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Vacuum energy from noncommutative models

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ABSTRACT

The vacuum energy is computed for a scalar field in a noncommutative background in several models of noncommutative geometry. One may expect that the noncommutativity introduces a natural cutoff on the ultraviolet divergences of field theory. Our calculations show however that this depends on the particular model considered: in some cases the divergences are suppressed and the vacuum energy is only logarithmically divergent, in other cases they are stronger than in the commutative theory.

1. Introduction

Noncommutative models [1,2] may improve the behavior of field theories in the ultraviolet region by smoothing or removing some of the singularities of commutative quantum field theory. In fact, they imply a lower bound on the length scales, given by the inverse of the noncommutativity parameter κ, or equivalently an upper bound on the energy scales, given by κ. The scale κ is usually assumed to of the order of the Planck mass $M_P \sim 10^{19}$ GeV (but lower values are not excluded), and may act as a natural cutoff on the divergences of quantum field theory, in contrast with the commutative case where the cutoff must be imposed by hand.

This possibility may be tested in the calculation of the vacuum energy of quantum fields. This computation has interesting implications on cosmology, since the vacuum energy is often identified with the cosmological constant [3]. Although this argument is almost certainly wrong, since it is not based on a well-defined theory and predicts a value that can be 120 orders of magnitudes greater than the observed one, it can still be interesting to check if the noncommutativity parameter can act as a natural cutoff and improve the ultraviolet behavior of the theory. Of course, if κ is of Planck scale, this does not change much the predictions from a phenomenological point of view, since in this context also the standard UV cutoff is usually assumed to have the same scale.

In this paper, we calculate the vacuum energy of a massless scalar field in noncommutative background, using the heat kernel method. This method allows to evaluate the one-loop effective action by calculating the integral of an operator, related to the solution of the heat equation on a Euclidean manifold. We shall follow the approach of [4], where models presenting a breaking of Lorentz invariance are studied. A review of the heat kernel formalism can be found for example in [5]. A calculation similar to the present one, but differing in several respects and based on a perturbative expansion in the noncommutativity parameter, has also been performed in [6].

We investigate a class of noncommutative models characterized by a deformation of the Heisenberg algebra, which in turn implies a deformation of the Poincaré symmetry and hence of the field equations. Also the measure of the Hilbert space must be adapted to the nontrivial representation of the deformed Heisenberg algebra, and these two effects combine to modify the value of the heat kernel integral in comparison with the commutative one.

We show that, contrary to naive expectations, noncommutativity does not completely regularize the theory, and only in some of the models examined the UV behavior is improved with respect to the commutative theory, while in other models it can be worsened. The best improvement occurs in the anti-Snyder model, where the trace of the heat kernel is finite, and the divergence of the vacuum energy is only logarithmic.

2. Heat kernel

Let us consider a field theory obeying the equation[1]

$$\mathcal{D}\phi = F(\partial_0, \partial_i)\phi = 0, \tag{1}$$

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[1] We adopt the following conventions: metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$; $\mu = 0, 1, 2, 3$; $i = 1, 2, 3$; $v^2 = v^\mu v_\mu$.

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where $\mathcal{D}$ is a differential operator that deforms the usual Laplacian $\partial^\mu\partial_\mu$ by a parameter κ, in such a way to preserve the invariance under spatial rotations.

It is known that for a quantum bosonic field in Euclidean space, with partition function defined as $Z = (\det\mathcal{D})^{-1/2}$, the one-loop effective action $W = \frac{1}{2}\ln\det\mathcal{D}$ can be written in terms of the heat kernel,

$$W = -\frac{1}{2}\int_{1/\epsilon^2}^{\infty}\frac{ds}{s}\,K(s), \tag{2}$$

where s is a real parameter and $K(s) = \int dx < x\,|\,\mathrm{e}^{-s\mathcal{D}}\,|\,x >$ is the trace of the heat kernel. The cutoff $1/\epsilon^2$, with $\epsilon \gg 1$, at the lower limit is introduced because in standard field theory the integral (2) is usually divergent for $s \to 0$ (UV divergence). We recall that the heat kernel $K(s,x,x') = < x\,|\,\mathrm{e}^{-s\mathcal{D}}\,|\,x' >$ is defined as a solution of the heat equation

$$(\partial_s + \mathcal{D})K(s,x,x') = 0, \qquad K(0,x,x') = \delta(x,x'). \tag{3}$$

The calculations are most easily performed in momentum space, where the solution of the heat equation is trivial. It follows that [4]

$$K(s) = \frac{V}{(2\pi)^d}\int_{-\infty}^{\infty} d^dp\ \mathrm{e}^{-sF(p_0,p_i)}, \tag{4}$$

where V is the volume of spacetime. In the special case of the undeformed Laplace operator, $K(s) = V/(4\pi s)^{d/2}$.

The effective action follows from eq. (2). The vacuum energy density λ is defined as

$$\lambda = -\frac{W}{V}, \tag{5}$$

and λ is often identified with the cosmological constant. Of course, in standard field theory the value of λ depends on the cutoff ϵ introduced to regularize the UV divergences of the effective action. In particular, in four dimensions $\lambda = \epsilon^4/64\pi^2$. One may hope that in noncommutative models these divergences might be regularized by the noncommutativity scale κ, so that the calculation gives a finite result without need of introducing an artificial cutoff. We want to study if this happens in some well-known cases. For ease of calculation, we consider massless fields, that may however lead to IR divergences. We shall always understand that these are regularized when one considers massive fields.

3. Noncommutative models

Noncommutative theories are based on the hypothesis that spacetime has a granular structure, implemented through the noncommutativity of spacetime coordinates, with a scale of length κ^{-1}, that is usually (but not necessarily) identified with the Planck length. Because of the presence of this fundamental scale, most noncommutative geometries are associated to a deformation of the action of Lorentz transformations on phase space, and hence of the Poincaré algebra.

It must be noted that their properties are not completely determined by the noncommutativity of spacetime coordinates, since the same noncommutative coordinates can be associated with different coproducts. Different realizations of a given noncommutative geometry are often called bases and usually lead to different physical predictions. The different bases can be characterized by specifying their deformed Heisenberg algebra, generated by the position operators x_μ and the momentum operators p_μ. From the knowledge of this algebra, one can obtain the coproduct of momenta and the other relevant quantities, using the methods developed in ref. [7].

In order to calculate the heat kernel one must first of all establish the field equations. The deformed invariance of the theory is preserved if the (momentum space) deformed Laplace equation is identified with the Casimir operator C of the deformed Poincaré algebra. However, this choice is not unique, because any function of C could be adopted.

Moreover, a nontrivial measure must be fixed on the Hilbert space, again invariant under deformed Lorentz transformations. To single out this measure uniquely, we also require that the position operators are symmetric in the representation chosen.

In this paper, we consider some specific models that lead to simple calculations of the heat kernel: the first one is the Snyder model [8]. Its main peculiarity is that it preserves the standard action of the Lorentz group on phase space, and it can be seen as dual to de Sitter spacetime. Its deformed Heisenberg algebra, in the original Snyder basis, is given by

$$[x_\mu, x_\nu] = i\frac{J_{\mu\nu}}{\kappa^2}, \quad [p_\mu, p_\nu] = 0, \quad [x_\mu, p_\nu] = i\left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{\kappa^2}\right), \tag{6}$$

where $J_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu$ are the generators of the Lorentz algebra. Since the Lorentz transformations are not deformed in this case, the Casimir operator is simply given by[2]

$$C = p^2 \equiv -p_0^2 + p_i^2, \tag{7}$$

with $m^2 = -p^2 < \kappa^2$. This implies an upper bound for the allowed particle masses in this model.

There is also the possibility of choosing the opposite sign in front of κ^2 in (6) (anti-Snyder geometry) [8,9]. In this case there is no upper bound on the particle masses. However, all the relations we shall discuss hold true, by simply changing the sign in front of κ^2.

The other examples belong to the κ-Poincaré class: one is the so-called Magueijo–Smolin model [10]. The nontrivial commutators of its Heisenberg algebra in the Granik basis [11] are

$$[x_i, x_0] = i\frac{x_i}{\kappa}, \quad [x_0, p_i] = i\frac{p_i}{\kappa}, \quad [x_i, p_j] = i\delta_{ij},$$
$$[x_0, p_0] = -i\left(1 - \frac{p_0}{\kappa}\right). \tag{8}$$

The Poincaré algebra is now deformed and its Casimir operator is

$$C = \frac{-p_0^2 + p_i^2}{\left(1 - \frac{p_0}{\kappa}\right)^2}. \tag{9}$$

In this case, the bound $p_0 < \kappa$ must hold.

The last one is the Majid–Ruegg (MR) model [12], describing the κ-Poincaré model [2] defined in the bicrossproduct basis. Its Heisenberg algebra reads

$$[x_i, x_0] = i\frac{x_i}{\kappa}, \quad [x_0, p_i] = i\frac{p_i}{\kappa}, \quad [x_i, p_j] = i\delta_{ij}, \quad [x_0, p_0] = -i. \tag{10}$$

Also in this case the Poincaré algebra is deformed, with Casimir operator

$$C = -\left(2\kappa\sinh\frac{p_0}{2\kappa}\right)^2 + \mathrm{e}^{\frac{p_0}{\kappa}}\,p_i^2. \tag{11}$$

[2] As noted above, this choice is not unique. Sometimes the choice $C = p^2/(1 - p^2/\kappa^2)$ is made.

The Euclidean version of this class of noncommutative models is usually obtained by an analytic continuation of the Lorentzian theory, with $p_0 \to ip_0$, $\kappa \to i\kappa$, see discussions in [13] for the κ-Poincaré model. This assumption appears natural considering that κ plays the role of an energy scale. Requiring $\kappa \to i\kappa$ also appears necessary in order to get a physically sensible interpretation of the Euclidean theory in the κ-Poincaré case, while in the Snyder case the situation is less clear.

For the class of models considered here, the calculation of the heat kernel differs in two ways from the standard case: first, as discussed above, the Laplace operator is chosen proportional to the Casimir operator, in order to be invariant under the deformed Lorentz transformations. Moreover, the measure of momentum space is not trivial, since the phase space operators satisfy a deformed Heisenberg algebra, and hence a nontrivial realization on the Hilbert space must be found.

4. Snyder model

We define the Euclidean Snyder model by the analytic continuation $p_0 \to ip_0$, $\kappa \to i\kappa$, as for the κ-Poincaré models. Keeping instead $\kappa \to \kappa$ would simply interchange the roles of Snyder and anti-Snyder Euclidean spaces. Our choice maintains the bound $m^2 < \kappa^2$ also in the Euclidean case.

In both instances, the Euclidean Casimir operator is given by

$$C = p_E^2 = p_0^2 + p_i^2. \tag{12}$$

We start by considering anti-Snyder space, because it gives rises to more interesting results. Euclidean anti-Snyder space in the basis (6) can be realized by a suitable choice of operators in a quantum representation. Two main choices can be found in the literature, in terms of a standard Hilbert space of functions of a canonical momentum variable P_μ, with Euclidean signature: the first one reads [8]

$$p_\mu = P_\mu, \qquad x_\mu = i\frac{\partial}{\partial P_\mu} + \frac{i}{\kappa^2} P_\mu P_\nu \frac{\partial}{\partial P_\nu}, \tag{13}$$

with $-\infty < P_\mu < \infty$. We require that in this representation the position operators x_μ be symmetric, i.e. that $<\psi\, |x_\mu|\, \phi> = <\phi\, |x_\mu|\, \psi>$. This occurs if one introduces a nontrivial measure in the P-space [14],

$$d\mu = \frac{d^4P}{(1+P^2/\kappa^2)^{\frac{d+1}{2}}}, \tag{14}$$

with d the dimension of the space.

We can now proceed to compute the trace of the heat kernel. In this representation, the Laplacian (12) is simply given by P^2. Hence, for $d = 4$, eq. (4) gives

$$K(s) = \frac{V}{16\pi^4} \int_{-\infty}^{\infty} \frac{d^4P}{(1+P^2/\kappa^2)^{5/2}}\, e^{-sP^2}. \tag{15}$$

Defining polar coordinates, with radial coordinate $\rho = \sqrt{P^2}$, after integrating on the angular variables, (15) becomes

$$K(s) = \frac{V}{8\pi^2} \int_0^{\infty} \frac{\rho^3 d\rho}{(1+\rho^2/\kappa^2)^{5/2}}\, e^{-s\rho^2}. \tag{16}$$

Performing the integral, one obtains

$$K(s) = \frac{\kappa^4 V}{24\pi^2} \left[2(1+\kappa^2 s) - \kappa\sqrt{\pi s}\,(3+2\kappa^2 s)\, e^{\kappa^2 s}\, \mathrm{erfc}(\kappa\sqrt{s})\right], \tag{17}$$

where $\mathrm{erfc}(x)$ is the complementary error function, $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt$.

In the limit $s \to 0$, $K(s)$ takes the finite value $\frac{\kappa^4 V}{12\pi^2}$, in contrast with the commutative theory, where it diverges as s^{-2}. Also in the limit $s \to \infty$ it takes a finite value. The evaluation of the vacuum energy (5) gives then for $s \to 0$ a logarithmic divergence, and the vacuum energy density reads

$$\lambda = \frac{\kappa^4}{12\pi^2} \ln \frac{\epsilon}{m}, \tag{18}$$

where m is the IR scale. The divergence is much milder than the commutative result, $\lambda \sim \epsilon^4$, although, if one assumes $\epsilon \sim \kappa$, the numerical value of λ is not much different in the two cases.

The same results can be obtained using another representation of the relations (6), again defined in terms of a standard Hilbert space of functions of a canonical momentum variable P_μ [9],

$$p_\mu = \frac{P_\mu}{\sqrt{1-P^2/\kappa^2}}, \qquad x_\mu = i\sqrt{1-P^2/\kappa^2}\, \frac{\partial}{\partial P_\mu}, \tag{19}$$

with $P^2 < \kappa^2$. In this case, the measure for which the operators x_μ are symmetric is given by [9]

$$d\mu = \frac{d^4P}{\sqrt{1-P^2/\kappa^2}}, \tag{20}$$

independently from the dimension of the space, while $p^2 = \frac{P^2}{1-P^2/\kappa^2}$. Hence,

$$K(s) = \frac{V}{16\pi^4} \int_{P^2<\kappa^2} \frac{d^4P}{\sqrt{1-P^2/\kappa^2}}\, e^{-\frac{sP^2}{1-P^2/\kappa^2}}. \tag{21}$$

In polar coordinates $\rho = \sqrt{P^2}$, this becomes after integration on the angular coordinates

$$K(s) = \frac{V}{8\pi^2} \int_0^{\kappa^2} \frac{\rho^3 d\rho}{\sqrt{1-\rho^2/\kappa^2}}\, e^{-\frac{s\rho^2}{1-\rho^2/\kappa^2}}. \tag{22}$$

By a change of variables $\rho \to \frac{\rho}{\sqrt{1-\rho^2/\kappa^2}}$, one finally recovers (16).

The Snyder model is obtained by replacing κ^2 with $-\kappa^2$ in (13) and (14), but now the calculation is more involved, since the integral for $K(s)$ does not converge on the boundary. In fact, the representation (13) becomes now

$$p_\mu = P_\mu, \qquad x_\mu = i\frac{\partial}{\partial P_\mu} - \frac{i}{\kappa^2} P_\mu P_\nu \frac{\partial}{\partial P_\nu}, \tag{23}$$

with $P^2 < \kappa^2$, and measure

$$d\mu = \frac{d^4P}{(1-P^2/\kappa^2)^{\frac{d+1}{2}}}. \tag{24}$$

The integral (16) becomes

$$K(s) = \frac{V}{8\pi^2} \int_0^{\kappa} \frac{\rho^3 d\rho}{(1-\rho^2/\kappa^2)^{5/2}}\, e^{-s\rho^2}, \tag{25}$$

that diverges at $\rho = \kappa$. One must therefore introduce an UV cutoff already at this stage, for example taking as upper limit of integration $\epsilon < \kappa$. This gives at leading order the constant value $K(s) \sim \frac{\kappa^5(\epsilon^2 - 2\kappa^2/3)}{(\kappa^2-\epsilon^2)^{3/2}}$. Taking the same cutoff ϵ for the integration over s, it follows that

$$\lambda \sim -\frac{\kappa^5(\epsilon^2 - 2\kappa^2/3)}{(\kappa^2-\epsilon^2)^{3/2}} \ln\frac{\epsilon}{m}. \tag{26}$$

Therefore, in this case the vacuum energy diverges as $(\kappa-\epsilon)^{-3/2}$, with $(\kappa-\epsilon) \ll 1$.

5. MS model

This model belongs to the κ-Poincaré class and considerations analogous to those of [13] suggest that the Euclidean theory can be defined through the prescription $p_0 \to ip_0$, $\kappa \to i\kappa$. This leads to the Euclidean Laplacian

$$C = \frac{p_0^2 + p_i^2}{\left(1-\frac{p_0}{\kappa}\right)^2}. \tag{27}$$

The action of the 4-dimensional rotations on phase space is deformed and only the action of the spatial rotations is preserved. However, the calculation can be performed in a way similar to the one of the previous section.

First of all, we notice that the MS model can be represented on a standard Hilbert space of functions of a canonical momentum variable P_μ as

$$p_\mu = \frac{P_\mu}{1+P_0/\kappa}, \qquad x_\mu = i(1+P_0/\kappa)\frac{\partial}{\partial P_\mu}, \tag{28}$$

where $-\infty < P_i < \infty$, $0 < P_0 < \infty$. In this representation, the measure for which the operators x_μ are symmetric is given by

$$d\mu = \frac{d^4P}{1+P_0/\kappa}. \tag{29}$$

The heat kernel integral becomes therefore

$$K = \frac{V}{16\pi^4}\int \frac{d^4P}{1+P_0/\kappa}\, \mathrm{e}^{-sP^2}, \tag{30}$$

and can be separated into

$$K = \frac{V}{16\pi^4}\int_0^\infty \frac{dP_0}{1+P_0/\kappa}\, \mathrm{e}^{-sP_0^2}\int_{-\infty}^{\infty} d^3P_i\, \mathrm{e}^{-sP_i^2}. \tag{31}$$

This gives

$$K = -\frac{\kappa V}{32\pi^{3/2}}\frac{\mathrm{e}^{-\kappa^2 s}}{s^{3/2}}\left[i\pi\,\mathrm{erf}(i\kappa\sqrt{s}) + \mathrm{Ei}(\kappa^2 s)\right], \tag{32}$$

where $\mathrm{erf}(x)$ is the error function and $\mathrm{Ei}(x)$ the exponential integral.

For $s \to 0$, $K \sim -\frac{\kappa V}{32\pi^{3/2}}s^{-3/2}\big(\ln(\kappa^2 s) + \gamma + O(s)\big)$, where γ is the Euler–Mascheroni constant. For $s \to \infty$, K vanishes. As one could have guessed from the structure of the integral, in this case only the integration on p_0 gives rise to a milder UV divergence, while the spatial part presents the usual divergence $s^{-3/2}$, with a further logarithmic factor. The calculation of the vacuum energy density gives at leading order

$$\lambda \sim \frac{1}{24\pi^{3/2}}\kappa\epsilon^3 \ln\frac{\epsilon}{\kappa}. \tag{33}$$

The UV divergence is milder than in the commutative case. With the natural identification $\epsilon = \kappa$, the logarithmic term vanishes, and the leading divergence is given by the next term in the expansion, with the standard κ^4 behavior.

6. MR model

Let us consider now the MR model. As discussed before, the Euclidean theory is obtained for $p_0 \to ip_0$, $\kappa \to i\kappa$. The Euclidean Laplacian is then

$$C = \left(2\kappa \sinh\frac{p_0}{2\kappa}\right)^2 + \mathrm{e}^{\frac{p_0}{\kappa}}\, p_i^2. \tag{34}$$

A representation of the Heisenberg algebra is given by

$$p_\mu = P_\mu, \qquad x_0 = i\frac{\partial}{\partial P_0} - \frac{i}{\kappa}P_i\frac{\partial}{\partial P_i}, \qquad x_i = i\frac{\partial}{\partial P_i}. \tag{35}$$

These operators are Hermitian for the measure [15]

$$d\mu = \mathrm{e}^{\frac{3P_0}{\kappa}}\, d^4P. \tag{36}$$

The heat kernel integral becomes then

$$K(s) = \frac{V}{16\pi^4}\int_{-\infty}^{\infty} d^4P\, \mathrm{e}^{\frac{3P_0}{\kappa}}\, \mathrm{e}^{-s[4\kappa^2\sinh^2\frac{P_0}{2\kappa} + \mathrm{e}^{P_0/\kappa}P_i^2]}, \tag{37}$$

or

$$K(s) = \frac{V}{16\pi^4}\int_{-\infty}^{\infty} dP_0\, \mathrm{e}^{\frac{3P_0}{\kappa}}\, \mathrm{e}^{-4\kappa^2 s\sinh^2\frac{P_0}{2\kappa}}\int_{-\infty}^{\infty} d^3P_i\, \mathrm{e}^{-s\,\mathrm{e}^{P_0/\kappa}P_i^2}, \tag{38}$$

and, after integration over the spatial coordinates,

$$K(s) = \frac{V}{16\pi^{3/2}}\frac{1}{s^{3/2}}\int_{-\infty}^{\infty} dP_0\, \mathrm{e}^{\frac{3P_0}{2\kappa}}\, \mathrm{e}^{-4\kappa^2 s\sinh^2\frac{P_0}{2\kappa}}. \tag{39}$$

The last integration gives

$$K(s) = \frac{V}{32\pi^2\kappa^2 s^3}(1+2\kappa^2 s). \tag{40}$$

In this case, the divergence for $s \to 0$ is worse than in the commutative case, while the expression converges for $s \to \infty$.

Computing the vacuum energy density we obtain

$$\lambda = \frac{\epsilon^6}{192\pi^2\kappa^2}\left(1+\frac{3\kappa^2}{\epsilon^2}\right). \tag{41}$$

Again, if one identifies ϵ with κ, $\lambda \propto \kappa^4$, like in the standard theory.

7. Conclusions

Using the heat kernel method, we have shown that in some cases noncommutativity can regularize the behavior of the vacuum energy of a scalar field theory. This is however not a universal property: it holds for the anti-Snyder model, but not necessarily in different instances, like the MR or the MS model. It is important to remark that our results are independent of the representation chosen in a Hilbert space. This has been shown explicitly for the Snyder model, but can be checked also in the other cases. It is however crucial to choose the correct measure in the Hilbert space.

The results obtained here are in agreement with explicit calculations of quantum field theory in Snyder space, which show an improvement of the divergences with respect to the commutative case [16]. Analogous conclusions concerning the energy of the vacuum in noncommutative theories have been obtained using a very different approach related to the Wheeler–deWitt equation, in ref. [17]. After completion of this paper, we become aware of further works treating related problems [18,19].

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