EPJ manuscript No.
(will be inserted by the editor)

Estimate of the charming penguin contributions to $B \to \pi\pi$

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Received: date / Revised version: date

Abstract. We consider the problem of factorization in B decays and illustrate the calculation of nonfactorizable contributions employing the QCD light-cone sum rule method. We present a more detailed calculation of the "charming penguin" contributions as a potential source of the substantial nonfactorizable $O(1/m_b)$ effects in the $B \to \pi\pi$ decay. Although the predicted corrections are not sizable, by calculating the CP asymmetry we illustrate how such corrections can accumulate to a visible effect. In conclusion, nonfactorizable contributions in nonleptonic B decays into charmonium are briefly discussed.

PACS. 13.25.Hw ,11.55.Hx,12.39.St

1 Nonleptonic B decays and factorization

For a long time the naive factorization method, in which the matrix element of the four-quark operator is approximated by the product of two matrix elements of the bilinear quark currents, was considered as a sufficiently precise tool for estimating matrix elements emerging in the amplitude of nonleptonic weak B decays. Nowadays, in order to make real use of the already very precise experimental data, we are forced to provide a more accurate estimate of nonleptonic decays, in particular of the nonperturbative part of the decay amplitude. Therefore, the question about the applicability of the factorization and the size of nonfactorizable corrections naturally emerged. The question was particularly raised in the work [1], where it was argued that in the charmless B decays there could exist large $\mathcal{O}(\Lambda_{QCD}/m_b)$ corrections and large strong phases coming from the "charming penguins".

There are several models which one can apply for the calculation of matrix elements of B-meson weak decays beyond the naive factorization [2,3,4]. By using the QCD factorization approach [3] one can show that the exclusive B-decay amplitude in the $m_b \to \infty$ limit can be expressed in terms of the factorizable part and the calculable $O(\alpha_s)$ nonfactorizable correction. However, because of the arguments given above, the nonfactorizable subleading effects in the decay amplitude, suppressed by inverse powers of m_b, could be important and have to be investigated. Estimates of nonfactorizable contributions in B decays, including the power-suppressed $O(1/m_b)$ contributions, can be obtained [4] using the method of QCD light-cone sum rules (LCSR). In particular, nonfactorizable contributions to nonleptonic decays such as $B \to \pi\pi$ and $B \to \pi K$ become interesting for a more precise constraint on the $\gamma = arg(V_{ub})$ angle of the CKM matrix. In connection with this problem, it is worth mentioning that there are also several strategies used to determine the γ angle from $B \to \pi\pi$ and $B \to \pi K$ based on the isospin and SU(3) relations. Unfortunately, the theoretical accuracy of these relations is limited and it has to be improved by calculating the SU(3) breaking effects, which can also be addressed by the LCSR method [5].

2 Matrix elements for $B \to \pi\pi$ from LCSR

The LCSR expression for the $B \to \pi\pi$ hadronic matrix element of the $\mathcal{O}_i$ operator of the weak Hamiltonian is derived by the procedure presented in detail in [4,8]. One starts by introducing the correlation function

$$F_\alpha^{(\mathcal{O}_i)} = i^2 \int d^4x\, e^{-i(p-q)x} \int d^4y\, e^{i(p-k)y} \times \langle 0 | T\{ j_{\alpha 5}^{(\pi)}(y) \mathcal{O}_i(0) j_5^{(B)}(x)\} | \pi^-(q)\rangle = (p-k)_\alpha F^{(\mathcal{O}_i)}(s_1, s_2, P^2) + \dots , \qquad (1)$$

where $j_{\alpha 5}^{(\pi)} = \overline{u}\gamma_\alpha\gamma_5 d$ and $j_5^{(B)} = i m_b \overline{b}\gamma_5 d$ are the quark currents interpolating the pion and the B meson, respectively. By employing the dispersion relation technique and by assuming the quark-hadron duality, we can write the LCSR expression for the hadronic matrix element $A^{(\mathcal{O}_i)}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv \langle \pi^-(p)\pi^+(-q)|\mathcal{O}_i|\bar{B}_d^0(p-q)\rangle$ as [4]

$$f_\pi f_B A^{(\mathcal{O}_i)}(\bar{B}_d^0 \to \pi^+\pi^-) e^{-m_B^2/M_2^2} = \int_{m_b^2}^{s_0^B} ds_2 e^{-s_2/M_2^2} \left\{ \int_0^{s_0^\pi} ds_1 e^{-s_1/M_1^2} \mathrm{Im}_{s_2}\mathrm{Im}_{s_1} F^{(\mathcal{O}_i)}(s_1, s_2, m_B^2) \right\}, \qquad (2)$$

where M_1 and M_2 are the Borel parameters in the pion and B-meson channels, respectively. The parameter s_0^π

(s_0^B) is the effective threshold parameter of the perturbative continuum in the pion (B-meson) channel. In the sum rule (2) the finite m_b corrections are taken into account, but numerically very small corrections of order s_0^π/m_B^2 are neglected.

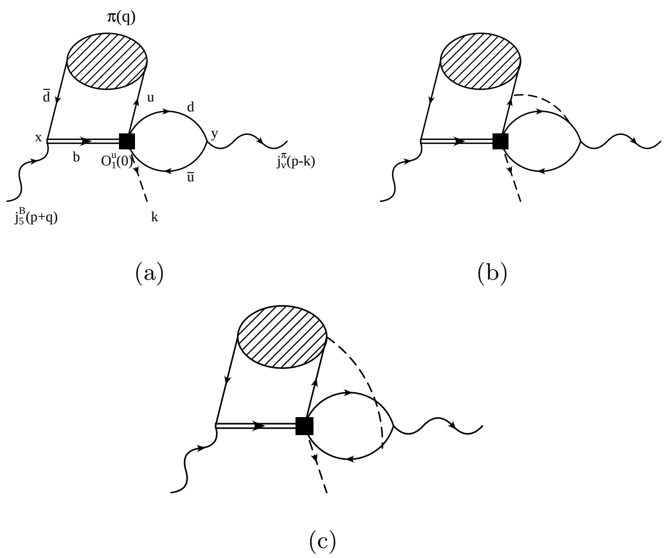


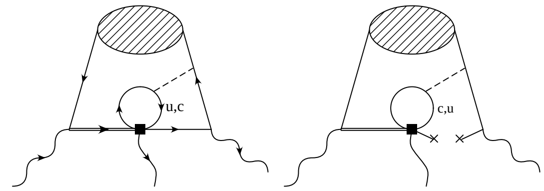
Fig. 1. Contributions in the emission topology: (a) factorizable contribution from the $\mathcal{O}_1^u$ operator; (b) an example of the $O(\alpha_s)$ correction; (c) an example of the nonfactorizable diagram from the $\tilde{\mathcal{O}}_1^u$ operator.

2.1 Emission topology

The leading contributions in the emission topology are shown in Fig.1. The factorizable part (Fig.1a) stems from the contribution of the leading operator $\mathcal{O}_1^u = (\overline{d}\Gamma_\mu u)(\overline{u}\Gamma^\mu b)$, mainly reproducing the naive factorization result in terms of the light-cone calculated f_π and $F_{B\to\pi}$ form factor. The hard corrections (Fig.1b) have not yet been addressed in LCSR, but the corresponding contributions calculated in the $m_b \to \infty$ limit in QCD factorization [3] appeared to be small. The soft nonfactorizable contribution due to the $\tilde{\mathcal{O}}_1^u$ operator (which is the color-octet part of $\mathcal{O}_2^u = 1/N_c\mathcal{O}_1^u + 2\tilde{\mathcal{O}}_1^u$ and expresses the exchange of a soft gluon between two pions (Fig.1c)) was shown [4] to be small, although of the same size as the $\mathcal{O}(\alpha_s)$ correction mentioned above.

2.2 Penguin topology

Types of the dominant penguin diagrams are shown in Fig.2 [7]. The main effect which we calculate arises from the c-quark loop annihilation into a hard gluon (Fig.2a). In addition, there is the quark-condensate contribution (Fig.2b) which after a detailed analysis appeared to be a natural upper limit to all neglected contributions of multiparticle distribution amplitudes (DA's). The effects of the



(a) (b)

Fig. 2. Contributions in the penguin topology: (a) an example of the $\mathcal{O}(\alpha_s)$ nonfactorizable penguin amplitude; (b) an example of the chirally enhanced twist-3 contribution to the penguin amplitude. The square stands for $\mathcal{O}_2^{u,c}$ and $\mathcal{O}_{1-6}$ operators. The leading contribution from the $\mathcal{O}_8$ operator proceeds without the quark loop [8, 7].

soft (low-virtuality) gluons coupled to the c-quark loop [7] in the sum rule approach manifest as multiparticle DA's and are therefore suppressed at least by $\mathcal{O}(\alpha_s/m_b)$. Therefore, the nonfactorizable $\mathcal{O}(1/m_b)$ corrections from penguin loops are mainly of perturbative origin, and both contributions from Fig.2. generate the strong rescattering phases in $B \to \pi\pi$ perturbatively by the well-known BSS mechanism [6].

3 CP asymmetry in $\overline{B}_d^0 \to \pi^+\pi^-$

Penguin contributions appear to produce a notable effect in the direct CP asymmetry and we take the CP asymmetry as a testing ground for the influence of the $1/m_b$ corrections in the charming penguin contributions. Following [7], we concentrate on the direct CP asymmetry in the $B_d^0 \to \pi^+\pi^-$ decay, which is given as

$$a_{\rm CP}^{\rm dir} \equiv (1-|\xi|^2)/(1+|\xi|^2)\,, \qquad (3)$$

where $\xi = e^{-2i(\beta+\gamma)}(1+R\,e^{i\gamma})/(1+R\,e^{-i\gamma})$ and $R \equiv -P/(R_b T)$. Here T is the contribution to the $B \to \pi\pi$ amplitude proportional to $V_{ub}V_{ud}^* = |V_{ub}V_{ud}^*|e^{-i\gamma}$. It contains the tree amplitude, the penguin-loop contractions of the current-current operators $\mathcal{O}_{1,2}^u$, and also the $V_{ub}V_{ud}^*$ proportional penguin $\mathcal{O}_{3-6}$ operator contractions. The remaining contributions, being proportional to $V_{cb}V_{cd}^*$, are included in P. The penguin-loop contractions of the current-current operators $\mathcal{O}_{1,2}^c$ represent the main contribution to this part. The factor $R_b = |V_{ub}||V_{ud}|/(|V_{cb}||V_{cd}|)$ is the ratio of the CKM matrix elements.

Both T and P amplitudes have strong phases; therefore, we have $T = |T|e^{i\delta_T}$ and $P = |P|e^{i\delta_P}$ and the CP asymmetry for $B_d^0 \to \pi^+\pi$ can be written as

$$a_{\rm CP}^{\rm dir} = \frac{-2|R|\,\sin(\delta_P-\delta_T)\,\sin\gamma}{1-2|R|\,\cos(\delta_P-\delta_T)\,\cos\gamma+|R|^2}\,. \qquad (4)$$

All contributions shown in Figs.1 and 2 are calculated in LCSR at finite m_b. We also include the LCSR result for the gluonic penguin contribution of the dipole operator $\mathcal{O}_{8g}$ [8]. The electroweak penguin contributions to $B \to \pi\pi$ are color-suppressed and negligible.

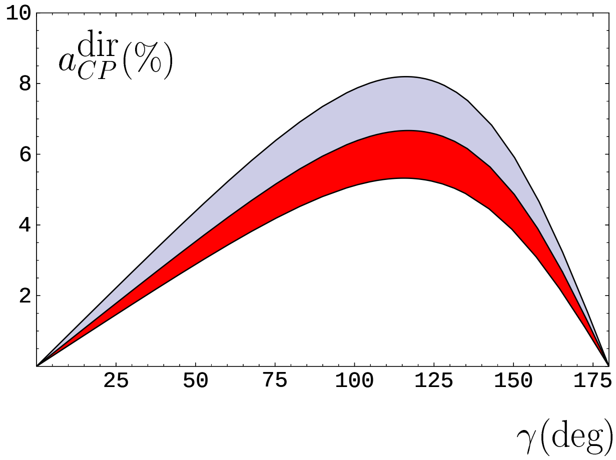


Fig. 3. Direct CP asymmetry in $B_d^0 \to \pi^+\pi^-$ as a function of the CKM angle γ. The uppermost curve is the result obtained for $m_b \to \infty$. The dark region is the LCSR result, with all uncertainties from the method included (uncertainties in the CKM matrix elements are not taken into account). The light region shows the deviation from the $m_b \to \infty$ limit result.

The hard $O(\alpha_s)$ corrections to T and P amplitudes are known in the $m_b \to \infty$ limit from QCD factorization [3]. We have examined the influence of these contributions to the phases δ_T and δ_P. It appears that they are highly suppressed in comparison with the phases emerging from the penguin-loop contributions. Therefore, we have neglected $O(\alpha_s)$ corrections in (4).

In Fig. 3 we show $a_{\text{CP}}^{\text{dir}}$ as a function of γ, calculated by using the penguin contributions estimated from LCSR at finite m_b (dark region) and compare the result in the infinite-mass limit that agrees with the QCD factorization prediction [3] (the uppermost curve). Both results are taken at the same scale $\mu_b \sim m_b/2$ as used in LCSR.

The prediction shown in Fig.3 is not final, annihilation effects are missing and the uncertainty in the CKM matrix elements is not taken into account either. However, the figure nicely illustrates the size of $O(1/m_b)$ corrections and the difference between the results obtained at the finite m_b and in the $m_b \to \infty$ limit (the light region in Fig.3).

4 Conclusion

We have discussed the factorization in nonleptonic B meson decays and have presented the LCSR calculation of the $B \to \pi\pi$ decay. It has been shown that the leading contribution factorizes, while the corrections beyond the factorization can be systematically approached. In the $m \to \infty$ limit, our result agrees with the QCD factorization prediction, while at finite m_b we have found $O(\alpha_s/m_b)$ effects which are numerically small, but accumulate to a sizable correction in the direct CP asymmetry. Large charming-penguin contributions *per se* are not predicted by this model.

Numerically, the nonfactorizable corrections in $B \to \pi\pi$ are not large, but the situation in the nonleptonic color-suppressed decays seems to be somewhat different. Recent measurements of the color-suppressed $\overline{B}^0 \to D^{(*)0}\pi^0$ decays, $B \to J/\psi K$ and particularly $B \to \chi_{cJ} K$ decays show large discrepancy with the naive factorization prediction and provide clear evidence for large nonfactorizable contributions. The $B \to \chi_{c0} K$ and $B \to \chi_{c2} K$ decays are particularly interesting because their branching ratios predicted in the naive factorization are exactly zero, and the measurements yield branching ratios $\sim \mathcal{O}(10^{-4})$ which are therefore comparable with $BR(B \to J/\psi K)$. This should not come out as a surprise, because the LCSR calculation shows the existence of large nonfactorizable corrections in $B \to J/\psi K$ of the order of 70% [9]. The corresponding large corrections are also expected for $B \to \chi_{cJ} K$ decays. Unfortunately, these large corrections still appear to be insufficient to reproduce the data, because the discrepancy between the the theory and the experiment is much larger (in the case of $B \to J/\psi K$, the data and the LCSR improved prediction still differ by a factor of two) and at the moment it is not clear how these discrepancies could be reduced.

Acknowledgments

I would like to thank the organizers for the invitation to participate at this very pleasant and stimulating conference and for their financial support. This work is also supported by the Ministry of Science and Technology of the Republic of Croatia under the contract 0098002.

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