_{13}\sin\phi_2 \\ &-\sin(2\theta_{13})\cos\theta_{12}\sin^2\theta_{23}\sin(\phi_2 - \delta),\end{aligned}$$
$$\begin{aligned}\rho_{\mu,6}(0) = &\sin(2\theta_{23})\cos\theta_{12}\cos\theta_{13}\cos(\phi_2 - \phi_1) \\ &-\sin(2\theta_{13})\sin\theta_{12}\sin^2\theta_{23}\cos(\phi_2 - \phi_1 - \delta),\end{aligned}$$
$$\begin{aligned}\rho_{\mu,7}(0) = &\sin(2\theta_{23})\cos\theta_{12}\cos\theta_{13}\sin(\phi_2 - \phi_1) \\ &-\sin(2\theta_{13})\sin\theta_{12}\sin^2\theta_{23}\sin(\phi_2 - \phi_1 - \delta),\end{aligned}$$

$$\rho_{\mu,8}(0) = \sqrt{3}\left(\frac{1}{3} - \sin^2\theta_{23}\cos^2\theta_{13}\right); \tag{19}$$

and finally for tau neutrino

$$\rho_{\tau,0}(0) = 1,$$
$$\begin{aligned}\rho_{\tau,1}(0) = &-\sin(2\theta_{12})\sin^2\theta_{23}\cos\phi_1 \\ &+\sin(2\theta_{23})\sin\theta_{13}\cos^2\theta_{12}\cos(\phi_1 - \delta) \\ &-\sin(2\theta_{23})\sin^2\theta_{12}\sin\theta_{13}\cos(\delta + \phi_1) \\ &+\sin(2\theta_{12})\cos^2\theta_{23}\sin^2\theta_{13}\cos\phi_1,\end{aligned}$$
$$\begin{aligned}\rho_{\tau,2}(0) = &-\sin(2\theta_{12})\sin^2\theta_{23}\sin\phi_1 \\ &+\sin(2\theta_{23})\sin\theta_{13}\cos^2\theta_{12}\sin(\phi_1 - \delta) \\ &-\sin(2\theta_{23})\sin^2\theta_{12}\sin\theta_{13}\cos(\delta + \phi_1) \\ &+\sin(2\theta_{12})\cos^2\theta_{23}\sin^2\theta_{13}\cos\phi_1 ,\end{aligned}$$
$$\begin{aligned}\rho_{\tau,3}(0) = &-1 + \cos^2\theta_{23}\cos^2\theta_{13} \\ &+2\sin^2\theta_{12}\sin^2\theta_{23} + 2\cos^2\theta_{12}\cos^2\theta_{23}\sin^2\theta_{13} \\ &-\sin(2\theta_{12})\sin(2\theta_{23})\sin\theta_{13}\cos\delta,\end{aligned}$$
$$\begin{aligned}\rho_{\tau,4}(0) = &\sin(2\theta_{23})\sin\theta_{12}\cos\theta_{13}\cos\phi_2 \\ &-\sin(2\theta_{13})\cos\theta_{12}\cos^2\theta_{23}\cos(\phi_2 - \delta) ,\end{aligned}$$
$$\begin{aligned}\rho_{\tau,5}(0) = &\sin(2\theta_{23})\sin\theta_{12}\cos\theta_{13}\sin\phi_2 \\ &-\sin(2\theta_{13})\cos\theta_{12}\cos^2\theta_{23}\sin(\phi_2 - \delta),\end{aligned}$$
$$\begin{aligned}\rho_{\tau,6}(0) = &-\sin(2\theta_{23})\cos\theta_{12}\cos\theta_{13}\cos(\phi_1 - \phi_2) \\ &-\sin(2\theta_{13})\sin\theta_{12}\cos^2\theta_{23}\cos(\delta + \phi_1 - \phi_2),\end{aligned}$$
$$\begin{aligned}\rho_{\tau,7}(0) = &\sin(2\theta_{23})\cos\theta_{12}\cos\theta_{13}\sin(\phi_1 - \phi_2) \\ &+\sin(2\theta_{13})\sin\theta_{12}\cos^2\theta_{23}\sin(\delta + \phi_1 - \phi_2),\end{aligned}$$
$$\rho_{\tau,8}(0) = \sqrt{3}\left(\frac{1}{3} - \cos^2\theta_{23}\cos^2\theta_{13}\right). \tag{20}$$

The neutrino oscillation probabilities, as said above, are obtained through the relation $P_{\nu_a \to \nu_b} = \text{Tr}\left[\rho_b(t) \cdot \rho_a(0)\right]$. By computing the transition probability in the case of a diagonal dissipator as in Eq. (15), for flavor preserving transitions we obtain

$$\Delta CP_{aa} = P_{\nu_a \to \nu_a} - P_{\bar{\nu}_a \to \bar{\nu}_a} = 0, \quad a = e, \mu, \tau. \tag{21}$$

In similar way, $\Delta T_{aa} = 0$. These result are the same of those obtained in the absence of decoherence. Moreover, like in the standard case, CP and T symmetries are violated because of the presence of the Dirac phase δ, while the presence of diagonal elements in the dissipation matrix only introduces a damping factor which is physically expected. For instance, the three channels responsible for CP violations read

$$\begin{aligned}\Delta CP_{e\mu} &= P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_e \to \bar{\nu}_\mu} \\ &= \sin\delta \cos^2\theta_{13}\sin(\theta_{12})\sin(2\theta_{23})\sin\theta_{13}\end{aligned}$$

$$\times \left[\sin(\Delta_{32}t)e^{-\gamma_{67}t} + \sin(\Delta_{21}t)e^{-\gamma_{12}t} \right.$$
$$\left. -\sin(\Delta_{31}t)e^{-\gamma_{45}t} \right],$$

$$\Delta CP_{e\tau} = P_{\nu_e \to \nu_\tau} - P_{\bar{\nu}_e \to \bar{\nu}_\mu} = -\Delta CP_{e\mu},$$
$$\Delta CP_{\mu\tau} = P_{\nu_\mu \to \nu_\tau} - P_{\bar{\nu}_\mu \to \bar{\nu}_\tau} = \Delta CP_{e\mu}. \quad (22)$$

Note that the sum of CP violations for fixed family is vanishing, as expected, i.e. we have

$$\Delta CP_{e\mu} + \Delta CP_{e\tau} = 0, \quad \Delta CP_{\mu e} + \Delta CP_{\mu\tau} = 0,$$
$$\Delta CP_{\tau e} + \Delta CP_{\tau\mu} = 0. \quad (23)$$

Moreover, they not depend on γ_3 and γ_8. Similar behaviors also manifest for T violating channels:

$$\Delta T_{e\mu} = P_{\nu_e \to \nu_\mu} - P_{\nu_\mu \to \nu_e} = \Delta CP_{e\mu}$$
$$\Delta T_{e\tau} = P_{\nu_e \to \nu_\tau} - P_{\nu_\tau \to \nu_e} = \Delta CP_{e\tau}$$
$$\Delta T_{\mu\tau} = P_{\nu_\mu \to \nu_\tau} - P_{\nu_\tau \to \nu_\mu} = \Delta CP_{\mu\tau}. \quad (24)$$

Hence, in presence of a diagonal dissipation matrix, CP and T are violated, but CPT is still preserved as in the standard case where no decoherence effects are present, i.e. $\Delta CP_{ab} = \Delta T_{ab}$.

Furthermore, it is clear that in such a case, the Majorana phases ϕ_1 and ϕ_2 do not play any role, indeed all oscillation formula are independent of them. The violation of CP and T is related only to the presence of the Dirac phase, indeed if we set $\delta = 0$ we recover CP and T invariance also in presence of a diagonal dissipator. Different results can be obtained for the following specific choices of the diagonal elements of the dissipator: $\gamma_1 \neq \gamma_2$, or $\gamma_4 \neq \gamma_5$, or $\gamma_6 \neq \gamma_7$. In these cases, one can show that the oscillation formulae and the CP and T violations depend on the Majorana phases.

4 Non-diagonal dissipator

We now study the scenario with a non-diagonal dissipator. In this paper, for simplicity, we consider the case for which only two symmetric off-diagonal elements are non-zero, in particular we focus on following form for the dissipator:

$$D_{11} = D_{22} = -\gamma_{12}, \quad D_{33} = -\gamma_3, \quad D_{44} = D_{55} = -\gamma_{45},$$
$$D_{66} = D_{77} = -\gamma_{67}, \quad D_{88} = -\gamma_8, \quad D_{12} = D_{21} = -\alpha_1 \quad (25)$$

while the remaining components are zero; we will also comment on what happens if other off-diagonal elements are switched on. For the dissipator in Eq. (25) by solving the set of equations (9), we obtain a system of differential equations similar to that in Eq.(16), where the components $\dot{\rho}_1$ and $\dot{\rho}_2$ reported in Eq. (16) are now replaced by the following differential equations

$$\dot{\rho}_1(t) = \Delta_{21}\rho_2(t) - \gamma_{12}\rho_1(t) - \alpha_1\rho_2(t),$$

$$\dot{\rho}_2(t) = -\Delta_{21}\rho_1(t) - \gamma_{12}\rho_2(t) - \alpha_1\rho_1(t). \quad (26)$$

respectively, and whose solutions read

$$\rho_1(t) = e^{-\gamma_{12}t} \left[\rho_1(0)\cosh(\Omega t) + \rho_2(0)\sinh(\Omega t)\frac{\Xi_+}{\Omega} \right],$$
$$\rho_2(t) = e^{-\gamma_{12}t} \left[\rho_1(0)\sinh(\Omega t)\frac{\Xi_-}{\Omega} + \rho_2(0)\cosh(\Omega t) \right],$$
$$(27)$$

while the other components are the same the ones in (17). We have defined the quantities $\Omega \equiv \sqrt{\alpha_1^2 - \Delta_{21}^2}$ and $\Xi_\pm \equiv \alpha_1 \pm \Delta_{21}$. The initial conditions $\rho_i(0)$ are the same as in Eqs. (18), (19) and (20) for electronic, muon and tau neutrinos, respectively.

Let us now distinguish two cases: (A) first, we consider a mixing matrix with zero Majorana phases to show the role played by the Dirac phase in the violation of CP and CPT symmetries; (B) subsequently, we compute the oscillation probabilities considering non-zero Majorana phases and analyze the effects on CP and CPT violations.

4.1 Zero Majorana phases

We set $\phi_1 = \phi_2 = 0$, which means that we work with the mixing matrix $\mathcal{U}_D$ in Eq. (2), i.e. with Dirac neutrinos. We have the following results for the transitions preserving the flavor:

$$\Delta CP_{ee} = P_{\nu_e \to \nu_e} - P_{\bar{\nu}_e \to \bar{\nu}_e} = 0,$$
$$\Delta CP_{\mu\mu} = P_{\nu_\mu \to \nu_\mu} - P_{\bar{\nu}_\mu \to \bar{\nu}_\mu}$$
$$= \frac{2\alpha_1 e^{-\gamma_{12}t}\sin\delta}{\Omega}\sin(2\theta_{23})\sin\theta_{13}\sinh(\Omega t)$$
$$\times \left[\cos^2(\theta_{23})\sin(2\theta_{12}) \right.$$
$$+\sin\theta_{13}\left(\cos(2\theta_{12})\sin(2\theta_{23})\right.$$
$$\left.\left. -\sin(2\theta_{12})\sin^2\theta_{23}\sin\theta_{13}\right) \right],$$
$$\Delta CP_{\tau\tau} = P_{\nu_\tau \to \nu_\tau} - P_{\bar{\nu}_\tau \to \bar{\nu}_\tau}$$
$$= \frac{2\alpha_1 e^{-\gamma_{12}t}\sin\delta}{\Omega}\sin(2\theta_{23})\sin\theta_{13}\sinh(\Omega t)$$
$$\times \left[\cos^2(\theta_{23})\sin^2\theta_{13}\sin(2\theta_{12}) \right.$$
$$\left. -\sin(2\theta_{12})\sin^2\theta_{23} + \cos(2\theta_{12})\sin\theta_{13}\sin(2\theta_{23}) \right].$$
$$(28)$$

In Eq. (28) it is shown that the violation of CP appears in the transitions $\nu_\mu \to \nu_\mu$ and $\nu_\tau \to \nu_\tau$. On the contrary, the transition $\nu_e \to \nu_e$ preserves such a symmetry. Notice that $\Delta CP_{\mu\mu}$ and $\Delta CP_{\tau\tau}$ does not appear either in absence of decoherence or in presence of a diagonal dissipator. As we will see in the next subsection, for Majorana neutrinos

$\Delta CP_{ee} \neq 0$. Therefore, the analysis of such a violation could be crucial in order to discriminate between Dirac and Majorana neutrinos in presence of an off-diagonal dissipation matrix.

Moreover, the CP violating channels for different neutrinos are modified as follows:

$$
\begin{aligned}
\Delta CP_{e\mu} &= P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_e \to \bar{\nu}_\mu} \\
&= \frac{\sin\delta}{\Omega} \cos^2\theta_{13} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin\theta_{13} \\
&\quad \times \left[\Omega \left(e^{-\gamma_{45}t} \sin(\Delta_{31}t) - e^{-\gamma_{67}t} \sin(\Delta_{32}t) \right) \right. \\
&\quad \left. - e^{-\gamma_{12}t} (\alpha_1 - \Delta_{21}) \sinh(\Omega t) \right],
\end{aligned}
$$

$$
\Delta CP_{e\tau} = P_{\nu_e \to \nu_\tau} - P_{\bar{\nu}_e \to \bar{\nu}_\tau} = -\Delta CP_{e\mu},
$$

$$
\begin{aligned}
\Delta CP_{\mu\tau} &= P_{\nu_\mu \to \nu_\tau} - P_{\bar{\nu}_\mu \to \bar{\nu}_\tau} \\
&= \frac{\sin\delta}{4\Omega} \sin\theta_{13} \sin(2\theta_{23}) \left[4\Omega\cos^2\theta_{13}\sin(2\theta_{12}) \right. \\
&\quad \times \left(-e^{-\gamma_{67}t}\sin(\Delta_{32}t) + e^{-\gamma_{45}t}\sin(\Delta_{31}t) \right) \\
&\quad - 2e^{-\gamma_{12}t}\left(\sin(2\theta_{12}) \left(2\Delta_{21}\cos^2(\theta_{13}) \right.\right. \\
&\quad \left. -\alpha_1\left(\cos(2\theta_{13}) - 3\right)\cos(2\theta_{23})\right) \\
&\quad \left.\left. +4\alpha_1\cos\delta\cos(2\theta_{12})\sin\theta_{13}\sin(2\theta_{23})\right) \sinh(\Omega t) \right].
\end{aligned}
$$
(29)

The T violations also differ from the diagonal case and are given by

$$
\begin{aligned}
\Delta T_{e\mu} &= P_{\nu_e \to \nu_\mu} - P_{\nu_\mu \to \nu_e} \\
&= \frac{\sin\delta}{\Omega} \cos^2\theta_{13} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin\theta_{13} \\
&\quad \times \left[\Omega \left(e^{-\gamma_{45}t}\sin(\Delta_{31}t) - e^{-\gamma_{67}t}\sin(\Delta_{32}t) \right) \right. \\
&\quad \left. - 2\Delta_{21} e^{-\gamma_{12}t}\sinh(t\Omega) \right],
\end{aligned}
$$

$$
\Delta T_{e\tau} = P_{\nu_e \to \nu_\tau} - P_{\nu_\tau \to \nu_e} = -\Delta T_{e\mu},
$$

$$
\Delta T_{\mu\tau} = P_{\nu_\mu \to \nu_\tau} - P_{\nu_\tau \to \nu_\mu} = \Delta T_{e\mu}.
$$
(30)

Therefore, unlike the case of a diagonal dissipator, when $\alpha_1 \neq 0$, not only CP and T are violated, but also CPT symmetry is not preserved:

$$
\Delta CP_{e\mu} \neq \Delta T_{e\mu}, \quad \Delta CP_{e\tau} \neq \Delta T_{e\tau},
$$

$$
\Delta CP_{\tau\mu} \neq \Delta T_{\tau\mu}.
$$
(31)

Such violations are related to the presence of the Dirac phase, indeed by setting $\delta = 0$, all the three symmetries are preserved even if $\alpha_1 \neq 0$. Let us point out that such an effect is not present in the two flavors case analyzed in [12] since in that case no Dirac phase is present and one can not find any relation between the phase δ and CPT violation. The CPT violation induced by Dirac phase is a new feature in presence of decoherence and dissipation. If we set $\alpha_1 = 0$ we recover the case of diagonal dissipator where CPT symmetry is preserved.

Let us emphasize that so far we have only considered one possible case of non-diagonal dissipator, in which only α_1

is non-zero. Of course, also other kinds of dissipation matrices can be studied in which other off-diagonal elements are non-zero. By making computations similar to those presented above, one can show that all the possible choices of the dissipator (12) lead to CP and T violations, as it also happens in the diagonal case. On the other hand, CPT is violated in most of the cases; however, there are some off-diagonal choices which still preserve it. Indeed, CPT symmetry is respected when the only non-zero off-diagonal element is one of the following: β_1, α_3, δ_3, ξ_3, η_1, ζ_2, χ_4, δ_5, β_6, α_7, γ_8.

4.2 Non-zero Majorana phases

In this subsection we repeat the previous analysis for the mixing matrix in Eq. (5) where the Majorana phases ϕ_1 and ϕ_2 are non-zero. We show that in presence of an off-diagonal dissipator, the oscillation formulae, the CP and T violations can depend on the Majorana phases, thus providing a new framework in which the real nature of neutrino can be challenged.

By working with the dissipator in Eq. (25) and using the parametrization in Eq. (5), we obtain the following CP violations for the transitions preserving the flavor:

$$
\begin{aligned}
\Delta CP_{ee}^M &= P_{\nu_e \to \nu_e}^M - P_{\bar{\nu}_e \to \bar{\nu}_e}^M \\
&= -\frac{2\alpha_1 e^{-\gamma_{12}t}\sin\delta}{\Omega} \\
&\quad \times \sin(2\phi_1)\cos^4\theta_{13}\sin^2(2\theta_{12})\sinh(\Omega t),
\end{aligned}
$$
(32)

$$
\begin{aligned}
\Delta CP_{\mu\mu}^M &= P_{\nu_\mu \to \nu_\mu}^M - P_{\bar{\nu}_\mu \to \bar{\nu}_\mu}^M \\
&= \frac{2\alpha_1 e^{-\gamma_{12}t}\sin\delta}{\Omega} \left[\cos\phi_1 \cos^2\theta_{23}\sin(2\theta_{12}) \right. \\
&\quad -\sin\theta_{13}\left(\cos\phi_1 \sin^2(2\theta_{23})\sin\theta_{13}\sin(2\theta_{12}) \right. \\
&\quad +\sin(2\theta_{23})\left(\cos(\delta+\phi_1)\sin^2\theta_{12} \right. \\
&\quad \left.\left.\left. -\cos(\delta-\phi_1)\cos^2\theta_{12}\right)\right)\right] \\
&\quad \times \left[\cos^2\theta_{12}\sin\theta_{13}\sin(2\theta_{23})\sin(\delta-\phi_1) \right. \\
&\quad -\cos^2\theta_{23}\sin(2\theta_{12})\sin\phi_1 \\
&\quad +\sin\theta_{13}\left(\sin\phi_1 \sin^2\theta_{23}\sin(2\theta_{12})\sin\theta_{13} \right. \\
&\quad \left.\left. +\sin^2\theta_{12}\sin(2\theta_{23})\sin(\delta+\phi_1)\right)\right] \sinh(\Omega t),
\end{aligned}
$$
(33)

$$
\begin{aligned}
\Delta CP_{\tau\tau}^M &= P_{\nu_\tau \to \nu_\tau}^M - P_{\bar{\nu}_\tau \to \bar{\nu}_\tau}^M \\
&= \frac{2\alpha_1 e^{-\gamma_{12}t}\sin\delta}{\Omega} \left[\cos\phi_1 \cos^2\theta_{23}\sin(2\theta_{12})\sin^2\theta_{13} \right. \\
&\quad -\cos\phi_1 \sin(2\theta_{12})\sin^2\theta_{23} \\
&\quad +\sin(2\theta_{23})\sin\theta_{13}\left(\cos(\delta-\phi_1)\cos^2\theta_{12} \right. \\
&\quad \left.\left. -\cos(\delta+\phi_1)\sin^2\theta_{12}\right)\right]
\end{aligned}
$$

$$\times \left[\cos^2\theta_{12}\sin\theta_{13}\sin(2\theta_{23})\sin(\delta-\phi_1) \right.$$
$$-\cos^2\theta_{23}\sin^2\theta_{13}\sin(2\theta_{12})\sin\phi_1$$
$$+\sin\phi_1\sin^2\theta_{23}\sin(2\theta_{12})$$
$$\left. +\sin^2\theta_{12}\sin(2\theta_{23})\sin\theta_{13}\sin(\delta+\phi_1) \right]\sinh(\Omega t). \tag{34}$$

Here, with the letter M we mean the transition probabilities for Majorana neutrinos. By comparing Eqs. (32), (33) and (34) with the analogue in Eq. (28), we can immediately note that the presence of non-zero Majorana phases introduces new terms in the formulae, and in particular, generate a CP violation also in the transition $\nu_e \rightarrow \nu_e$. This violation is absent for Dirac neutrinos, and depends on ϕ_1 for the dissipator considered.

The transition (32) has a very peculiar meaning: unlike the case of zero Majorana phases, here ΔCP_{ee} turns out to be non-vanishing, and becomes zero only when $\phi_1 = 0$ (and then the neutrinos are Dirac particles). Such a feature is crucial in order to discriminate between Dirac and Majorana neutrinos and could provide a completely new way to test the real nature of neutrinos in future experiments. Indeed, by considering the mixing matrix in Eq. (5) and the dissipator in Eq. (25), we have $\Delta CP_{ee} = 0$ for Dirac neutrinos and $\Delta CP_{ee}^M \neq 0$ for Majorana neutrinos. Let us also clarify that such a difference in the CP violation for $\nu_e \rightarrow \nu_e$ transition, with respect to the other two, depends on the form of the dissipation matrix and on the representation of the mixing matrix for Majorana neutrinos. For instance, a different correlation between the Dirac phase δ and the mixing angles can give different formula for the oscillation probabilities.

The possibility to violate the CP symmetry in the transitions flavor preserving, here revealed, is a new result which can indicate the presence of decoherence and allow us to fix the form of the mixing matrix, besides the neutrino nature. We emphasize that the result $\Delta CP_{ee} \neq 0$, exhibited in the Majorana case, is not replicated by the Dirac case for any value of the parameters ($\alpha, \gamma_{ij}, \phi_i$, etc) and for any value of any of the coefficients entering in three flavor neutrino mixing. It characterizes only Majorana neutrinos and it is absent for Dirac neutrinos, for which $\phi_i = 0$.

The CP violations for transitions between different neutrinos are:

$$\Delta CP_{e\mu}^M = P_{\nu_e \rightarrow \nu_\mu}^M - P_{\bar\nu_e \rightarrow \bar\nu_\mu}^M$$
$$= -\frac{\cos\theta_{13}}{2\Omega} \left[\Omega\sin\delta \left(-e^{-\gamma_{67}t}\sin(\Delta_{32}t) \right. \right.$$
$$\left. +e^{-\gamma_{45}t}\sin(\Delta_{31}t) \right)\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})$$
$$+\frac{e^{-\gamma_{12}t}}{2} \left(\alpha_1\cos\theta_{13} \left(2\cos^2\theta_{13} - (\cos(2\theta_{13}) - 3) \right. \right.$$
$$\left. \times \cos(2\theta_{23}) \right)\sin(2\phi_1)\sin^2(2\theta_{12})$$
$$\left. +(-2(\Delta_{21} + \alpha_1\cos(2\phi_1))\cos\delta\sin(4\theta_{12}))\sin(2\theta_{13}) \right.$$

$$\times \sin(2\theta_{23})) \sinh(\Omega t)],$$
$$\Delta CP_{e\tau}^M = P_{\nu_e \rightarrow \nu_\tau}^M - P_{\bar\nu_e \rightarrow \bar\nu_\tau}^M = -\Delta CP_{e\mu}^M, \tag{35}$$

We do not report explicitly the expression of $\Delta CP_{\mu\tau}^M$ because of its length. Its behavior is depicted in the left panel of Fig. 1.

The T violating channels are not affected by the Majorana phases for our choice of the dissipator, indeed they are the same as in Eq. (30):

$$\Delta T_{e\mu}^M = \Delta T_{e\mu}, \quad \Delta T_{e\tau}^M = \Delta T_{e\tau}, \quad \Delta T_{\mu\tau}^M = \Delta T_{\mu\tau}. \tag{36}$$

This fact induces an extra violation of the CPT symmetry since we have

$$\Delta CP_{e\mu}^M \neq \Delta T_{e\mu}, \quad \Delta CP_{e\tau}^M \neq \Delta T_{e\tau},$$
$$\Delta CP_{\tau\mu}^M \neq \Delta T_{\tau\mu}. \tag{37}$$

In presence of an off-diagonal dissipator, Dirac and Majorana phases induce two independent CPT violations. The results here presented are obtained by considering the non-diagonal dissipator in which only α_1 is non-zero; see Eq. (25). Other kinds of dissipation matrices can be studied with other off-diagonal elements switched on. Like for the mixing matrix in Eq. (2), also for the matrix (5), CP and T are always violated, while CPT can be still preserved for some non-zero off-diagonal elements. Indeed, CPT is respected if the only non-zero off-diagonal element is one of among these: $\beta_1, \alpha_3, \delta_3, \xi_3, \eta_1, \zeta_2, \chi_4, \delta_5, \beta_6, \alpha_7, \gamma_8$.

Notice also that other choices of the Majorana matrix would give different results. For instance the mixing matrix $\mathcal{U}_M$ in Eq. (4) give different expressions for the oscillation formula as compared to Eq. (5). This implies that the physical results depend on the chosen parametrization of the Majorana mixing matrix.

Summarizing, in presence of an off-diagonal dissipator, the neutrino oscillation formula depend on the parametrization of the mixing matrix. A physical implication is that Dirac and Majorana neutrinos are two totally distinct entities and their nature, together with CPT violation, could be tested in future experiments.

5 Comparison between Dirac and Majorana neutrinos

In this section we make a more quantitative comparison between Dirac and Majorana neutrinos considering some specific transition probability.

Let us emphasize that a more realistic phenomenological analysis would require the introduction of matter effects.[1] The Earth is not charge-symmetric (it contains electrons,

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[1] It is worthwhile mentioning that in Ref. [47], the Mikheyev–Smirnov–Wolfenstein (MSW) effect was generalized in presence of decoherence.
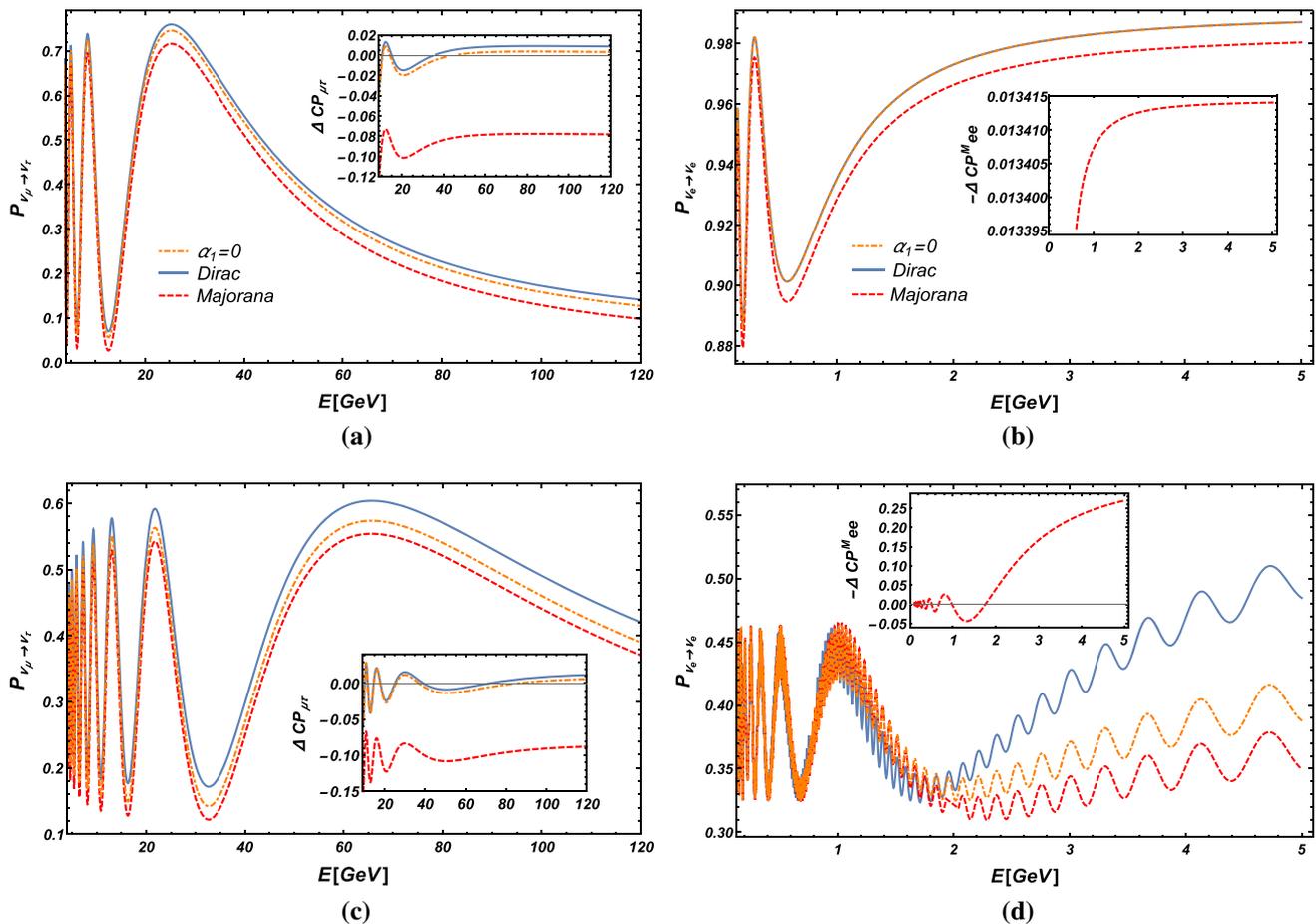
Fig. 1 **a** Plots of the transition probability $P_{\nu_\mu \to \nu_\tau}$ as a function of the energy E for a diagonal dissipator $\alpha_1 = 0$, for which Dirac and Majorana neutrinos have identical behavior (orange dot-dashed line) and for the off-diagonal dissipator for Dirac (blue solid line) and Majorana (red dashed line) neutrinos. Inset: corresponding plots of $\Delta CP_{\mu\tau}$. We consider the energy range (0–120) GeV corresponding to the accessible energies in the IceCube DeepCore experiment and set $t = 6.58 \times 10^{22}$ GeV^{-1}. **b** Plots of $P_{\nu_e \to \nu_e}$ as a function of the energy for an off–diagonal dissipator in the case of Dirac (blue solid line) and Majorana (red dashed line) neutrinos. Notice that for this transition, the behavior of Dirac neutrinos in the off–diagonal dissipator case is identical to that of the neutrinos in the case of a diagonal dissipator. Inset: plot of the CP violation in the channel $\nu_e \leftrightarrow \nu_e$ for

Majorana neutrinos. We consider the energy values (0.3–5) GeV characteristic of DUNE experiment and we set $t = 1.49 \times 10^{21}$ GeV^{-1}. In both the plots we assume $\phi_1 = \pi/4$, $\phi_2 = \pi/3$, $\delta = -\pi/2$, we consider the following values of the elements of the dissipator: $\gamma_{12} = 1.2 \times 10^{-23}$ GeV, $\gamma_{45} = 4 \times 10^{-24}$ GeV, $\gamma_{67} = 4.7 \times 10^{-24}$ GeV, $\gamma_3 = \gamma_8 = 7.9 \times 10^{-24}$ GeV, $\alpha_1 = 1.3 \times 10^{-24}$ GeV, and use the following experimental values for the mixing angles: $\sin^2 \theta_{23} = 0.51$, $\Delta m_{23}^2 = 2.55 \times 10^{-3}$ eV^2, $\Delta m_{12}^2 = 7.56 \times 10^{-5}$ eV^2. Moreover, in the **c** and **d** we have plotted the same oscillation formulae with the same values of parameters except the time travel that is now equal to $t \simeq 1.7 \times 10^{23}$ GeV^{-1}, corresponding to the distance from the surface of Earth and the geostationary orbit (35,786 km)

protons and neutrons but contain their antiparticles), then the oscillations in matter involving electron neutrino already induce the CP and CPT violations also in absence of decoherence. Therefore, one has to be careful to identify the right contribution responsible for violations purely induced by decoherence. Since we are mainly interested in highlighting the effects of the decoherence from a theoretical point of view, we consider the neutrino oscillations in vacuum. Indeed, our study provides a starting point for future improvements.

To make the comparison, we choose three different set of parameters as possible examples: the first two are taken from

the available data of IceCube [48] and DUNE [49], while the third is inspired by a hypothetical experiment in which a detector is located at the geostationary orbit. In the following we approximate $x \approx t$ in Natural units.

In Fig. 1a, we plot the $\nu_\mu \to \nu_\tau$ oscillations in vacuum and $\Delta CP_{\mu\tau}$ as functions of the neutrino energy, by using the range of energy of the IceCube DeepCore experiment $E \in (6-120)$ GeV [48,50] and a distance equal to Earth diameter $x = 1.3 \times 10^4$ km, corresponding to $t = 6.58 \times 10^{22}$ GeV^{-1}. We draw the oscillation formula $P_{\nu_\mu \to \nu_\tau}$ and the quantity $\Delta CP_{\mu\tau}$ obtained by using the diagonal and the

off-diagonal dissipators with zero and non-zero Majorana phases, respectively.

In panel (b), we plot the oscillation formula $P_{\nu_e \to \nu_e}$ and ΔCP_{ee} in the energy range (0.3–5) GeV which is typical of DUNE experiment [46]. We consider the time scale $t = 1.49 \times 10^{21} \text{GeV}^{-1}$. For both the plots, we assume $\phi_1 = \pi/4$, $\phi_2 = \pi/3$, $\delta = -\pi/2$; and we use the following values for the elements of the dissipator: $\gamma_{12} = 1.2 \times 10^{-23}$ GeV, $\gamma_{45} = 4.0 \times 10^{-24}$ GeV, $\gamma_{67} = 4.7 \times 10^{-24}$ GeV, $\gamma_3 = \gamma_8 = 7.9 \times 10^{-24}$ GeV, $\alpha_1 = 1.3 \times 10^{-24}$ GeV, which are compatible with the experimental upper bounds on γ_i [46,50]. Moreover, we consider the following experimental values of the parameters: $\sin^2\theta_{23} = 0.51$, $\Delta m_{23}^2 = 2.55 \times 10^{-3} \text{eV}^2$, $\Delta m_{12}^2 = 7.56 \times 10^{-5} \text{eV}^2$ [50].

From the plots it is clear that, by using the current upper bounds on the elements of the dissipation matrix, 5–10% relative differences appear between Dirac and Majoranas oscillation formulae, providing smoking-gun signatures of the real nature of neutrinos. We believe that with an improved and more realistic phenomenological analysis these effects could be detected in next long baseline experiments.

It is worthwhile mentioning that, although matter effects are typically important and can even dominate over decoherence, there can be configurations in which the opposite happens. In fact, in Ref. [12], at least for two neutrino families, it was shown that some predicted matter effects in presence of decoherence (see Ref. [47]) are negligible in the range of energies $E \in [0.3–1]$ GeV in the context of Dune experiment. Of course, further investigations are needed to strengthen our argument for three neutrino families. Furthermore, let us emphasize that in short baseline experiments like reactors our predicted effects would be negligible; long baseline experiments are instead necessary for our purpose. A promising scenario in which decoherence effects in vacuum could be relevant is given by spacetime fluctuations [30–36] motivated by quantum gravity models. In this case, neutrinos would propagate in vacuum and only feel their interaction with the quantum fluctuations of the geometric background. The configuration chosen in panels (c) and (d) is indeed motivated by this last possibility, namely a hypothetical experiment could consist in a detector located at the geostationary orbit (35,786 km from Earth surface), and neutrinos that start propagating from Earth (in vacuum) with time travel $t \simeq 0.12 \, \text{s} \simeq 1.7 \times 10^{23} \, \text{GeV}^{-1}$.

6 Summary and conclusions

In this work we have analyzed the physical implications of decoherence and dissipation in the context of three flavors neutrino mixing. We have computed the transition probabilities for Dirac and Majorana neutrinos in the cases of a diagonal and an off-diagonal dissipation matrix. By analyz-

ing Dirac neutrinos, we have shown that in presence of a diagonal dissipator, the oscillation formula do not depend on the parametrization of the mixing matrix and CPT symmetry is still preserved. Subsequently, we have switched on an off-diagonal elements in the dissipation matrix, and shown that for Dirac neutrinos the oscillation formula can depend on the parametrization of the mixing matrix. Moreover, we have revealed the possibility of a CP violation in the neutrino transitions preserving the flavor and the existence of a CPT violation due to the Dirac phase δ. By performing analogue computations for Majorana neutrinos, we have shown that in presence of an off-diagonal dissipation matrix, the oscillation formulae can depend on the Majorana phases ϕ_i. These formulae depend on the choices of the parametrization of the Majorana mixing matrix. Indeed, different parametrizations of the Majorana mixing matrix lead to different formulae. We have also revealed a CPT violation term purely induced by the Majorana phases, which generalize the result in [12] obtained for two flavors neutrinos.

We have only tuned some of the available free parameters, but the distinction between the two type of neutrinos is still true also by tuning other parameters. Let us emphasis that, in presence of an off-diagonal dissipator with only $\alpha_1 \neq 0$, we have shown that $\Delta CP_{ee} = P_{\nu_e \to \nu_e} - P_{\bar{\nu}_e \to \bar{\nu}_e}$ is zero for Dirac neutrinos, while it is non-vanishing for Majorana ones. This result holds for any value of the parameters (α, γ_{ij}, ϕ_i, etc) and for any value of any of the coefficients entering in three flavor neutrino mixing: $\Delta CP_{ee} \neq 0$ characterizes only Majorana neutrinos.

Before concluding let us emphasize that a detection of CPT violation induced by decoherence effects could be attributed to fluctuations of the space-time [30–36], thus representing a signature of quantum gravity. Moreover, the studies on neutrino mixing in curved space [51–54] could be also generalized by including in them the decoherence and dissipation effects here presented. Therefore, our study might open new windows of opportunity to address several open questions in fundamental physics. It is worthwhile note that, non-perturbative field theoretical effects of particle mixing [55–75] can be neglected in the our treatment.

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