LCSR analysis of exclusive two-body B decay into charmonium

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Abstract

We analyze $B \to KH_c$ (where $H_c = \eta_c, J/\psi, \chi_{cJ(J=0,1)}$) decays estimating nonfactorizable contributions from the light-cone sum rules (LCSR). Nonfactorizable contributions are sizable for $B \to KJ/\psi$ and particularly for $B \to K\chi_{c1}$ decay. For the B decays into a (pseudo)scalar charmonia we find small nonfactorizable contributions which cannot accommodate relatively large branching ratios obtained by measurements. This specially concerns the puzzling $B \to K\chi_{c0}$ decay.

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1 Introduction

Last years there was a considerable progress in the measurements of B decays into diverse charmonium final states. These decays governed by a color-suppressed $b \to c$ transition could provide a valuable information on the factorization properties of B-meson decays.

The first observation of $B^- \to K^-\chi_{c0}$ decay a year ago by Belle collaboration [1], Table 1, clearly shows the breakdown of the naive factorization assumption in the color-suppressed B decays into charmonium. Namely, this decay, and the corresponding $B^- \to K^-\chi_{c2}$ decay are forbidden in the factorization approach [2], due to the V-A structure of the weak vertex, i.e.

$$\langle \chi_{c0} | (\bar{c}c)_{V\mp A} | 0 \rangle = \langle \chi_{c2} | (\bar{c}c)_{V\mp A} | 0 \rangle = 0 \,. \tag{1}$$

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Table 1
Experimental summary on branching ratios of B decays into charmonium [1]. The first result for a particular decay mode is Belle result, the second one is from BaBar Collaboration, and the third one (when exists) is from the CLEO experiment.

decay mode	$\mathcal{B}(10^{-4})$
$B^- \to K^- \eta_c$	$12.5 \pm 1.4^{+1.0}_{-1.2} \pm 3.8$
	$15.0 \pm 1.9 \pm 1.5 \pm 4.6$
	$6.9^{+2.6}_{-2.1} \pm 0.8 \pm 2.0$
$B^- \to K^- J/\psi$	$10.1 \pm 0.3 \pm 0.8$
	$10.1 \pm 0.3 \pm 0.5$
$B^- \to K^- \chi_{c0}$	$6.0^{+2.1}_{-1.8} \pm 1.1$
	2.7 ± 0.7
$B^- \to K^- \chi_{c1}$	$6.1 \pm 0.6 \pm 0.6$
	$7.5 \pm 0.8 \pm 0.8$
$\frac{\mathcal{B}(B^- \to K^- \chi_{c0})}{\mathcal{B}(B^- \to K^- J/\psi)}$	$0.60^{+0.21}_{-0.18} \pm 0.05 \pm 0.08$

More surprisingly the branching ratio of $B^- \to K^- \chi_{c0}$ is comparable with the branching ratios of the decays $B^- \to K^- J/\psi$ and $B^- \to K^- \chi_{c1}$, which posses the nonvanishing factorizable amplitudes. Another nonfactorizable B decay into χ_{c2} charmonium was observed inclusively with a large branching fraction

$$\mathcal{B}(B \to X \chi_{c2}) = (1.80^{+0.23}_{-0.28} \pm 0.26) \cdot 10^{-3}\,, \tag{2}$$

which is of the order of the branching fraction of the factorizable $B \to X \chi_{c1}$ decay.

However, even for the B decays into charmonium states which can be calculated by the factorization assumption there is a problem of theoretically too low predictions which cannot accommodate the experimental data. Therefore, for all B decays into charmonium states one expects large nonfactorizable effects.

In this letter we would like to approach the calculation of the nonfactorizable contributions to the B decay branching ratios into charmonium from the LCSR point of view. The method was developed for the calculation of the soft nonfactorizable corrections in $B \to \pi\pi$ [3] and was later extended on the $B \to K J/\psi$ decay [4]. Here we will made a short presentation of the method, and the interested reader should have a look in the above references for the further details.

Table 2
Charmonium states considered in the paper and their properties.

$H_c(J^{PC})$	j_{H_c}	$\langle 0\|j_{H_c}\|H_c\rangle$	m_{H_c}[GeV][6]	f_{H_c}[MeV]	$\sqrt{s_{H_c}}$[GeV][7]
$\eta_c(0^{-+})$	$i\bar{c}\gamma_5 c$	$f_{\eta_c} m_{\eta_c}$	3.0	420 ± 50 [8]	3.8 ± 0.2
$J/\psi(1^{--})$	$\bar{c}\gamma_\mu c$	$f_{J/\psi} m_{J/\psi} \epsilon_\mu$	3.1	405 ± 14 [4]	3.8 ± 0.2
$\chi_{c0}(0^{++})$	$\bar{c}c$	$f_{\chi_{c0}} m_{\chi_{c0}}$	3.4	360 [9]	4.0 ± 0.2
$\chi_{c1}(1^{++})$	$\bar{c}\gamma_\mu\gamma_5 c$	$i f_{\chi_{c1}} m_{\chi_{c1}} \epsilon_\mu$	3.5	335 [9]	4.0 ± 0.2

The effective weak Hamiltonian relevant for our discussion on the $b \to c\bar{c}s$ transition

$$H_W = \frac{G_F}{\sqrt{2}} \Big\{ V_{cb} V_{cs}^* [C_1(\mu)\mathcal{O}_1 + C_1(\mu)\mathcal{O}_1] - V_{tb} V_{ts}^* \sum_i C_i(\mu)\mathcal{O}_i \Big\} + h.c. \quad (3)$$

contain the current-current operators

$$\mathcal{O}_1 = (\bar{c}b)_{V-A}(\bar{s}c)_{V-A} \, , \quad \mathcal{O}_2 = (\bar{c}c)_{V-A}(\bar{s}b)_{V-A} \, , \quad (4)$$

and the QCD penguin operators $\mathcal{O}_{3-6}$, see the review [5] for precise definitions. For the considerations in this paper, one can safely neglect contributions of the penguin operators and consider only contributions of the leading $\mathcal{O}_{1,2}$ operators. The $\mathcal{O}_1$ operator can be projected to a color-singlet state as

$$\mathcal{O}_1 = \frac{1}{N_c}\mathcal{O}_2 + 2\tilde{\mathcal{O}}_2 \quad (5)$$

where $\tilde{\mathcal{O}}_2 = (\bar{c}T^a\gamma_\mu(1-\gamma_5)c)(\bar{s}T^a\gamma^\mu(1-\gamma_5)b)$.

For $H_c = \eta_c, J/\psi, \chi_{c0}, \chi_{c1}$, the decay rate can then be written as

$$\Gamma(B \to K H_c) = \frac{G_F^2}{32\pi} |V_{cb}|^2 |V_{cs}^*|^2 \frac{1}{m_B} \left(1 - \frac{m_{H_c}^2}{m_B^2}\right) |\mathcal{M}_{H_c}|^2 \quad (6)$$

and

$$\begin{aligned} \mathcal{M}_{H_c} &= \mathcal{M}_{H_c}^{nonfact} + \mathcal{M}_{H_c}^{fact} \\ &= \left(C_2(\mu) + \frac{C_1(\mu)}{3}\right) \langle H_c K|\mathcal{O}_2|B\rangle + 2C_1(\mu)\langle H_c K|\tilde{\mathcal{O}}_2|B\rangle \, . \end{aligned} \quad (7)$$

The first part of (7) can be calculated by factorizing the matrix element of the $\mathcal{O}_2$ operator as

$$\langle H_c K|\mathcal{O}_2|B\rangle = \langle H_c|(\bar{c}c)_{V-A}|0\rangle\langle K|(\bar{s}b)_{V-A}|B\rangle\,, \quad (8)$$

and using the corresponding expressions for the $\langle H_c|(\bar{c}c)_{V-A}|0\rangle$ from Table 2. Note that due to the reason stated in introduction, Eq.(1), there is no factorizable contribution to the $B \to K\chi_{c0}$ decay. The $B \to K$ matrix element from (8) is defined by the decomposition

$$\langle K(q)|\bar{s}\gamma_\mu b|B(p+q)\rangle = \\ (2q+p)_\mu F^+_{BK}(p^2) + \frac{m_B^2 - m_K^2}{p^2} p_\mu \left(-F^+_{BK}(p^2) + F^0_{BK}(p^2)\right)\,, \quad (9)$$

while the estimation of the form factors F^+_{BK} and F^0_{BK} in the LCSR approach [10] gives the following values needed in our calculation:

$$F^+_{BK}(m^2_{J/\psi}) = 0.60\,, \quad F^+_{BK}(m^2_{\chi_{c1}}) = 0.74\,, \quad F^0_{BK}(m^2_{\eta_c}) = 0.42\,, \quad (10)$$

with the theoretical uncertainty of 15%.

For a particular charmonium considered, the expression (7) can be then brought into the following form:

$$\begin{aligned}
&\mathcal{M}_{J/\psi} = \\
&\quad 2\epsilon\cdot p_K\, f_{J/\psi} m_{J/\psi} F^+_{BK}(m^2_{J/\psi}) \left[\left(C_2(\mu) + \frac{C_1(\mu)}{3}\right) + 2C_1(\mu)\frac{\tilde{F}_{J/\psi}(\mu)}{F^+_{BK}(m^2_{J/\psi})}\right]\,, \\
&\mathcal{M}_{\chi_{c1}} = \\
&\quad i\, 2\epsilon\cdot p_K\, f_{\chi_{c1}} m_{\chi_{c1}} F^+_{BK}(m^2_{\chi_{c1}}) \left[\left(C_2(\mu) + \frac{C_1(\mu)}{3}\right) + 2C_1(\mu)\frac{\tilde{F}_{\chi_{c1}}(\mu)}{F^+_{BK}(m^2_{\chi_{c1}})}\right]\,, \\
&\mathcal{M}_{\eta_c} = i\, m_B^2 f_{\eta_c} F^0_{BK}(m^2_{\eta_c}) \left[\left(C_2(\mu) + \frac{C_1(\mu)}{3}\right) + 2C_1(\mu)\frac{\tilde{F}_{\eta_c}(\mu)}{F^0_{BK}(m^2_{\eta_c})}\right]\,, \\
&\mathcal{M}_{\chi_{c0}} = 2C_1(\mu) m_B^2 f_{\chi_{c0}} \tilde{F}_{\chi_{c0}}(\mu)\,. \qquad (11)
\end{aligned}$$

$\tilde{F}_{H_c=(\eta_c,J/\psi,\chi_{c0},\chi_{c1})}$ represents the nonfactorizable part directly proportional to the contribution of the $\tilde{O}_2$ operator in (7). This contribution vanishes under the factorization assumption. Below we present the calculation of the nonfactorizable soft contributions in the LCSR approach.

2 Nonfactorizable contributions in $B \to KH_c$ ($H_c = \eta_c, J/\psi, \chi_{cJ(J=0,1)}$) decays from LCSR

The starting point of the calculation using the LCSR method is the correlation function, defined as

$$\mathcal{F} = i^2 \int dx^4 \int dy^4 e^{-ip_B x + i(p_{H_c}-k)y} \langle K(p_K)| j_{H_c}(y) \tilde{\mathcal{O}}_2(0) j_B(x)|0\rangle \,, \tag{12}$$

where $p_K^2 = m_K^2 = 0$, $k^2 = 0$ and $p_B^2 = m_B^2$, $p_{H_c}^2 = m_{H_c}^2$. The interpolating current of a B^- meson is given as $j_B = im_b \bar{b}\gamma_5 u$, whereas the choice of the interpolating charmonium current has to be done according to the definite J, P, and C quantum numbers of a particular meson H_c. The considered charmonium currents, together with some properties of charmonium needed in the sum rule analysis are summarized in Table 2. For the consistency, we also include discussion on the $B \to KJ/\psi$ decay which was already extensively presented in [4] and will only quote here the numerical result. For the rest of the B decays into charmonium, the calculation closely follows the LCSR approach developed in [3] and [4] and we refer to these references for all details. Including the twist-3 and twist-4 nonfactorizable contributions, we can write first the nonfactorizable contribution to the $B \to K\chi_{c1}$ decay analogously to the result for $B \to KJ/\psi$, Eq.(65) in [4]:

$$\begin{aligned}
\tilde{F}_{\chi_{c1}}(\mu_b) = & \frac{1}{4\pi^2 f_{\chi_{c1}}^2} \int_{4m_c^2}^{s_0^{\chi_{c1}}} ds \frac{(m_{\chi_{c1}}^2 + Q_0^2)^{n+1}}{(s+Q_0^2)^{n+1}} \\
& \frac{1}{2m_B^2 f_B} \int_{u_0^B}^{1} \frac{du}{u} e^{(m_B^2 - (m_b^2 - m_{\chi_{c1}}^2(1-u))/u)/M^2} \int_0^{1-\frac{4m_c^2}{s}} \frac{dy}{2\sqrt{y}} \frac{m_b}{m_B^2 - m_{\chi_{c1}}^2} \\
& \left\{ \frac{f_{3K}}{2} \left[\int_0^u \frac{dv}{v^2} \phi_{3K}(1-u, u-v, v) \left(\frac{m_b^2 - m_{\chi_{c1}}^2}{u}(2v - X) + s - \frac{4m_c^2}{1-y} \right) \right.\right. \\
& \qquad \left. - \left(s - \frac{4m_c^2}{1-y} \right) \left(\frac{1}{v^2} \phi_{3K}(1-u, u-v, v) \right)_{v=0} X \right] \\
& \qquad \left. + m_b f_K \int_0^u \frac{dv}{v} \tilde{\phi}_\perp(1-u, u-v, v) \left[3 - \frac{2}{v} X \right] \right\} ,
\end{aligned} \tag{13}$$

where $s_0^{H_c}$ and s_0^B are the effective threshold parameters of the perturbative continuum in the H_c and B channel, respectively, and $u_0^B = (m_b^2 - m_{H_c}^2)/(s_0^B - m_{H_c}^2)$, here specified for $H_c = \chi_{c1}$ charmonium. M, the Borel parameter in the B channel, and the parameter $n = 1, 2, \ldots$ in the charmonium channel have to be chosen in such a way that a reliable perturbative calculation is possible, but on the other hand that excited and continuum states in a given channel are suppressed. The function X appearing above is $X = x(s, y, m_B^2) = (s - 4m_c^2/(1-y))/(s - m_B^2)$ and the expansion up to $O(X^2)$ is performed. The twist-3, ϕ_{3K}, and the twist-4, $\tilde{\phi}_\perp$, three-particle kaon distribution amplitudes are defined as usual [11]. The scale at which $\tilde{F}_{H_c}$'s are calculated is $\mu_b \sim m_b/2 \sim 2.4$ GeV.

As for the B decays into the scalar and pseudoscalar charmonium the calculation yields

$$\begin{aligned}
\tilde{F}_{\eta_c}(\mu_b) = & \frac{1}{f_{\eta_c} m_B^2} \frac{1}{4\pi^2 f_{\eta_c}} \int_{4m_c^2}^{s_0^{\eta_c}} ds \frac{(m_{\eta_c}^2 + Q_0^2)^{n+1}}{(s+Q_0^2)^{n+1}} \\
& \frac{1}{2m_B^2 f_B} \int_{u_0^B}^1 \frac{du}{u} e^{(m_B^2 - (m_b^2 - m_{\eta_c}^2(1-u))/u)/M^2} \int_0^{1-\frac{4m_c^2}{s}} \frac{dy}{(1-y)\sqrt{y}} \frac{m_b m_c}{m_{\eta_c}} \\
& \left\{ f_{3K} \left[\int_0^u \frac{dv}{v} \phi_{3K}(1-u, u-v, v) \left(-\frac{m_b^2 - m_{\eta_c}^2}{u} \right) \right] \right. \\
& \left. + 3 m_b f_K \int_0^u \frac{dv}{v} \tilde{\phi}_\perp(1-u, u-v, v) \right\},
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
\tilde{F}_{\chi_{c0}}(\mu_b) = & \frac{1}{f_{\chi_{c0}} m_B^2} \frac{1}{4\pi^2 f_{\chi_{c0}}} \int_{4m_c^2}^{s_0^{\chi_{c0}}} ds \frac{(m_{\chi_{c0}}^2 + Q_0^2)^{n+1}}{(s+Q_0^2)^{n+1}} \\
& \frac{1}{2m_B^2 f_B} \int_{u_0^B}^1 \frac{du}{u} e^{(m_B^2 - (m_b^2 - m_{\chi_{c0}}^2(1-u))/u)/M^2} \int_0^{1-\frac{4m_c^2}{s}} \frac{dy}{(1-y)\sqrt{y}} \frac{m_b m_c}{m_{\chi_{c0}}} \\
& \times \left\{ f_{3K} \left[\int_0^u \frac{dv}{v} \phi_{3K}(1-u, u-v, v) \left(\frac{m_b^2 - m_{\chi_{c0}}^2}{u} \right) \left(y + (1-y)\frac{X}{v} \right) \right] \right. \\
& \left. + 3 m_b f_K \int_0^u \frac{dv}{v} \tilde{\phi}_\perp(1-u, u-v, v) \left(y + (1-y)\frac{X}{v} \right) \right\}.
\end{aligned} \tag{15}$$

3 Numerical predictions and discussions

Let us first specify the numerical values of the needed parameters. For parameters in the B channel we use $m_B = 5.28$ GeV and the values taken from [12]: $f_B = 180 \pm 30$ MeV, $m_b = 4.7 \pm 0.1$ GeV, and $s_0^B = 35 \pm 2\,\text{GeV}^2$. For the charmonium states we use the parameters from Table 2 and $m_c = 1.25 \pm 0.10$. The K meson decay constant is taken as $f_K = 0.16$ GeV. For parameters which enter the coefficients of the twist-3 and twist-4 kaon wave functions we suppose that $f_{3\pi} \simeq f_{3K}$ and $\delta_K^2 \simeq \delta_\pi^2$, and take $f_{3K} = 0.0026$ GeV, $\delta^2(\mu_b) = 0.17$

Table 3
Theoretical results for the $B \to KH_c$ decays calculated in this paper. $\tilde{F}_{H_c}(\mu_b)$ is the nonfactorizable contribution, $\mathcal{M}_{H_c}^{nonfact}/\mathcal{M}_{H_c}^{fact}$, Eq.(7), is the ratio of the nonfactorizable and factorizable amplitudes for a particular mode $B \to KH_c$, whereas $\mathcal{B}$ is the branching ratio, all calculated at $\mu = \mu_b$. Large scale-dependent uncertainties pertinent to the factorizable amplitude are not included.

decay mode	$\tilde{F}_{H_c}(\mu_b)$	$\mathcal{M}_{H_c}^{nonfact}/\mathcal{M}_{H_c}^{fact}$	$\mathcal{B}(10^{-4})$
$B^- \to K^-\eta_c$	0.0015-0.0019	0.08-0.10	2.0 ± 0.1
$B^- \to K^-J\psi$	0.011-0.018 [4]	0.40-0.70 [4]	3.3 ± 0.6
$B^- \to K^-\chi_{c0}$	-(0.0007-0.0008)	-	0.0017 ± 0.0002
$B^- \to K^-\chi_{c1}$	0.044 - 0.052	1.30-1.50	5.1 ± 0.5

GeV [11]. The stability region for the Borel parameter is found in the interval $M^2 = 10 \pm 2\,\text{GeV}^2$, known also from other LCSR calculations of B meson properties. Concerning the sum rules in the charmonium channels, the calculation is rather stable on the change of n in the interval $n = 4 - 7$. Q_0^2 is parameterized by $Q_0^2 = 4m_c^2\xi$, where in order that the lowest resonances dominate ξ takes values from 0.5 to 1 for the B decays into s-wave charmonia, while ξ is between 1 and 2.5 in the decays into p-wave charmonia [7].

The results of our calculation are summarized in Table 3. Comparing the numbers from Table 1 and Table 3, it is important to note that in general the calculated branching ratios are still too low to accommodate the data, except maybe for the $B \to K\chi_{c1}$ decay. The nonfactorizable correction to $B \to KJ/\psi$ and particularly to $B \to K\chi_{c1}$ is large, for the $B \to K\chi_{c1}$ decay this correction is even larger than the factorizable contribution, Table 3. On the other hand, in B decays into (pseudo)scalar charmonia the nonfactorizable contributions are small. The reason is the cancellation of the twist-3 and twist-4 contributions in these decays. However, even without this cancellation, the nonfactorizable effects produced by the exchange of a soft gluon between a kaon and χ_{c0} as calculated here, could not be able to account for such a large branching ratio of $B \to K\chi_{c0}$ as measured experimentally. These would demand contributions which have to be at least an order of magnitude larger than those typically occurring in a B decay into charmonium state as estimated by the LCSR method. Therefore it is very unlikely that the mechanism of the nonfactorizable soft gluon exchange is the reason for a relatively large branching ratio of the $B \to K\chi_{c0}$ decay. A possible speculative explanation for large discrepancies between the theoretical results for the B decays into (pseudo)scalar charmonia and the experimental data one could find in the nonperturbative effects of the instanton type. They could appear due to the light quark admixtures in the η_c and χ_{c0} mesons, yet it would be hard to account for such contributions reliably.

The $B \to K\chi_{c0}$ decay was also analyzed within QCD factorization method [13] and it was observed [14] that there is a problem of logarithmic divergences of the decay amplitude already at the leading-twist order. In another approach, by studying the mechanism of the rescattering of charmed intermediate states in $B \to K\chi_{c0}$, the authors of Ref.[15] show that the rescattering effects could provide the large part of the $B \to K\chi_{c0}$ amplitude.

To summarize, we have studied the soft nonfactorizable contributions to $B \to KH_c(H_c = \eta_c, J/\psi, \chi_{c0}, \chi_{c1})$ decays by using the LCSR approach. Inspite of the expected large contributions which could explain large discrepancy between the factorizable predictions and the experimental data, we were not able to confirm these expectations, except for the $B \to K\chi_{c1}$ decay, for which large nonfactorizable corrections are found. The other predicted B decays into charmonium receive nonfactorizable soft contributions too small to accommodate the data. Unfortunately, this is particularly true for the puzzling $B \to K\chi_{c0}$ decay that factorized amplitude vanishes and the LCSR mechanism considered in this paper cannot explain its relatively large branching ratio.

Note added:
While this work has been prepared, Ref. [16] appeared, where the authors discuss nonfactorizable soft contributions in the $B \to K\eta_c$ and $B \to K\chi_{c0}$ decays within the LCSR approach. There are some differences observed in the approach as well as in the results between [16] and our paper. As for the $B \to K\eta_c$ decay, the authors of [16] choose the pseudovector current for η_c. In that case, the result can be easily derived from Eq.(13) according to the approach taken from [4]. The differences show in the twist-4 part. Apart from this, by taking the pseudovector current to describe η_c one has also to include the mixing with the $\chi_{c1}(1^{++})$ state explicitly [17] which was not considered in [16]. More importantly, for the problematic $B \to K\chi_{c0}$ decay, apart from the superfluous factor x in Eq.(49) of [16], we agree analytically in the twist-3 part. In the second version of [16], the authors have corrected numerical errors and have included the twist-4 contribution to their result for the nonfactorizable $B \to K\chi_{c0}$ amplitude. Although we now agree that the soft nonfactorizable contribution in the $B \to K\chi_{c0}$ decay is small and cannot accommodate the experimental data, we still disagree in the twist-4 part of the nonfactorizable contributions, which renders the numerical results from this paper somewhat different from those obtained in [16].

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References

[1] F. Fang, "B to charmonium: Mini-summary," in Proc. of the Flavor Physics and CP Violation (FPCP), Philadelphia, Pennsylvania, 16-18 May 2002, Ed. R.G.C. Oldeman, eConf C020516, [arXiv:hep-ex/0207004].

[2] P. F. Harrison and H. R. Quinn [BABAR Collaboration], "The BaBar physics book: Physics at an asymmetric B factory," SLAC-R-0504.

[3] A. Khodjamirian, Nucl. Phys. B **605** (2001) 558 [arXiv:hep-ph/0012271].

[4] B. Melić, Phys. Rev. D **68** (2003) 034004 [arXiv:hep-ph/0303250].

[5] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125 [arXiv:hep-ph/9512380].

[6] K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66** (2002) 010001.

[7] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1.

[8] D. S. Hwang and G. H. Kim, Z. Phys. C **76** (1997) 107 [arXiv:hep-ph/9703364].

[9] V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Phys. Rept. **41** (1978) 1.

[10] P. Ball, JHEP **9809** (1998) 005 [arXiv:hep-ph/9802394]; A. Khodjamirian, R. Ruckl, S. Weinzierl, C. W. Winhart and O. I. Yakovlev, Phys. Rev. D **62** (2000) 114002 [arXiv:hep-ph/0001297].

[11] V. M. Braun and I. E. Filyanov, Z. Phys. C **48** (1990) 239.

[12] V. M. Belyaev, A. Khodjamirian and R. Ruckl, Z. Phys. C **60** (1993) 349 [arXiv:hep-ph/9305348].

[13] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591** (2000) 313 [arXiv:hep-ph/0006124].

[14] Z. Z. Song and K. T. Chao, Phys. Lett. B **568** (2003) 127 [arXiv:hep-ph/0206253].

[15] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B **542** (2002) 71 [arXiv:hep-ph/0207061].

[16] Z. G. Wang, L. Li and T. Huang, "Nonfactorizable soft contributions in the $B \to \eta_c K, \chi_{c0} K$ decays with the light-cone sum rules approach," arXiv:hep-ph/0311296.

[17] A. Khodjamirian, private communication.