Quarkonia decays into two photons induced by space-time noncommutativity

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In this article we propose standard model strictly forbidden decay modes, quarkonia ($\bar{Q}Q_{1^{--}} = J/\psi, \Upsilon$) decays into two photons, as a possible signature of space-time noncommutativity. An experimental discovery of $J/\psi \to \gamma\gamma$ and/or $\Upsilon \to \gamma\gamma$ processes would certainly indicate a violation of the Landau-Pomeranchuk-Yang theorem and a definitive appearance of new physics.

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I. INTRODUCTION

A general feature of gauge theories on noncommutative (NC) space-time is the appearance of many new interactions which can lead to standard model (SM) forbidden processes. In this paper we use the noncommutative standard model (NCSM) to estimate decay of heavy quarkonia ($\bar{Q}Q_{1^{--}} = J/\psi,\ \Upsilon$), i.e. quarkonia annihilation into two photons, which is strictly forbidden in the SM by angular momentum conservation and Bose statistics, known as Landau-Pomeranchuk-Yang (LPY) theorem. Since the violation of the LPY theorem represents in fact the violation of Lorentz invariance, which is intrinsically embedded in noncommutative theories, such decays can in principle serve as an signature of space-time noncommutativity. This proposal represents an attempt to obtain the bound on the NC scale $\Lambda_{\rm NC}$ from hadronic physics.

A method for implementing nonabelian $SU(N)$ Yang-Mills theories on noncommutative space-time was proposed in [1–4]. In [5–8] this method was applied to the full SM of particle physics resulting in a minimal noncommutative extension of the SM with the same structure group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and with the same fields and number of coupling parameters as in the original SM. It is the only known approach that allows to build models of the electroweak sector directly based on the structure group $SU(2)_L \times U(1)_Y$ in a noncommutative background. We call this model NCSM and it represents an effective, anomaly free [9], noncommutative field theory. Space-time noncommutativity can be parameterized by the constant antisymmetric matrix $\theta_{\mu\nu}$,

$$x_\mu * x_\nu - x_\nu * x_\mu = i\theta_{\mu\nu}, \qquad (1)$$

where $*$ denotes the product of noncommutative structure, while $\theta^{\mu\nu} = c^{\mu\nu}/\Lambda_{NC}^2$, with $c^{\mu\nu}$ being dimensionless coefficients presumably of order unity.

Violation of Lorentz symmetry is introduced by virtue of nonzero $\theta^{\mu\nu}$. Furthermore, the analysis of discrete symmetry properties of the NCSM [10] shows that θ transforms under C, P, T in such a way that it preserves these symmetries in the action. However, considering θ as a fixed spectator field, there will be spontaneous breaking of C, P and/or CP (relative to the spectator). In the process of interest the C symmetry is violated.

An alternative proposal for the construction of noncommutative generalizations of the standard model has been put forward in [11].

Signatures of noncommutativity have been discussed from the point of view of collider physics [12,13], including SM forbidden $Z \to \gamma\gamma, gg$ decays [6,14], neutrino astrophysics [15] and neutrino physics [16], as well as low-energy nonaccelerator experiments [17–19]. Note that the Lorentz violating operators considered in [17,18] do not appear in the NCSM [2–8,14–16] considered in this article. Furthermore, the rather high bound on noncommutativity obtained in [19] is based on a particular operator contribution appearing in NC QCD that, as discussed in [20], is canceled by the contribution of other terms in this model. In the NCSM [2–8,14–16] the existing bound of $|\theta| \lesssim (10\ \text{TeV})^{-2}$ comes from a rather crude model estimate obtained in [21]. Finally, research of bound state decays in the framework of noncommutative QED/QCD [22–24] was performed by computating the lifetimes of ortho and para positronium [25] as well as the corrections to gluonic decays of heavy quarkonia [26]. For reviews, see [27–29].

Experimental discovery of the kinematically allowed decays $J/\psi \to \gamma\gamma$ and $\Upsilon \to \gamma\gamma$, as well as $Z \to \gamma\gamma$ and $Z \to gg$, would certainly prove a violation of the LPY theorem and could serve as a possible indication/signal for space-time noncommutativity.

In Sec. II we briefly review the ingredients of the NCSM relevant to this work. In Sec. III the amplitudes for the

$\overline{Q}Q_{1^{--}} \to \gamma\gamma$ process are worked out, while in Sec. IV the decay rates are determined. Section V is devoted to the discussion of numerical results and concluding remarks.

II. THE NON-COMMUTATIVE STANDARD MODEL

The general action of the NCSM is

$$S_{\rm NCSM} = S_{\rm fermions} + S_{\rm gauge} + S_{\rm Higgs} + S_{\rm Yukawa}, \quad (2)$$

where for explicit expressions of particular contributions we refer to [7].

For the simple case of quark QED interactions, which is relevant to SM forbidden decays of quarkonia into two photons, the expansion up to the first order in the NC parameter θ reads

$$\begin{aligned} S_{\psi,\rm QED} &= \int d^4x \bar{\hat{\psi}} * (i\hat{\not{D}} - m_q)\hat{\psi} \\ &= \int d^4x \bigg[\bar{\psi}(i\not{D} - m_q)\psi \\ &\quad - \frac{1}{4}\bar{\psi}A_{\mu\nu}(i\theta^{\mu\nu\rho}D_\rho - m_q\theta^{\mu\nu})\psi \bigg], \end{aligned} \quad (3)$$

where $\theta^{\mu\nu\rho} = \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu$ and $A_{\mu\nu}$ is the photon field strength tensor. The kinetic part in (3) comes from $S_{\rm fermion}$, while the mass contribution originates from $S_{\rm Yukawa}$ which, in the case of QED, interactions takes this simple form [7]. At the order θ, the electroweak interactions have additional, much more involved mass contributions, but only the SM electroweak quark interactions are relevant to our calculation.

In the gauge sector of the action (2), we have freedom in the choice of traces in kinetic terms for gauge fields. Two different choices were under consideration in [7] producing two different actions in the gauge sector, corresponding to the so-called minimal (mNCSM) and the nonminimal (nmNCSM) model, respectively. The matter sector of the action is not affected by the change of the gauge part; the quark-gauge boson interactions remain the same in both models.

The mNCSM adopts the following choice for the traces in $S_{\rm gauge}^{\rm mNCSM}$: a sum of three traces over the $U(1)_Y$, $SU(2)_L$, $SU(3)_C$ sectors with $Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ in the definition of Tr_1 and the fundamental representation for $SU(2)_L$ and $SU(3)_C$ generators in Tr_2 and Tr_3, respectively. Up to the first order in the θ expansion, the following gauge terms in the mNCSM are obtained:

$$\begin{aligned} S_{\rm gauge}^{\rm mNCSM} &= -\frac{1}{4}\int d^4x \mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \\ &\quad - \frac{1}{2}\mathrm{Tr}\int d^4x \bigg[B_{\mu\nu}B^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} \\ &\quad - 2g_S\theta^{\mu\nu}\bigg(\frac{1}{4}G_{\mu\nu}G_{\rho\sigma} - G_{\mu\rho}G_{\nu\sigma}\bigg)G^{\rho\sigma}\bigg]. \end{aligned} \quad (4)$$

Here, $\mathcal{A}_{\mu\nu}$, $B_{\mu\nu}(= B^a_{\mu\nu}T^a_L)$ and $G_{\mu\nu}(= G^a_{\mu\nu}T^a_S)$ denote the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ field strengths, respectively. At order θ there are no corrections nor new interactions involving electroweak fields.

One can pick up the other representation of the gauge sector, such that the trace is chosen over all particle multiplets on which covariant derivatives act and which have different quantum numbers. In the SM, these are five multiplets for each generation of fermions and one Higgs multiplet. New triple neutral gauge boson interactions, usually forbidden by Lorentz invariance, angular moment conservation and the LPY theorem, arise quite naturally in the framework of nmNCSM model [6,14].

In the case of quarkonia decays into two photons, the gauge sector in the nmNCSM gives important contributions through the appearances of novel triple gauge boson $\gamma\gamma\gamma$ and $Z\gamma\gamma$ couplings. Here we give parts relevant to this paper:

$$\begin{aligned} \mathcal{L}_{\gamma\gamma\gamma}^{\rm nmNCSM} &= \frac{e}{4}\sin 2\theta_W K_{\gamma\gamma\gamma}\theta^{\rho\sigma}A^{\mu\nu}(A_{\mu\nu}A_{\rho\sigma} - 4A_{\mu\rho}A_{\nu\sigma}), \\ \mathcal{L}_{Z\gamma\gamma}^{\rm nmNCSM} &= \frac{e}{4}\sin 2\theta_W K_{Z\gamma\gamma}\theta^{\rho\sigma}[2Z^{\mu\nu}(2A_{\mu\rho}A_{\nu\sigma} - A_{\mu\nu}A_{\rho\sigma}) \\ &\quad + 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\sigma} - Z_{\rho\sigma}A_{\mu\nu}A^{\mu\nu}]. \end{aligned} \quad (5)$$

For details of the nmNCSM construction and the allowed values of the constants $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$, see [6,14]. Parameters in the nmNCSM can be restricted by considering GUTs on noncommutative space-time [10].

III. AMPLITUDES FOR $\bar{Q}Q_{1^{--}} \to \gamma\gamma$ DECAYS

In our model, for computations of relevant matrix elements, i.e. for computations of the diagrams from Figs. 1 and 2, we employ Feynman rules derived in [7]. A helpful property of the action, important for check of calculations, is its symmetry under ordinary gauge transformations in addition to noncommutative ones. Furthermore, in the computations of the diagrams from Figs. 1 and 2, applying the following prescription for quarkonia to the vacuum transition matrix element of the operator $q_i^\alpha \overline{q}_j^\beta$ ($q = c, b$

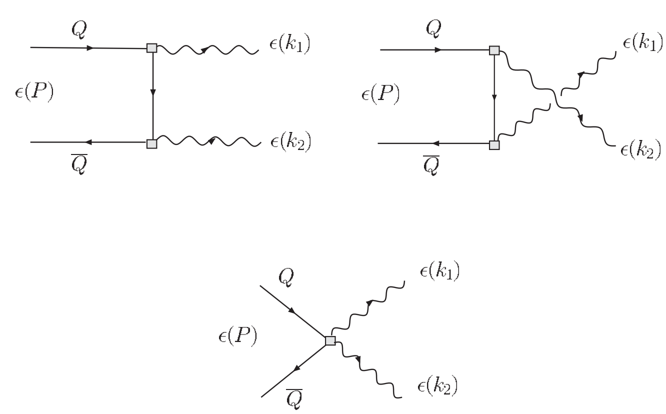

FIG. 1. Contributions to the $\bar{Q}Q_{1^{--}} \to \gamma\gamma$ amplitude in the mNCSM, up to the first order in θ.

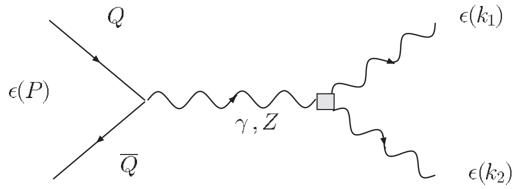

FIG. 2. Additional contributions to the $\bar{Q}Q_{1^{--}} \to \gamma\gamma$ amplitude in the nmNCSM, up to the first order in θ.

and i, j are color indices)

$$\langle 0|q_i^\alpha \bar{q}_j^\beta|\bar{Q}Q_{1^{--}}(P)\rangle = -\frac{|\Psi_{\bar{Q}Q}(0)|}{\sqrt{12M}}[(\not{P} + M)\not{\epsilon}]^{\alpha\beta}\delta_{ij}, \tag{6}$$

we actually hadronize free quarks into a quarkonium bound state and calculate the amplitude for the quarkonia decay into two photons. Here, $|\Psi_{\bar{Q}Q}(0)|$ represents the quarkonia wave function at the origin

$$|\Psi_{\bar{Q}Q}(0)|^2 = \frac{\Gamma(\bar{Q}Q_{1^{--}} \to \ell^+\ell^-)M^2}{16\pi\alpha^2 e_Q^2}. \tag{7}$$

In the mNCSM, to order θ, the diagrams displayed in Fig. 1 contribute to the amplitude $\mathcal{M}_1(\bar{Q}Q_{1^{--}} \to \gamma\gamma)$ given by

$$\begin{aligned}\mathcal{M}_1 = {} & i\pi 4\sqrt{3M}\alpha e_Q^2|\Psi_{\bar{Q}Q}(0)|\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P) \\ & \times \bigg\{-(k_1 - k_2)^\rho\bigg[\theta^{\mu\nu} - 2g^{\mu\nu}\frac{(k_1\theta k_2)}{M^2}\bigg] \\ & + 2g^{\mu\rho}\bigg[(k_1\theta)^\nu - 2k_1^\nu\frac{(k_1\theta k_2)}{M^2}\bigg] \\ & + 2g^{\nu\rho}\bigg[(k_2\theta)^\mu + 2k_2^\mu\frac{(k_1\theta k_2)}{M^2}\bigg]\bigg\},\end{aligned} \tag{8}$$

where $(k_i\theta)^\mu = k_{i\nu}\theta^{\nu\mu}$ and $k_1\theta k_2 = k_{1\mu}\theta^{\mu\nu}k_{2\nu}$, while M and P are the mass and the total momentum of the discussed quarkonium state, respectively.

Additional diagrams displayed in Fig. 2 contribute in the nmNCSM. The corresponding amplitude $\mathcal{M}_2(\bar{Q}Q_{1^{--}} \to \gamma\gamma)$ reads

$$\begin{aligned}\mathcal{M}_2 = {} & -i\pi\frac{16\sqrt{3M}}{M^2}\alpha|\Psi_{\bar{Q}Q}(0)|\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P) \\ & \times \Theta_3((\mu, k_1), (\nu, k_2), (\rho, P))\bigg[e_Q \sin 2\theta_W K_{\gamma\gamma\gamma} \\ & + \bigg(\frac{M}{M_Z}\bigg)^2 c_V^Q K_{Z\gamma\gamma}\bigg],\end{aligned} \tag{9}$$

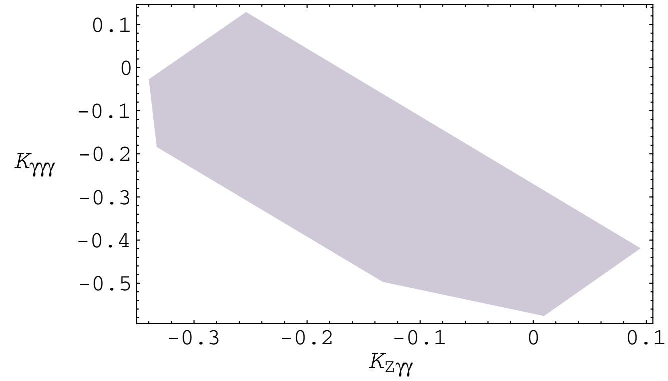

FIG. 3 (color online). The allowed region for the $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants [14].

where the needed vector couplings of c and b quarks are given as [30]

$$c_V^c = \frac{1}{2}\bigg(1 - \frac{8}{3}\sin^2\theta_W\bigg), \qquad c_V^b = -\frac{1}{2}\bigg(1 - \frac{4}{3}\sin^2\theta_W\bigg), \tag{10}$$

and Θ_3 is the triple gauge boson function defined in [7,8]. The values of $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants evaluated at the M_Z scale are restricted by the six-sided polygon, as shown in Fig. 3, taken from [14].

While only the quark exchange diagrams from Fig. 1 contribute to the mNCSM amplitude $\mathcal{M}^{\text{mNCSM}} = \mathcal{M}_1$, the nmNCSM amplitude for the quarkonia decay into two photons amounts to the sum of contributions from the diagrams in Figs. 1 and 2, $\mathcal{M}^{\text{nmNCSM}} = \mathcal{M}_1 + \mathcal{M}_2$.

At the end of this section, note that the NCSM free quark amplitude obtained from the sum of diagrams in Fig. 1 is invariant under the electromagnetic gauge transformation, as it should be. Each of the γ and Z exchange diagrams from Fig. 2 is also electromagnetically gauge invariant, as expected.

IV. RATES OF QUARKONIA DECAYS INTO TWO PHOTONS

The amplitude squared for the quarkonia decay derived in the nmNCSM from the amplitudes (8) and (9) is

$$\sum_{\text{pol.}}|\mathcal{M}^{\text{nmNCSM}}(\bar{Q}Q_{1^{--}} \to \gamma\gamma)|^2 = \sum_{\text{pol.}}|\mathcal{M}_1 + \mathcal{M}_2|^2, \tag{11}$$

from which, after the phase space integration, we obtain the decay rate,

$$\Gamma^{\text{nmNCSM}}(\bar{Q}Q_{1^{--}} \to \gamma\gamma) = \frac{4\alpha^2\pi}{3}|\Psi_\Upsilon(0)|^2\frac{M^2}{\Lambda_{\text{NC}}^4}\bigg[7\vec{E}_\theta^2 + 3\vec{B}_\theta^2\bigg]\bigg[\frac{e_Q^2}{2} - e_Q\sin 2\theta_W K_{\gamma\gamma\gamma} - \bigg(\frac{M}{M_Z}\bigg)^2 c_V^Q K_{Z\gamma\gamma}\bigg]^2. \tag{12}$$

The coupling constants appearing in Eq. (12) are evaluated at the M_Z scale [14,30]. Analogously one obtains the decay rate for mNCSM, $(\bar{Q}Q_{1^{--}} \to \gamma\gamma)$. It corresponds to setting $K_{\gamma\gamma\gamma} = K_{Z\gamma\gamma} = 0$ in (12).

In the above computations we have used the following identities:

$$\theta^2 = (\theta^2)^\mu_\mu = \theta_{\mu\nu}\theta^{\nu\mu} = \frac{2}{\Lambda^4_{\rm NC}}\left(\sum_{i=1}^{3}(c^{0i})^2 - \sum_{i,j=1;i<j}^{3}(c^{ij})^2\right) \equiv \frac{2}{\Lambda^4_{\rm NC}}(\vec{E}^2_\theta - \vec{B}^2_\theta). \tag{13}$$

To maximize the rates (12) we can assume that the dimensionless quantities $\vec{E}^2_\theta$ and $\vec{B}^2_\theta$ are of order one [1]. Normalizing the obtained decay rate to the decay of $\bar{Q}Q_{1^{--}}$ into lepton pairs, by using (7) we find

$$\frac{\Gamma^{\rm nmNCSM}(\bar{Q}Q_{1^{--}} \to \gamma\gamma)}{\Gamma(\bar{Q}Q_{1^{--}} \to \ell^+\ell^-)} = \frac{5}{24}e_Q^2\left(\frac{M}{\Lambda_{\rm NC}}\right)^4\left[1 - \frac{2}{e_Q}\sin 2\theta_W K_{\gamma\gamma\gamma} - \frac{2}{e_Q^2}\left(\frac{M}{M_Z}\right)^2 c_V^Q K_{Z\gamma\gamma}\right]^2. \tag{14}$$

Hence, in the case of mNCSM couplings, we obtain the following ratios for Υ and J/ψ decays:

$$\frac{\Gamma^{\rm mNCSM}(\Upsilon \to \gamma\gamma)}{\Gamma(\Upsilon \to \ell^+\ell^-)} = \frac{5}{216}\left(\frac{M_\Upsilon}{\Lambda_{\rm NC}}\right)^4 \tag{15}$$

and

$$\frac{\Gamma^{\rm mNCSM}(J/\psi \to \gamma\gamma)}{\Gamma(J/\psi \to \ell^+\ell^-)} = \frac{5}{54}\left(\frac{M_{J/\psi}}{\Lambda_{\rm NC}}\right)^4. \tag{16}$$

Note here that choosing $\vec{E}^2_\theta = 0$ and $\vec{B}^2_\theta \simeq 1$, as it is favored by the string theory, would produce result (14) multiplied by 3/10, which does not change the final conclusion in a serious way.

The range of the scale of noncommutativity, $1 \geq \Lambda_{\rm NC}/{\rm TeV} \geq 0.25$, was choosen because it produces experimentally reachable quarkonia decay to two photons rates. Since the rate (14) depends on $1/\Lambda^4_{\rm NC}$ it is quite clear that any larger $\Lambda_{\rm NC}$ would dramatically decrease possibility to see the signal for the noncommutativity of space-time via quarkonia to two photons decay at present and near-future experiments.

The choosen range of the scale of noncommutativity then gives

$$2 \times 10^{-10} \lesssim \frac{\Gamma^{\rm mNCSM}(\Upsilon \to \gamma\gamma)}{\Gamma(\Upsilon \to \ell^+\ell^-)} \lesssim 5 \times 10^{-8} \tag{17}$$

and

$$9 \times 10^{-12} \lesssim \frac{\Gamma^{\rm mNCSM}(J/\psi \to \gamma\gamma)}{\Gamma(J/\psi \to \ell^+\ell^-)} \lesssim 2 \times 10^{-9}. \tag{18}$$

From the experiment one has $\Gamma^{\rm exp.}(\Upsilon(1S) \to e^+e^-) = (1.314 \pm 0.029)$ keV and $\Gamma^{\rm exp.}_{\rm tot}(\Upsilon(1S)) = (53.0 \pm 1.5)$ keV [31], which then leads to

$$5 \times 10^{-12} \lesssim BR^{\rm mNCSM}(\Upsilon(1S) \to \gamma\gamma) \lesssim 10^{-9}. \tag{19}$$

For the J/ψ case, $\Gamma^{\rm exp.}(J/\psi \to e^+e^-) = (5.4 \pm 0.15 \pm 0.07)$ keV and $\Gamma^{\rm exp.}_{\rm tot}(J/\psi) = (91.0 \pm 3.2)$ keV [31], which, with $\Lambda_{\rm NC}$ in the above range, gives the following range for the J/ψ branching ratio:

$$5 \times 10^{-13} \lesssim BR^{\rm mNCSM}(J/\psi \to \gamma\gamma) \lesssim 10^{-10}. \tag{20}$$

For the nmNCSM, with the choice of the triple gauge boson couplings $K_{\gamma\gamma\gamma} = -0.576$ and $K_{Z\gamma\gamma} = 0.010$ the rates for the $\Upsilon \to \gamma\gamma$ and $J/\psi \to \gamma\gamma$ decays reach maximal values. In the same range of the scale of noncommutativity as above, $1 \geq \Lambda_{\rm NC}/{\rm TeV} \geq 0.25$, we then have

$$7 \times 10^{-10} \lesssim \frac{\Gamma^{\rm nmNCSM}(\Upsilon \to \gamma\gamma)}{\Gamma(\Upsilon \to \ell^+\ell^-)} \lesssim 2 \times 10^{-7} \tag{21}$$

and

$$5 \times 10^{-11} \lesssim \frac{\Gamma^{\rm nmNCSM}(J/\psi \to \gamma\gamma)}{\Gamma(J/\psi \to \ell^+\ell^-)} \lesssim 10^{-8}. \tag{22}$$

Using the aforementioned experimental values for Υ and J/ψ we obtain the following ranges for relevant branching ratios:

$$2 \times 10^{-11} \lesssim BR^{\rm nmNCSM}(\Upsilon \to \gamma\gamma) \lesssim 4 \times 10^{-9}, \tag{23}$$

$$3 \times 10^{-12} \lesssim BR^{\rm nmNCSM}(J/\psi \to \gamma\gamma) \lesssim 8 \times 10^{-10}. \tag{24}$$

V. DISCUSSION AND CONCLUSION

In this paper we have considered decays of two quarkonia states: $\bar{Q}Q_{1^{--}} = J/\psi, \Upsilon(1S)$ into two photons which violate the LPY theorem.

Theoretically, in the Υ case, the addition of triple neutral gauge boson couplings via a photon and a Z-boson exchange diagram in Fig. 2, contributes to the nmNCSM amplitude both constructively and destructively, depending on the specific values of the $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants from the area in Fig. 3. As a consequence, the branching ratio for the $\Upsilon \to \gamma\gamma$ decay increases or decreases with respect to the values obtained in the mNCSM. This is illustrated in Fig. 4 which shows the branching ratio $BR^{\rm nmNCSM}(\Upsilon \to \gamma\gamma)$ as a function of the triple neutral gauge boson constants $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$. Since the constructive – destructive contributions are spread approximatively equally within the allowed range of the $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$ constants, we conclude that additional uncertainty in the prediction of the $\Upsilon(1S) \to \gamma\gamma$ branching ratio, due to the

[1]The parameterization of the $\theta^{\mu\nu}$ matrix elements used here is taken from [12].

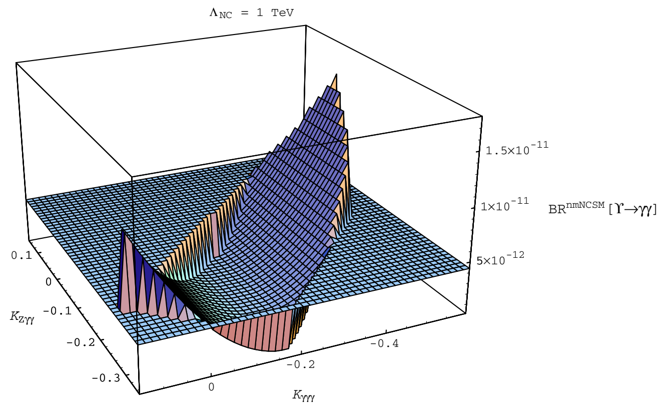

FIG. 4 (color online). Branching ratio $BR^{\mathrm{nmNCSM}}(\Upsilon \to \gamma\gamma)$ as a function of $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants, at the scale of noncommutativity $\Lambda_{\mathrm{NC}} = 1$ TeV. The horizontal plane at the value of 4.7×10^{-12} indicates the $BR^{\mathrm{mNCSM}}(\Upsilon \to \gamma\gamma)$, which one obtains by setting $K_{\gamma\gamma\gamma} = K_{Z\gamma\gamma} = 0$ in (12).

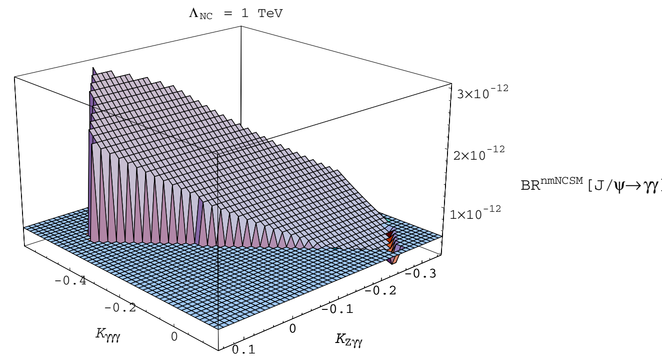

FIG. 5 (color online). Branching ratio $BR^{\mathrm{nmNCSM}}(J/\psi \to \gamma\gamma)$ as a function of $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants, at the scale of noncommutativity $\Lambda_{\mathrm{NC}} = 1$ TeV. The horizontal plane at the value of 5.1×10^{-13} indicates the $BR^{\mathrm{mNCSM}}(J/\psi \to \gamma\gamma)$, which one obtains by setting $K_{\gamma\gamma\gamma} = K_{Z\gamma\gamma} = 0$ in (12).

NCSM model building, makes the $\Upsilon(1S) \to \gamma\gamma$ decay a less favorite candidate for distinguishing the mNCSM from the nmNCSM.

However, for the J/ψ decay, because of the positive charm quark charge, the addition of the photon and Z-boson exchange diagrams (Fig. 2) to the quark exchange diagrams in Fig. 1 contributes constructively to the dominant part of the $K_{\gamma\gamma\gamma} - K_{Z\gamma\gamma}$ area, producing rates larger by a factor of $\simeq 5$ with respect to those gained in the mNCSM. Destructive contributions are small, covering about a few % of the $K_{\gamma\gamma\gamma} - K_{Z\gamma\gamma}$ area. This is illustrated in Fig. 5 showing that the nmNCSM contributions to the $J/\psi \to \gamma\gamma$ decay are almost always constructive, thus effectively enhancing its branching ratio.

Experimentally [31,32], concerning the $\Upsilon(1S)$ decay, at CLEO-III there is the largest sample of $\Upsilon(1S)$ resonances produced in e^+e^- collisions, about 21×10^6. However, it will be very difficult to observe the $\Upsilon(1S) \to \gamma\gamma$ decay because of the larger QED background from the nonresonant $e^+e^- \to \gamma\gamma$ process. The resonant cross-section for $e^+e^- \to \Upsilon(1S)$ is of the same order of magnitude as the background cross-section. Thus it seems that the detection of $BR(\Upsilon(1S) \to \gamma\gamma)$ below 10^{-3} will be hopeless with the present data. Considering the $J/\psi \to \gamma\gamma$ decay, there are much better chances since the resonant cross-section is much higher. The existing limit, which comes from a very old experiment $BR(J/\psi \to \gamma\gamma) < 5 \times 10^{-4}$ [30], can be improved but probably the NC limits given in (20) and (24) are unreachable today. Unfortunately, the above experimental limits are too weak to set any reliable bound on the noncommutative scale from our model estimate.

Finally, note that quarkonia and the Z-boson to two photon decay processes are related in the nmNCSM, via the $Z\gamma\gamma$ interaction [6] of the strength $2e\sin 2\theta_W K_{Z\gamma\gamma}$ determined in [14]. We have found that the $\bar{Q}Q_{1^{--}} \to \gamma\gamma$ decay rates become maximal for the values $K_{\gamma\gamma\gamma} = -0.576$ and $K_{Z\gamma\gamma} = 0.01$. The same value $K_{Z\gamma\gamma} = 0.01$, produces the minimal value of $BR(Z \to \gamma\gamma)$ via Eq. (17) from [6] and Table 1 from [14]. On the other hand, the value of $K_{Z\gamma\gamma} = -0.34$, for any $-0.03 > K_{\gamma\gamma\gamma} > -0.19$ (see Table 1 in [14]), maximizes the $Z \to \gamma\gamma$ decay rate and, at the same time, minimizes the $\bar{Q}Q_{1^{--}} \to \gamma\gamma$ branching ratios (see Figs. 4 and 5). The combination of all three decays would certainly narrow the parameter space of unknown constants of our model, like $\theta^{\mu\nu}$, $K_{Z\gamma\gamma}$, etc.

In conclusion, if the future experiments measure any of the $Z \to \gamma\gamma$, $J/\psi \to \gamma\gamma$ and $\Upsilon(1S) \to \gamma\gamma$ branching ratios, the appearances of the physics beyond the SM would then be strongly indicated. We hope that the importance of a possible discovery of space-time noncommutativity will convince experimentalists to look for SM forbidden decays in hadronic physics.

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