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$K \to \pi\gamma$ decays and space-time noncommutativity

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We propose the $K \to \pi\gamma$ decay mode as a signature of the violation of the Lorentz invariance and the appearance of new physics via space-time noncommutativity.

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In this proposal we assume space-time noncommutativity (NC) to compute the $K \to \pi\gamma$ decay forbidden by the Lorentz invariance.

The dynamics of the standard model (SM) forbidden flavor-changing weak decays is described in the framework of the noncommutative SM (NCSM), where the content of particles and symmetries is the same as in the usual commutative space-time. Gauge symmetry is included in an infinitesimal form, thus SU(N) gauge symmetry is implementable. All NC fields are expressed in terms of the usual fields via the Seiberg-Witten (SW) map by expansion up to first order in the NC parameter $\theta^{\mu\nu}$. As in other particle physics models on noncommutative space-time, a general feature of the NCSM action is the violation of space-time symmetries, in particular, of angular momentum conservation and discrete symmetries like P, CP, and possibly even CPT. This symmetry breaking is spontaneous in the sense that it is broken with respect to a fixed noncommutative background.

The method for implementing non-Abelian SU(N) theories on noncommutative space-time, based on the Seiberg-Witten map [1], has been proposed in [2]. In [3–6] this method has been applied to the standard model of particle physics resulting in the noncommutative extension of the SM, called NCSM action, with the same structure group $SU(3)c \times SU(2)_L \times U(1)_Y$ and with the same fields and number of coupling parameters as in the original SM. It represents a $\theta^{\mu\nu}$-expanded effective action

$$S_{\rm NCSM} = S_{\rm fermions} + S_{\rm gauge} + S_{\rm Higgs} + S_{\rm Yukawa}, \quad (1)$$

valid at very short distances, that leads to an anomaly free theory [7]. For expressions of particular contributions we refer to [5,6]. The $\theta^{\mu\nu} = c^{\mu\nu}/\Lambda_{\rm NC}^2$ is the constant, antisymmetric tensor, where $c^{\mu\nu}$ are dimensionless coefficients of order unity and $\Lambda_{\rm NC}$ is the scale of noncommutativity. The matter sector of the action (1), relevant to this work, is not affected by the freedom of choosing traces in the gauge kinetic part; the quark-gauge boson interactions remain the same [5,6]. The above action is symmetric under ordinary gauge transformations in addition to noncommutative ones.

An alternative NCSM proposal was presented in [8].

Signatures of noncommutativity have been discussed within collider physics [9–12], SM forbidden decays [4,13–16], neutrino astrophysics [17,18], in [19], as well as for low-energy nonaccelerator experiments [20–23].

This paper represents an estimate of the $K \to \pi\gamma$ decay branching ratio, based on the complete analysis of the $S_{\rm NCSM}$ action presented in [5]. From $S_{\rm Higgs}$ we find the contribution proportional to the M_W^2 [5] for the θ correction to the SM vertex $A_\mu(q)W_\nu^-(p)W_\rho^+(k)$:

$$\begin{aligned} V^{\mu\nu\rho}_{\gamma W^- W^+} &= ie[g^{\mu\nu}(q-p)^\rho + g^{\nu\rho}(p-k)^\mu + g^{\rho\mu}(k-q)^\nu \\ &\quad + \frac{i}{2}M_W^2(\theta^{\mu\nu}q^\rho + \theta^{\mu\rho}q^\nu + g^{\mu\nu}(\theta q)^\rho \\ &\quad - g^{\nu\rho}(\theta q)^\mu + g^{\rho\mu}(\theta q)^\nu)], \end{aligned} \quad (2)$$

while explicit expressions containing important Yukawa terms for $\bar{q}^{(i)}q^{(j)}\gamma$ and $\bar{q}^{(i)}q^{(j)}\gamma W^+$ vertices are given by Eqs. (76), (78), and (86) of Ref. [5]. There is also an additional contribution to the vertex (2) from Eq. (89) in [5], but owing to the symmetry this term vanishes.

The free quark amplitude $\mathcal{M}$ contributing to the $K^+ \to \pi^+\gamma$ decay arises from the Feynman diagrams displayed in Fig. 1 and is given as

$$\mathcal{M} = (\mathcal{M}^{\rm SM}_{(a+b)} + \mathcal{M}^{\theta}_{(a+b+c)})_\mu \varepsilon^\mu(q). \quad (3)$$

The hadronic matrix element $\langle \pi^+(p)|\mathcal{M}|K^+(k)\rangle$ responsible for $K^+ \to \pi^+\gamma$ decay contains the 4-quark-operators from Fig. 1; in fact, it represents the nonperturbative quantity which has been computed by using the vacuum saturation approximation and the partial conservation of the axial-vector current (PCAC):

$$\langle \pi^+(p)|\bar{u}\gamma_\mu\gamma_5 d|0\rangle = -ip_\mu f_\pi. \quad (4)$$

In this way we have hadronized free quarks into pseudoscalar π^+- and K^+-meson bound states. Since $\langle \pi^+(p)|(\mathcal{M}^{\rm SM}_{(a+b)})_\mu|K^+(k)\rangle\varepsilon^\mu(q) = 0$, the Lorentz invariance is satisfied for the $K^+ \to \pi^+\gamma$ process computed in the SM, as it should be.

We obtain the following $K^+ \to \pi^+\gamma$ decay amplitude:

$$\begin{aligned} \mathcal{A}^\theta(K^+ \to \pi^+\gamma) &= \langle \pi^+(p)|(\mathcal{M}^\theta_{(a+b+c)})_\mu|K^+(k)\rangle\varepsilon^\mu(q) \\ &= i\kappa(\mathcal{A}^\theta_{(a+b+c)})_\mu\varepsilon^\mu(q), \end{aligned} \quad (5)$$

where κ is a dimensionless constant

$$\kappa = \frac{eG_F}{4\sqrt{2}} V_{ud} V_{us}^{\dagger} f_{\pi} f_K, \tag{6}$$

while particular contributions originating from diagrams in Fig. 1(a)–1(c) are

$$\begin{aligned}
(\mathcal{A}^{\theta}_{(a)})_{\mu} &= 2(k^2(\theta p)_{\mu} - p^2(\theta k)_{\mu} - 2(q\theta k)k_{\mu}), \\
(\mathcal{A}^{\theta}_{(b)})_{\mu} &= \frac{kp}{kq}[(Q_u + Q_s)((q\theta k)k_{\mu} - (kq)(\theta k)_{\mu}) - (Q_u + Q_d)((q\theta k)p_{\mu} - (kq)(\theta p)_{\mu})] - R_{\pi}[(Q_u - Q_s)(kq)(\theta p)_{\mu} \\
&\quad - i(Q_u + Q_s)\epsilon_{\mu\nu\rho\tau} q^{\nu}(\theta p)^{\rho} k^{\tau}] + R_K[(Q_u - Q_d)(kq)(\theta k)_{\mu} + i(Q_u + Q_d)\epsilon_{\mu\nu\rho\tau} q^{\nu}(\theta k)^{\rho} p^{\tau}], \qquad (7) \\
(\mathcal{A}^{\theta}_{(c)})_{\mu} &= (Q_u + Q_d)(p^2(\theta k)_{\mu} - (kp)(\theta p)_{\mu} + (q\theta k)p_{\mu}) + 2(m_d Q_u + m_u Q_d)\frac{p^2(\theta k)_{\mu}}{m_d + m_u} \\
&\quad - (Q_u + Q_s)(k^2(\theta p)_{\mu} - (kp)(\theta k)_{\mu} - (q\theta k)k_{\mu}) - 2(m_s Q_u + m_u Q_s)\frac{k^2(\theta p)_{\mu}}{m_s + m_u},
\end{aligned}$$

with $Q_u = 2/3$, $Q_d = Q_s = -1/3$, and with the following kinematics and notation: $k = p + q$, $k^2 = m_K^2$, $p^2 = m_{\pi}^2$, $q^2 = 0$, $(\theta k)_{\mu} = \theta_{\mu\nu} k^{\nu}$, and $q\theta k = q_{\mu}\theta^{\mu\nu}k_{\nu}$. The mass ratios R_{π} and R_K,

$$R_{\pi} = \frac{p^2}{kq}\frac{m_d - m_u}{m_d + m_u}, \qquad R_K = \frac{k^2}{kq}\frac{m_s - m_u}{m_s + m_u}, \tag{8}$$

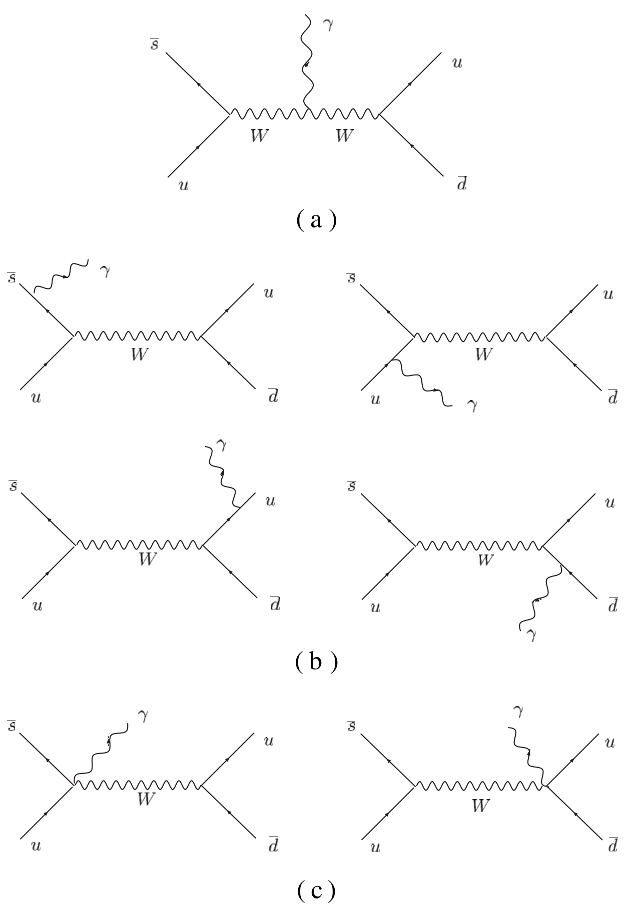

FIG. 1. Feynman diagrams contributing to the free quark amplitude $\mathcal{M}$ responsible for the $K^+ \to \pi^+\gamma$ decay.

are evaluated for $(m_d - m_u)/(m_d + m_u) \simeq 1/3.5$ and $(m_s - m_u)/(m_s + m_u) \simeq 1$ [24]. However, due to the numerical insignificance, in the following we neglect all terms proportional to R_{π}.

Certain contributions to the amplitudes (7) from the Yukawa parts of (b) and (c) classes of Feynman diagrams in Fig. 1 combine through the ''charge-mass'' interplay in such a way as to manifest the SU(2) and SU(3) symmetry breaking via $m_d - m_u$ and $m_s - m_u$ mass differences and, more importantly, to maintain the classical gauge invariance of the total amplitude (5).

Taking the kaon at rest and performing the phase-space integrations we find the following rate:

$$\begin{aligned}
\mathrm{BR}(K^+ \to \pi^+\gamma) &= \tau_{K^+}\Gamma(K^+ \to \pi^+\gamma) \\
&\simeq \tau_{K^+}\frac{\alpha}{128} G_F^2 f_{\pi}^2 f_K^2 |V_{ud}V_{us}^{\dagger}|^2 \frac{m_K^5}{\Lambda_{\mathrm{NC}}^4} \\
&\quad \times \left(1 - \frac{m_{\pi}^2}{m_K^2}\right)\left[1 - \frac{50}{27}\frac{m_{\pi}^2}{m_K^2} + \frac{25}{27}\frac{m_{\pi}^4}{m_K^4}\right] \\
&\simeq 0.8 \times 10^{-16}(1\ \mathrm{TeV}/\Lambda_{\mathrm{NC}})^4,
\end{aligned} \tag{9}$$

where τ_{K^+} is the K^+ meson mean life.

Considering the $K^+ \to \pi^+\gamma$ experiment we report on the Brookhaven collaboration E949 who recently published a new upper limit on the branching ratio $\mathrm{BR}(K^+ \to \pi^+\gamma) < 2.3 \times 10^{-9}$, at 90% C.L. [25]. This result is based on the data analysis primarily used to extract a $K^+ \to \pi^+\gamma\gamma$ result near the π^+ kinematic end point to test unitarity corrections of the chiral perturbation theory. Having the $K^+ \to \pi^+\gamma\gamma$ background under control, the limit achieved on the $K^+ \to \pi^+\gamma$ branching ratio is about 150 times better [25] with respect to the previous results of the E787 Collaboration [26].

Here we have presented theoretical computation of the $K^+ \to \pi^+\gamma$ branching ratio in the NCSM. The inclusion of the NC parts of the classes (a) and (b) of Feynman diagrams (Fig. 1) into the total amplitude $\mathcal{A}^{\theta}_{(a+b+c)}$ represents the novel feature in comparison with previous estimate [27] (see also the relevant Feynman rules in Ref. [5]).

Main enhancement of the rate (9) with respect to the result of Ref. [27] is coming from the class (a) of Feynman diagrams in Fig. 1 via θ correction to the SM vertex $A_\mu W_\nu^- W_\rho^+$ (2) discovered through analysis of the θ-expanded Higgs sector action $S_{\rm Higgs}$ in [5]. The rate (9) is about an order of magnitude higher with respect to the first, incomplete estimate [27] based only on the gauge invariant part of the amplitude $\mathcal{A}^\theta_{(c)}$ from (7). Our prediction is correct within the approximation made, i.e., by neglecting hardly controllable corrections ($1/N_c$, etc.) to the vacuum saturation approximation and to the PCAC.

In the framework of NCSM, i.e. the minimal NC extension of SM, another SM forbidden decay was recently examined, quarkonia $\to \gamma\gamma$ [16]. Although the SM forbidden decays of this kind could serve as a potentially good place for the discovery of space-time noncommutativity, the existing experimental limits are too weak to set a meaningful bound on $\Lambda_{\rm NC}$ (for other estimates from the literature, see [16] and references therein). Collider scattering experiments could offer another "laboratory," also very sensitive to the space-time NC signals. The first limits on noncommutative QED from an e^+e^- collider experiment, yielding $\Lambda_{\rm NC} > 141$ GeV at 95% confidence level, was obtained by the OPAL Collaboration [28]. The high precision of the future linear colliders could enable searches for noncommutativity by measuring deviations from the SM polarization observables [9–12]. In such a way a bound on NC parameters could be set more restrictively, since the near-future collider experiments will be sensitive to energy scales corresponding to $\Lambda_{\rm NC} \sim 1$ TeV.

To conclude, concerning the considered $K \to \pi\gamma$ decay and the possibility that the space-time noncommutativity is observed in such a decay, our theoretically predicted signature is relatively small. However, the arrival of new facilities should be encouraging, because further machines are expected to yield a production of $\bar{K}K$ pairs that might be larger by a number of orders of magnitude [29].

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