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Neutrino nature, total and geometric phase

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Abstract. We study the total and the geometric phase associated with neutrino mixing and we show that the phases produced by the neutrino oscillations have different values depending on the representation of the mixing matrix and on the neutrino nature. Therefore the phases represent a possible probe to distinguish between Dirac and Majorana neutrinos.

1. Introduction

The phenomenon of neutrino mixing and oscillation, that has been proved experimentally [1]-[6], implies that the neutrino has a mass. Then the neutrino, being a neutral particle, can be a Majorana particle (a fermion that is its own antiparticle), or a Dirac particle (a fermion different from its antiparticle). At the moment the neutrino nature is not established.

A Majorana field is characterized by the presence of the Majorana phases ϕ_i in the mixing matrix which violate the CP symmetry. These phases cannot be eliminated since the Lagrangian of Majorana neutrinos is not invariant under $U(1)$ global transformation. By contrast, for Dirac neutrino, the Lagrangian is invariant under $U(1)$ global transformation and the ϕ_i phases can be removed. The mixing matrices for Majorana U_M and for Dirac neutrinos U_D can be related for example by the equation, $U_M = U_D \cdot diag(1, e^{i\phi_1}, e^{i\phi_2}, ..., e^{i\phi_{n-1}})$, where $i = 1, ..., n-1$. Other representations of U_M can be obtained by the rephasing the lepton charge fields in the charged current weak-interaction Lagrangian [7]. For example, in two flavor neutrino mixing case, one can consider the following mixing matrices for Majorana neutrinos

$$U_1 = \begin{pmatrix} \cos\theta & \sin\theta\, e^{i\phi} \\ -\sin\theta & \cos\theta\, e^{i\phi} \end{pmatrix}, \qquad or \qquad U_2 = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ -\sin\theta\, e^{i\phi} & \cos\theta \end{pmatrix}, \tag{1}$$

where θ is the mixing angle, and ϕ is the Majorana phase. It should be noted that, neglecting the dissipation [8], the Majorana phases do not affect the neutrino oscillation formulae, being such formulae equivalent for Majorana and for Dirac neutrinos [9]. Therefore, the oscillation formulae are not useful in the study of the neutrino nature.

Recently, the study of the geometric phase has attracted also a great attention. The geometric phase appears in the evolution of any quantum state describing a system characterized by a Hamiltonian defined on a parameter space [10]–[25]. This phase arises in many physical systems [26]–[41] and it has been observed experimentally.

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In this paper, we report the results of the study on the total and geometric phases of neutrino presented in Ref.[42] and we show that, unlike the oscillation formulae, the total phase (and the dynamical one), generated by the transition between different flavors, depends on the choice of the matrix U. Indeed, different choices of U lead to different values of the total phases. In particular, considering the two flavor neutrino mixing case, we show that the use of the matrix U_2 in Eq.(1) (and of that corresponding to oscillations in a medium), generates values of the phases which are different for Majorana and for Dirac neutrinos. By contrast, if we consider the U_1 matrix, all the phases are independent from ϕ and Majorana neutrinos cannot be distinguished from Dirac neutrinos.

The paper is organized as follows. In Section 2 we analyze the total and the geometric phase for neutrinos by using different mixing matrces. In Section 3 we report a numerical analysis on the neutrino phases and in Section 4 we give our conclusions.

2. Total and geometric phases for neutrinos

We analyze the neutrinos propagation in vacuum and through a medium. The matter effects, are taken into account by replacing in the flavor states in vacuum, Δm^2 with $\Delta m_m^2 = \Delta m^2 R_\pm$, and $\sin 2\theta$ with $\sin 2\theta_m = \sin 2\theta / R_\pm$. The coefficients $R_\pm$ are, $R_\pm = \sqrt{\left(\cos 2\theta \pm \frac{2\sqrt{2} G_F n_e E}{\Delta m^2}\right)^2 + \sin^2 2\theta}$, with $+$ for oscillation of antineutrinos and $-$ for oscillations of neutrinos [43, 44]. In the following, we consider the flavor states $|\nu_e(z)\rangle$ and $|\nu_\mu(z)\rangle$ at the z distance given by the mixing matrix U_2, with θ replaced by θ_m,

$$
\begin{aligned}
|\nu_e(z)\rangle &= \cos\theta_m e^{i\frac{\Delta m_m^2}{4E}z}|\nu_1\rangle + e^{-i\phi}\sin\theta_m e^{-i\frac{\Delta m_m^2}{4E}z}|\nu_2\rangle, \\
|\nu_\mu(z)\rangle &= -e^{i\phi}\sin\theta_m e^{i\frac{\Delta m_m^2}{4E}z}|\nu_1\rangle + \cos\theta_m e^{-i\frac{\Delta m_m^2}{4E}z}|\nu_2\rangle.
\end{aligned} \tag{2}
$$

and we derive the total and the non–cyclic geometric phase [18]. For a quantum system whose state vector is $|\psi(s)\rangle$, the geometric phase is defined as the difference between the total phase $\Phi_\psi^{tot} = \arg\langle\psi(s_1)|\psi(s_2)\rangle$ and the dynamic phase $\Phi_\psi^{dyn} = \Im\int_{s_1}^{s_2}\langle\psi(s)|\dot{\psi}(s)\rangle ds$, i.e. $\Phi^g = \Phi_\psi^{tot} - \Phi_\psi^{dyn}$. Here, s is a real parameter such that $s \in [s_1, s_2]$, and the dot denotes the derivative with respect to s. For electron neutrino, the geometric phase is

$$
\begin{aligned}
\Phi_{\nu_e}^g(z) &= \arg\left[\langle\nu_e(0)|\nu_e(z)\rangle\right] - \Im\int_0^z \langle\nu_e(z')|\dot{\nu}_e(z')\rangle dz' \\
&= \arg\left[\cos\left(\frac{\Delta m_m^2 z}{4E}\right) + i\cos 2\theta_m \sin\left(\frac{\Delta m_m^2 z}{4E}\right)\right] - \frac{\Delta m_m^2 z}{4E}\cos 2\theta_m\,.
\end{aligned} \tag{3}
$$

For muon neutrino we have $\Phi_{\nu_\mu}^g(z) = -\Phi_{\nu_e}^g(z)$. Eq.(3) holds both for Majorana and for Dirac neutrinos, indeed it does not depend on the CP violating phase ϕ and thus it is independent on the choice of the mixing matrix. However, we can also consider the following phases due to the neutrino transitions between different flavors,

$$
\Phi_{\nu_e\to\nu_\mu}(z) = \arg\left[\langle\nu_e(0)|\nu_\mu(z)\rangle\right] - \Im\int_0^z \langle\nu_e(z')|\dot{\nu}_\mu(z')\rangle dz'\,, \tag{4}
$$

$$
\Phi_{\nu_\mu\to\nu_e}(z) = \arg\left[\langle\nu_\mu(0)|\nu_e(z)\rangle\right] - \Im\int_0^z \langle\nu_\mu(z')|\dot{\nu}_e(z')\rangle dz'\,. \tag{5}
$$

Eqs.(4) and (5) represent the differences between the total and the dynamic phases generated by the transitions $\nu_e \to \nu_\mu$ and $\nu_\mu \to \nu_e$, respectively. By using the Majorana neutrino states in

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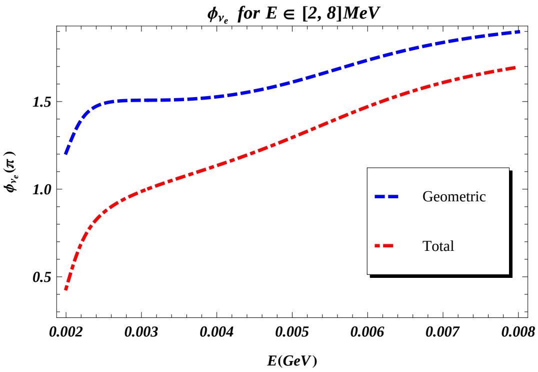

Figure 1. *(Color online) Plots of the total (the red dot dashed line) and the geometric phases (the blue dashed line) of ν_e, as a function of the neutrino energy E, for a distance length $z = 100km$.*

Eqs.(2), we have

$$\Phi_{\nu_e\to\nu_\mu}(z) = \frac{3\pi}{2} + \phi + \left(\frac{\Delta m_m^2}{4E}\sin 2\theta_m \cos\phi\right) z\,, \tag{6}$$

$$\Phi_{\nu_\mu\to\nu_e}(z) = \frac{3\pi}{2} - \phi + \left(\frac{\Delta m_m^2}{4E}\sin 2\theta_m \cos\phi\right) z\,. \tag{7}$$

Then, $\Phi_{\nu_e\to\nu_\mu} \neq \Phi_{\nu_\mu\to\nu_e}$. Although both the total and the dynamic phases depend on ϕ, the asymmetry between the transitions $\nu_e \to \nu_\mu$ and $\nu_\mu \to \nu_e$ is due to the total phases. Indeed, we have $\Phi^{tot}_{\nu_e\to\nu_\mu} = \frac{3\pi}{2} + \phi$ and $\Phi^{tot}_{\nu_\mu\to\nu_e} = \frac{3\pi}{2} - \phi$ (whereas $\Phi^{dyn}_{\nu_e\to\nu_\mu} = \Phi^{dyn}_{\nu_\mu\to\nu_e} = \left(\frac{\Delta m_m^2}{4E}\sin 2\theta_m \cos\phi\right) z$). By contrast, for Dirac neutrinos we have

$$\Phi_{\nu_e\to\nu_\mu}(z) = \Phi_{\nu_\mu\to\nu_e}(z) = \frac{3\pi}{2} + \left(\frac{\Delta m_m^2}{4E}\sin 2\theta_m\right) z\,, \tag{8}$$

and the total phases reduce to $\Phi^{tot}_{\nu_e\to\nu_\mu}(z) = \Phi^{tot}_{\nu_\mu\to\nu_e}(z) = \frac{3\pi}{2}$. The phases defined in Eqs.(4) and (5) and the total phases depend on the choice of the mixing matrix. Indeed, if we consider the mixing matrix obtained by U_1 by replacing θ with θ_m, the result of Eq.(8) is obtained also for Majorana neutrinos. Similar results are found for oscillation in vacuum. Therefore the phases $\Phi_{\nu_e\to\nu_\mu}$ and $\Phi_{\nu_\mu\to\nu_e}$ and the total phases $\Phi^{tot}_{\nu_e\to\nu_\mu}$ and $\Phi^{tot}_{\nu_\mu\to\nu_e}$ discriminate between the two matrices U_1 and U_2.

3. Numerical analysis.

In order to connect of our results with experiments, we plot in Figs.1 and 2 the total, the geometric phases and the phases defined in Eqs.(4) and (5) by using the characteristic values of experiments such as RENO [2] and T2K [4]. In Fig.1 we report the total and geometric phases associated with the evolution of ν_e. We consider the neutrino propagation through the matter and the values of the parameters of RENO experiment [2]: neutrino energy $E \in [2-8]MeV$, electron earth density $n_e = 10^{24}cm^{-3}$, $\Delta m^2 = 7.6\times10^{-3}eV^2$ and distance $z = 100km$. In Fig.2 we report the phases $\Phi_{\nu_e\to\nu_\mu}$ and $\Phi_{\nu_\mu\to\nu_e}$, by assuming $E \sim 1GeV$ and $z = 300km$, which are values compatible with the parameters of $T2K$ experiment [4]. Moreover we consider $\phi = 0.3$, and the values of n_e and Δm^2 considered above.

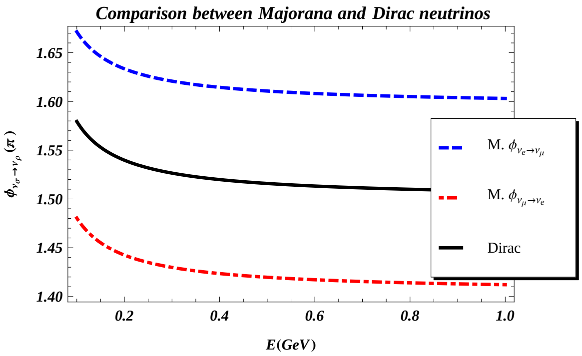

Figure 2. *(Color online) Plot of the phases $\Phi_{\nu_e\to\nu_\mu}$ (the blue dashed line) and $\Phi_{\nu_\mu\to\nu_e}$ (the red dot dashed line) for Majorana neutrinos as a function of the neutrino energy E, for a distance length $z = 300km$. The phases $\Phi_{\nu_e\to\nu_\mu} = \Phi_{\nu_\mu\to\nu_e}$ for Dirac neutrinos is represented by the black solid line.*

4. Conclusions.

We analyzed the total and the geometric phases generated in the evolution of the neutrino. We have shown that for Majorana neutrinos the phases due to a transition between different neutrino flavors take different values depending on the representation of the mixing matrix and on the nature of neutrinos. By considering the mixing matrix U_2, we obtained for Majorana neutrinos, $\Phi_{\nu_e\to\nu_\mu} \neq \Phi_{\nu_\mu\to\nu_e}$ (and $\Phi^{tot}_{\nu_e\to\nu_\mu} \neq \Phi^{tot}_{\nu_\mu\to\nu_e}$), that reveals an asymmetry in the transitions $\nu_e \to \nu_\mu$ and $\nu_\mu \to \nu_e$. This asymmetry disappears for Dirac neutrinos. On the contrary, by using U_1, we have $\Phi_{\nu_e\to\nu_\mu} = \Phi_{\nu_\mu\to\nu_e}$, (and $\Phi^{tot}_{\nu_e\to\nu_\mu} = \Phi^{tot}_{\nu_\mu\to\nu_e}$) both for Dirac and Majorana neutrinos and nothing can be said on the neutrino natures. We presented a numerical analysis by using the characteristic parameters of RENO and T2K experiments and we have obtained values for the neutrino phases which, in principle, are detectable. Our results pave the way for a completely new method to study the nature of neutrinos. In our discussion, the quantum field theory effects on particle mixing [45]-[60], can be safely neglected [47].

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