## Lifetime-difference pattern of heavy hadrons

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## Abstract

The preasymptotic effects originating from the four-quark operators are disentangled from the other contributions by considering appropriate combinations of inclusive decay rates. Under the assumption of the isospin and heavy-quark symmetry, a set of relations connecting charmed and beauty decays is obtained without invoking specific models. The results are compared with other approaches and confronted with experiment.

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It is astonishing that a lot of physical observables, decay rates of semileptonic, nonleptonic, and radiative decays are described in the framework of the inverse heavy-quark mass expansion in terms of relatively few basic quantities, e.g., quark masses and hadronic expectation values (HEV) of several leading local operators [1, 2, 3]. The underlying theory is based on a few field-theory basics, such as the operator product expansion, quark-hadron duality, and certain well-known symmetries. Even more surprisingly, the theory seems to work rather well in the case of charmed hadrons, where the expansion parameter  $\sqrt{\mu_G^2(D)/m_c^2} \simeq 0.5$  is by no means very small. A systematic analysis leads to very clear lifetime-pattern predictions, the agreement with experiment being reasonable even for absolute lifetime values [4, 5].

In beauty hadron decays, one expects that the whole theory should work much better, since the expansion parameter,  $\sqrt{\mu_G^2(B)/m_b^2} \simeq 0.13 \ll 1$ , is significantly smaller than in the case of charmed hadrons. Still, in spite of the overall agreement between theory and experiment [1], there are some questions to be answered. The main question is: does the quark-hadron duality work? Unfortunately, for a reliable study of duality, one needs complete control over nonperturbative phenomena, which is out of reach of the present theory. Nevertheless, the validity of duality was

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studied in exactly solvable 't Hooft model in  $QCD_2$  [6, 7], leading to a perfect matching, which is certainly encouraging. Still, one would be happy to find stronger support for duality in the (3+1) theory, too.

The question is: could one try to reduce the uncertanties of the calculation, like the strong sensibility to the value of  $m_Q$ , and/or find the way to disentangle various preasymptotic effects which cause lifetime differences?

The decay rate of a decaying hadron is generally of the type

$$\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3} |V|^2 \frac{1}{2M_{H_Q}} \left[ \sum_{D=3}^{D_c} c_D^f \frac{\langle H_Q | O_D | H_Q \rangle}{m_Q^{D-3}} + \mathcal{O}(1/m_Q^{D_c-2}) \right], \tag{1}$$

where  $c_D^f$  are the Wilson coefficients and  $\langle H_Q|O_D|H_Q\rangle$  are the matrix elements of the D-dimensional operators which are suppressed by the inverse power of mass  $1/m_Q^{D-3}$ .

The following comments are in order:

- 1. It would be desirable to extract a combination of rates  $\Gamma'$  that depends only on the operators with D=6, since they are the operators responsible for lifetime differences.
- 2. Since the D=6 operators are suppressed by  $m_Q^{-3}$  with respect to the leading D=3 operator, this would reduce the mass dependence of  $\Gamma'$  to  $m_Q^2$ , thus significantly reducing the errors coming from the uncertainty of  $m_Q$ .
- 3. Even truncating the sum in (1) at D = 6, one would presumably be able to test the role of preasymptotic effects with satisfactory accuracy, relying on heavy-quark symmetry. That is the aim of this paper.

As both the decays in the b and c sectors of hadrons are described by the same formalism, it seems worthwhile to investigate the possibility of establishing connections between these two sectors. In a recent paper [8], Voloshin demonstrated one of such possible connections. The strength of that approach is in avoiding the model dependence assuming SU(3) flavor and heavy-quark symmetry (HQS). One can extend this approach to obtain more predictivity. Using the matrix elements extracted from charmed baryons, a parameter  $F_B^{eff}$  (parametrizing four-quark contributions to beauty baryons) can be determined and concrete numerical predictions can be obtained in this way [9]. This procedure brings the ratio  $\tau(\Lambda_b)/\tau(B_d^0)$  to better agreement with experiment and gives predictions for the lifetimes of beauty baryons.

In the present paper we adopt a similar, yet different strategy from that followed in [8]. In [8], the author, when establishing the connection between c and b baryons, always relates baryons with the same light-quark content, i.e., mutually related baryons differ in the flavor of the heavy quark. Also, the author explicitly extracts values of four-quark operator matrix elements and then applies them elsewhere. Our approach differs in both these respects. The heavy hadrons that we relate neither contain the same heavy quark(s) nor the same light (anti)quarks – they are affected by the same type of the four-quark operator contribution. Furthermore, we form such combinations that the four-quark operator matrix elements get reduced.

In forming such combinations, we consider the leading modes of the decay of c and b quarks with respect to the CKM matrix elements. For the c quark, we then have only one nonleptonic mode,  $c \to s\overline{d}u$ , and one semileptonic mode per lepton family,  $c \to s\overline{l}\nu_l$ . In the case of b quark, there are two nonleptonic modes,  $b \to c\overline{u}d$  and  $b \to c\overline{c}s$ , as well as one semileptonic mode per

lepton family,  $b \to cl\overline{\nu_l}$ . As we consider b baryons containing only light quarks along with b quarks, the semileptonic Cabibbo leading modes for the decay of the b quark do not appear. We also disregard mass corrections of four-quark operators coming from the massive particles in final decay states. This is a better approximation for c decays (as  $m_s^2/m_c^2 \sim 0.01$ ) than for b decays (where  $m_c^2/m_b^2 \sim 0.1$ ).

Traditionally, the effects of four-quark operators are called the positive Pauli interference, the negative Pauli interference and, W exchange or annihilation [10, 11]. We consider pairs of one charmed and one beauty particle which contain the same type of four-quark contribution. It is important to notice that in b and c decays different light-quark flavors participate in the same type of four-quark contributions. Using isospin (SU(2) flavor) symmetry and HQS we are able to form appropriate decay-rate differences which lead us to the final result.

We start our considerations with heavy mesons. The first pair of considered particles are  $D^+(c\overline{d})$  and  $B^-(b\overline{u})$ . In both these mesons the effect of negative Pauli interference occurs. The contributions of four-quark operators to the decay rates of these two particles are described by the same type of operators. Moreover, the operators for the b case can be obtained from those in the c case by making the substitution  $c \to b$ ,  $\overline{d} \to \overline{u}$ . The second pair of particles form  $D^0(c\overline{u})$  and  $B^0(b\overline{d})$ . In both these mesons the effect of W exchange occurs.

Next, we would like to isolate the effects of four-quark operators, i.e., we have to eliminate the contributions of the operators of dimensions 5 and lower. We achieve this goal by taking differences of the decay rates of D and B mesons assuming the isospin symmetry. This procedure leads us to the following relations:

$$\Gamma(D^{+}) - \Gamma(D^{0}) = \frac{G_F^2 m_c^2}{4\pi} |V_{cs}|^2 |V_{ud}|^2 [\langle D^{+}|P^{cd}|D^{+}\rangle - \langle D^{0}|P^{cu}|D^{0}\rangle], \qquad (2)$$

$$\Gamma(B^{-}) - \Gamma(B^{0}) = \frac{G_F^2 m_b^2}{4\pi} |V_{cb}|^2 |V_{ud}|^2 [\langle B^{-}|P^{bu}|B^{-}\rangle - \langle B^{0}|P^{bd}|B^{0}\rangle], \qquad (3)$$

where P's denote the appropriate four-quark terms from the heavy-quark effective Lagrangian, see, e.g., Ref. [8].

Using HQS and isospin symmetry we can express the terms describing the negative Pauli interference as

$$P^{cd} = \sum_{i=1}^{2} c_i^{cd}(\mu) O_i^{cd}(\mu) = \sum_{i=1}^{2} c_{i,negint}^{Qq}(\mu) O_{i,negint}^{Qq}(\mu) + \mathcal{O}(1/m_c),$$
(4)

$$P^{bu} = \sum_{i=1}^{2} c_i^{bu}(\mu) O_i^{bu}(\mu) = \sum_{i=1}^{2} c_{i,negint}^{Qq}(\mu) O_{i,negint}^{Qq}(\mu) + \mathcal{O}(1/m_b),$$
 (5)

where Q denotes a heavy quark in the heavy-quark limit  $m_Q \to \infty$ . For the matrix elements of the above mentioned operators we obtain (by applying HQS and isospin symmetry again)  $\langle D^+|P^{cd}|D^+\rangle = A_{negint} + \mathcal{O}'(1/m_c)$  and  $\langle B^-|P^{bu}|B^-\rangle = A_{negint} + \mathcal{O}'(1/m_b)$ , where the matrix element is  $A_{negint} = \langle M_{Q\overline{q}}|\sum_{i=1}^2 c_{i,negint}^{Qq}(\mu)O_{i,negint}^{Qq}(\mu)|M_{Q\overline{q}}\rangle$  and  $M_{Q\overline{q}}$  denotes the mesonic state with one heavy quark Q and one light antiquark  $\overline{q}$  in the heavy-quark limit. In the preceding relations we have also stated that heavy-quark mass suppressed corrections to the operators and their matrix elements do not have to be the same.

In an analogous manner we can obtain the expressions for the operators describing W exchange using the notation explained above:

$$P^{cu} = \sum_{i=1}^{4} c_i^{cu}(\mu) O_i^{cu}(\mu) = \sum_{i=1}^{4} c_{i,exch}^{Qq}(\mu) O_{i,exch}^{Qq}(\mu) + \mathcal{O}(1/m_c),$$
 (6)

$$P^{bd} = \sum_{i=1}^{4} c_i^{bd}(\mu) O_i^{bd}(\mu) = \sum_{i=1}^{4} c_{i,exch}^{Qq}(\mu) O_{i,exch}^{Qq}(\mu) + \mathcal{O}(1/m_b).$$
 (7)

The matrix elements of the operators given by (6) and (7) are  $\langle D^0|P^{cu}|D^0\rangle = A_{exch} + \mathcal{O}'(1/m_c)$  and  $\langle B^0|P^{bd}|B^0\rangle = A_{exch} + \mathcal{O}'(1/m_b)$ , with  $A_{exch} = \langle M_{Q\overline{q}}|\sum_{i=1}^4 c_{i,exch}^{Qq}(\mu)O_{i,exch}^{Qq}(\mu)|M_{Q\overline{q}}\rangle$ . In all relations given above, the dependence of the coefficients  $c_i$  and the operators  $O_i$  on the scale  $\mu$  is explicitly displayed. Since the dependences of the operators and the coefficients on  $\mu$  cancel each other, there is no need for an explicit value for the  $\mu$  scale, and also no hybrid renomalization is required. That scale, however, should be small compared with the heavy-quark masses, to allow for the proper heavy-quark expansion and the reduction of the four-quark operator contribution in the heavy-quark limit, as shown below.

Combining all preceding expressions, we obtain for the decay rate differences

$$\Gamma(D^{+}) - \Gamma(D^{0}) = \frac{G_F^2 m_c^2}{4\pi} |V_{cs}|^2 |V_{ud}|^2 [A_{negint} - A_{exch} + \mathcal{O}(1/m_c)],$$
(8)

$$\Gamma(B^{-}) - \Gamma(B^{0}) = \frac{G_F^2 m_b^2}{4\pi} |V_{cb}|^2 |V_{ud}|^2 [A_{negint} - A_{exch} + \mathcal{O}(1/m_b)]. \tag{9}$$

As  $m_b \gg m_c$ , it is clear that the approximation made above should work much better for beauty particles than for charmed particles. Still, we assume that the approximations made are justified in both c and b decays and that the corrections of order  $\mathcal{O}(1/m_{c,b})$  are not large. In this case, we obtain the final relation for mesons:

$$r^{BD} \equiv \frac{\Gamma(B^{-}) - \Gamma(B^{0})}{\Gamma(D^{+}) - \Gamma(D^{0})} = \frac{m_b^2}{m_c^2} \frac{|V_{cb}|^2}{|V_{cs}|^2} + \mathcal{O}(1/m_c, 1/m_b).$$
 (10)

Let us consider relation (10) in more detail. By forming the lifetime differences (2) and (3) we have not only eliminated the effects of the leading operators in the mesonic decay rates, but applied the HQS first at the subleading level of  $\mathcal{O}(1/m_{c,b}^3)$ . This has in turn enabled us to eliminate the dependence on four-quark matrix elements in relation (10). Also, the sensitivity to the choice of heavy quark masses is now significantly reduced since in expression (10) we have only the second power of masses. The procedure, however, has its limits. It enables us to reduce the matrix elements, but cannot be extended to CKM suppressed modes. Also, mass corrections due to massive particles in the final decay states spoil the procedure and, therefore, have been left out. Nevertheless, both of these corrections are under good theoretical control. Moreover, the contributions from mass corrections and suppressed modes are smaller by more than one order of magnitude than the leading contribution and cannot significantly change relation (10).

Starting from expression (10), we may check the standard formalism of inclusive decays against experimental data, especially if the four-quark operator contributions are sufficient to explain the lifetime differences of heavy mesons. To perform such a check, we determine the quantity  $r^{BD}$  using

the experimental values for lifetimes and theoretically (using relation (10)) and then compare the values. Taking the experimental values from [12], we obtain

$$r_{exp}^{BD} = 0.030 \pm 0.011. (11)$$

The large error of  $r_{exp}^{BD}$  comes from the fact that the experimental errors of lifetime measurements of  $B^-$  and  $B^0$  are large compared with the difference between central values of the results for  $B^-$  and  $B^0$ . Inspection of [12] shows that the experimental situation in the sector of B mesons is far from being settled and more precise measurements of B meson lifetimes are needed to reduce the uncertainty in the value of  $r_{exp}^{BD}$ .

The theoretical value  $r_{th}^{BD}$  is calculated from expression (10). The numerical values for heavy-quark masses are taken to be  $m_c(m_c) = 1.25 \pm 0.1 \, GeV$  [13] and  $m_b(1 \, GeV) = 4.59 \pm 0.08 \, GeV$  [14] for the reasons discussed in [9]. The values for the matrix elements of the CKM matrix are taken from [12] to be  $|V_{cs}| = 1.04 \pm 0.16$  and  $|V_{cb}| = 0.0402 \pm 0.0019$ . Using these numerical values we obtain

$$r_{th}^{BD} = 0.020 \pm 0.007. (12)$$

The relatively large error in (12) comes predominantly from the large experimental error of the  $|V_{cs}|$  CKM matrix element.

Direct comparison of the numerical results (11) and (12) indicates that these are consistent within errors. Since the relative errors of both results are rather large and experimental data are still fluid, it is possible that these results will experience some change. Nevertheless, we expect that the two results will remain comparable and even closer to each other.

These results confirm, in a model-independent way, that four-quark operators can account for the greatest part of decay rate differences of heavy mesons, i.e., that the contributions of other operators of dimension 6 and higher-dimensional operators are not so important. Of course, these conclusions should be interpreted in the light of the approximations made.

The procedure presented for heavy mesons can be extended to heavy baryons. First, we consider the system of singly heavy baryons. These baryons contain two light quarks, which introduces two types of four-quark operator contributions for each baryon. Again, we choose two pairs of singly heavy baryons, each pair containing one charmed and one beauty baryon related by the same dominant type of four-quark contributions. The first pair contains  $\Xi_c^+(cus)$  and  $\Xi_b^-(bds)$ , while the second pair contains  $\Xi_c^0(cds)$  and  $\Xi_b^0(bus)$ . The first pair exhibits the negative Pauli interference and different nonleptonic and semileptonic contributions of four-quark operators containing s quark fields. The second pair comprises the W-exchange effect and the same contributions of four-quark operators containing s quark fields as in the first pair of baryons. It is important to notice that the four-quark operators that contain s quark fields do not describe the same type of effects in s and s decays in our pairs (and therefore are not given by the contributions of the same type in the effective Lagrangian). Nevertheless, we form decay-rate differences in such a manner that the contributions of four-quark operators containing the s-quark field cancel (assuming s-quark type four-quark operators significant mass corrections appear because of two massive s-quarks in the final state.

In the singly heavy sector we, therefore, form the following differences:

$$\Gamma(\Xi_c^+) - \Gamma(\Xi_c^0) = \frac{G_F^2 m_c^2}{4\pi} |V_{cs}|^2 |V_{ud}|^2 [\langle \Xi_c^+ | P^{cu} | \Xi_c^+ \rangle - \langle \Xi_c^0 | P^{cd} | \Xi_c^0 \rangle], \tag{13}$$

$$\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0) = \frac{G_F^2 m_b^2}{4\pi} |V_{cb}|^2 |V_{ud}|^2 [\langle \Xi_b^- | P^{bd} | \Xi_b^- \rangle - \langle \Xi_b^0 | P^{bu} | \Xi_b^0 \rangle]. \tag{14}$$

Following the procedure displayed in relations (4) to (10) and using the *isospin* symmetry and HQS again, we obtain the expressions (note that the four-quark operators which contribute to the negative Pauli interference in mesons contribute to W exchange in baryons and vice versa [10, 11]):

$$\Gamma(\Xi_c^+) - \Gamma(\Xi_c^0) = \frac{G_F^2 m_c^2}{4\pi} |V_{cs}|^2 |V_{ud}|^2 [B_{negint} - B_{exch} + \mathcal{O}(1/m_c)],$$
(15)

and

$$\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0) = \frac{G_F^2 m_b^2}{4\pi} |V_{cb}|^2 |V_{ud}|^2 [B_{negint} - B_{exch} + \mathcal{O}(1/m_b)],$$
(16)

where the matrix elements of the four-quark operators between singly heavy baryons  $B_{Qqq'}$  have the form  $B_{negint} = \langle B_{Qqq'} | \sum_{i=1}^4 c_{i,negint}^{Qq}(\mu) O_{i,negint}^{Qq}(\mu) | B_{Qqq'} \rangle$  and  $B_{exch} = \langle B_{Qqq'} | \sum_{i=1}^2 c_{i,exch}^{Qq}(\mu) O_{i,exch}^{Qq}(\mu) | B_{Qqq'} \rangle$ . The final relation for singly heavy baryons looks like

$$r^{bc} \equiv \frac{\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0)}{\Gamma(\Xi_c^+) - \Gamma(\Xi_c^0)} = \frac{m_b^2}{m_c^2} \frac{|V_{cb}|^2}{|V_{cs}|^2} [1 + \mathcal{O}(1/m_c, 1/m_b)]. \tag{17}$$

This relation can be used to test the present model-dependent calculation of heavy-baryon lifetimes. Lifetime splittings both in the charmed and in the beauty sector have been investigated in several papers [2, 3, 4, 8, 9, 15]. We have recalculated our predictions from [4] for charmed baryon lifetimes and from [9] for beauty baryon lifetimes with the numerical parameters preferred in this paper and have used the recalculated predictions to test the relation (17), which we express in units  $\frac{m_b^2}{m_c^2} \frac{|V_{cb}|^2}{|V_{cs}|^2}$  thus reducing the dependence on heavy-quark masses. In the case when the same approximations (neglecting mass corrections and Cabibbo-suppressed modes) are made, the above relation differs from unity by -12%. This number shows the deviation of the model-dependent calculation (where the contributions of four-quark operators are explicitly evaluated) from the model-independent prediction given only by the ratios of heavy-quark masses and Cabibbo matrix elements. The complete calculation with the mass corrections and Cabibbo-suppressed modes gives 0.79 for the above ratio, which indicates the order of neglected corrections to be less than 10%.

There are several important implications of relation (17). We can obtain a model-independent prediction for the still unmeasured lifetime difference between singly beauty baryons in the isospin doublet. Using the same numerical values for the parameters entering the right-hand side as before, and taking the experimentally measured lifetimes of singly-charmed baryons  $\Xi_c^+$  and  $\Xi_c^0$  from [12], we obtain

$$\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0) = -(0.14 \pm 0.06) \text{ ps}^{-1}.$$
 (18)

This lifetime difference can be compared with the model-independent prediction obtained by using the HQS and SU(3) symmetry in [8],  $\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0) = -(0.11 \pm 0.03) \mathrm{ps}^{-1}$ , and some moderate model-dependent prediction from [9],  $\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0) = -0.094 \,\mathrm{ps}^{-1}$ . It is also worth mentioning that owing to the neglected mass corrections in final decay states of beauty baryons in the derivation of the ratios (17) and (18), there is no splitting between the nonleptonic rates of  $\Xi_b^0$ and  $\Lambda_b$ . Since the Cabibbo-suppressed modes have also been neglected, the total decay rates of  $\Xi_b^0$  and  $\Lambda_b$  appear to be equal at this level. Therefore, the predictions from (17) are also valid for the  $\Gamma(\Xi_b^-) - \Gamma(\Lambda_b)$  difference. We can see that the results for the predicted splitting in the isospin doublet of beauty baryons obtained using different methods are all consistent.

It is interesting to examine the explicit expression taken from [8], eq.(23), which can be rewritten as

$$\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0) = \frac{m_b^2}{m_c^2} \frac{|V_{cb}|^2}{|V_{cs}|^2} [0.91\Gamma(\Xi_c^+) - 0.85\Gamma(\Xi_c^0) - 0.06\Gamma(\Lambda_c)],$$
(19)

and which is obtained by applying SU(3) symmetry, and with Cabibbo subleading effects included in charmed baryon decays. The appearance of the total decay rate for  $\Lambda_c$  is the direct consequence of the SU(3) relations used. Otherwise, we can note a very similar structure to our relation (17) obtained within the SU(2) approximation. The first two coefficients in front of the rates in (19) differ from one by less than 15%, where about 5 – 10% difference comes from the Cabibbosuppressed modes included, and the rest comes presumably from the difference between the SU(3)and SU(2) approximations applied.

In the last case, we apply our procedure to the system of doubly heavy baryons. Again, we form pairs of one doubly charmed and one doubly beauty baryon. The first pair includes  $\Xi_{cc}^{++}$  (ccu) and  $\Xi_{bb}^{-}$  (bbd). The decays of both baryons in this pair include the effect of negative Pauli interference. The second pair is formed from  $\Xi_{cc}^{+}$  (ccd) and  $\Xi_{bb}^{0}$  (bbu). Both of these baryons exhibit the effect of W exchange. There are no Cabibbo-leading semileptonic four-quark contributions in doubly heavy baryons. Appropriate decay-rate differences are given by the following expressions:

$$\Gamma(\Xi_{cc}^{++}) - \Gamma(\Xi_{cc}^{+}) = \frac{G_F^2 m_c^2}{4\pi} |V_{cs}|^2 |V_{ud}|^2 [\langle \Xi_{cc}^{++} | P^{cu} | \Xi_{cc}^{++} \rangle - \langle \Xi_{cc}^{+} | P^{cd} | \Xi_{cc}^{+} \rangle], \qquad (20)$$

$$\Gamma(\Xi_{bb}^{-}) - \Gamma(\Xi_{bb}^{0}) = \frac{G_F^2 m_b^2}{4\pi} |V_{cb}|^2 |V_{ud}|^2 [\langle \Xi_{bb}^{-} | P^{bd} | \Xi_{bb}^{-} \rangle - \langle \Xi_{bb}^{0} | P^{bu} | \Xi_{bb}^{0} \rangle]. \tag{21}$$

Using isospin symmetry and HQS, we obtain, per analogiam with above derivations, relations for the decay-rate differences of doubly heavy baryons which lead to the final relation for doubly heavy baryons

$$r^{bbcc} \equiv \frac{\Gamma(\Xi_{bb}^{-}) - \Gamma(\Xi_{bb}^{0})}{\Gamma(\Xi_{cc}^{++}) - \Gamma(\Xi_{cc}^{+})} = \frac{m_b^2}{m_c^2} \frac{|V_{cb}|^2}{|V_{cs}|^2} [1 + \mathcal{O}(1/m_c, 1/m_b)]. \tag{22}$$

Again, we have achieved the reduction of the matrix elements of four-quark operators between doubly heavy baryons in the heavy quark limit.

Relation (22) enables us to estimate the difference in the decay rates of doubly-beauty baryons using recent results [5] for doubly-charmed baryon lifetimes. Using the calculated values  $\Gamma(\Xi_{cc}^{++}) = 0.952 \, ps^{-1}$  and  $\Gamma(\Xi_{cc}^{+}) = 5 \, ps^{-1}$  with the values of the parameters  $m_c = 1.35 \, GeV$ ,  $m_b = 4.7 \, GeV$ ,  $|V_{cb}| = 0.04$  and  $|V_{cs}| = 1.04$  gives

$$\Gamma(\Xi_{bb}^{-}) - \Gamma(\Xi_{bb}^{0}) = -0.073 \,\mathrm{ps}^{-1} \,.$$
 (23)

So far we have applied our reduction procedure separately to heavy mesons, singly-heavy baryons, and doubly-heavy baryons. Still, by inspection of relations (10), (17), and (22) we see that  $r^{BD}$ ,  $r^{bc}$ , and  $r^{bbcc}$  are the same up to  $\mathcal{O}(1/m_c, 1/m_b)$ , i.e.,

$$\frac{\Gamma(B^{-}) - \Gamma(B^{0})}{\Gamma(D^{+}) - \Gamma(D^{0})} = \frac{\Gamma(\Xi_{b}^{-}) - \Gamma(\Xi_{b}^{0})}{\Gamma(\Xi_{c}^{+}) - \Gamma(\Xi_{c}^{0})} = \frac{\Gamma(\Xi_{bb}^{-}) - \Gamma(\Xi_{bb}^{0})}{\Gamma(\Xi_{cc}^{++}) - \Gamma(\Xi_{cc}^{+})} = \frac{m_{b}^{2}}{m_{c}^{2}} \frac{|V_{cb}|^{2}}{|V_{cs}|^{2}}.$$
(24)

Thus, we obtain a relation in (24) which clearly indicates certain universal behavior in the decays of all heavy hadrons. The existence of some universality could have been anticipated from the fact that the same expression (1) describes the decays of all heavy baryons. Still, that universality attains its concrete, model-independent form in the relation (24). This relation connects all sectors of heavy hadrons that are usually treated separately: mesonic and baryonic, charmed and beauty. Also, this relation brings some order in the otherwise rather intricate pattern of heavy-hadron lifetimes. An advantage of this relation is that by knowing some decay rates, one can calculate or give constraints on some other decay rates. Also, knowing decay rates from experiment, one can test the findings of the theory in a model-independent fashion. These results will help to establish the limitations of the present standard method of calculating inclusive processes and to test its underlying assumptions.

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