Perturbative reduction of derivative order in EFT

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Higher derivative (H.D.) theories

- Simple definition: theories which contain in their action irreducible time derivatives of order higher than two

$$\int dt\, X\ddot{X}\,, \qquad \int dt\, Y^3\ddot{X}\,, \qquad \int dt\, \dot{X}\ddot{X}\,, \tag{1}$$

$$\int dt\, \ddot{X}^2\,, \qquad \int dt\, \dot{Y}\ddot{X} \sim -\int dt\, \ddot{Y}\dot{X} \tag{2}$$

- Typically lead to higher order equations of motion (in time derivatives)

- Interesting for cosmological phenomenology (Starobinsky inflation, Horndeski theories and dark energy)

Problems with higher derivatives

- Why not H.D. theories?

- Unphysical properties: Ostrogradsky ghosts
- Propagate more degrees of freedom (d.o.f.)
- Hamiltonian unbounded from below (for unconstrained theories)
- Runaway solution (e.g. self-accelerating electron in Abraham-Lorentz theory)

Higher derivative oscillator

$$S[X] = \int dt \left\{ \frac{1}{2}\dot{X}^2 - \frac{\omega^2}{2}X^2 + \frac{\epsilon}{2\omega^2}\ddot{X}^2 \right\} \tag{3}$$

- 4-th order equation of motion:

$$\ddot{X} + \omega^2 X = \frac{\epsilon}{\omega^2} X^{(4)} \tag{4}$$

- 4 initial conditions $\rightarrow$ 2 degrees of freedom
- 4 linearly independent solutions

$$X(t) = A_1 \times e^{i\omega_1 t} + B_1 \times e^{-i\omega_1 t} + A_2 \times e^{\omega_2 t} + B_2 \times e^{-\omega_2 t} \tag{5}$$

$$\omega_1 = \omega \sqrt{\frac{1}{2\epsilon}\left[-1 + \sqrt{1+4\epsilon}\right]} \approx \omega + \mathcal{O}(\epsilon) \tag{6}$$

$$\omega_2 = \omega \sqrt{\frac{1}{2\epsilon}\left[1 + \sqrt{1+4\epsilon\omega^2}\right]} \approx \frac{\omega}{\sqrt{\epsilon}}\left[1 + \mathcal{O}(\epsilon)\right] \tag{7}$$

Higher derivative oscillator

- Time translation invariance $\rightarrow$ energy as Noether current

$$E = \frac{1}{2}\sqrt{1-4\epsilon}\,\omega_1^2(A_1^2 + B_1^2) - \frac{1}{2}\sqrt{1-4\epsilon}\,\omega_2^2(A_2^2 + B_2^2) \qquad (8)$$

- Energy not bounded from below!
- Runaway solutions?
- Fine-tune initial conditions to remove ghosts
  $\Rightarrow$ solutions not stable under perturbations

- General theorem for unconstrained H.D. theories: Hamiltonian is linear in some momenta – unbounded

  Woodard arXiv:1506.02210 [hep-th]

Effective (field) theories

- Decoupling principle: low energy physics independent from high energy physics
- Only low-energy effective degrees of freedom
- High energy physics encoded in effective interactions of low energy degrees of freedom (d.o.f.)
- Prescription: write down systematically all the local higher dimensional corrections (operators) respecting given symmetries, suppressed by some high scale M

$$S[X] = \int dt \left\{ \left[\frac{1}{2}\dot{X}^2 - \frac{\omega^2}{2} X^2 \right] + \frac{1}{M^2} \left[\alpha_1 \dot{X}^4 + \alpha_2 \ddot{X}^2 \right] + \frac{1}{M^4} \left[\alpha_3 \dot{X}^6 + \alpha_4 \ddot{X}^3 + \alpha_5 \ddot{X}^3 \right] + \dots \right\} \tag{9}$$

EFTs and higher derivatives

- EFTs are generically higher derivative theories
- Are all EFTs unstable?

  $\Rightarrow$ naively yes

- Does the number of d.o.f. grow with each order?
- Where does local EFT come from?

  $\Rightarrow$ from small nonlocal corrections

- Examples where UV physics is know suggests this – take this attitude to UV-incomplete theories (gravity)

Two coupled oscillators: integrating out

- Consider two oscillators with hierarchy $\Omega^2 \gg \omega^2$

$$\ddot{X} + \omega^2 X = -\lambda Y \quad (10)$$

$$\ddot{Y} + \Omega^2 Y = -\lambda X \quad (11)$$

- Solve for the stiff oscillator approximately

$$Y = \frac{-\lambda}{\partial_t^2 + \Omega^2} X = -\frac{\lambda}{\Omega^2} X + \frac{\lambda}{\Omega^4} \ddot{X} - \frac{\lambda}{\Omega^6} X^{(4)} + \frac{\lambda}{\Omega^8} X^{(6)} + \ldots \quad (12)$$

- leads to H.D. equation of motion

$$\left(1 + \frac{\lambda^2}{\Omega^4}\right) \ddot{X} + \left(\omega^2 - \frac{\lambda^2}{\Omega^2}\right) X = \frac{\lambda^2}{\Omega^6} X^{(4)} - \frac{\lambda^2}{\Omega^8} X^{(6)} \ldots\ldots \quad (13)$$

Two coupled oscillators: IR local effective theory

- Two degrees of freedom turn into arbitrarily many!
- The same type of story applies to quantum "integrating out"
- Local derivative expansion introduces errors we need to pay attention to!

$\Rightarrow$ Treat higher derivative corrections in the same perturbative spirit in which they arise

H.D. oscillator

- Local corrections arise as expansion in $\epsilon \sim \Omega^{-2}$
  $\Rightarrow$ Implicit assumption: solutions analytic in ϵ

$$\ddot{X} + \omega^2 X = \epsilon X^{(4)} + \mathcal{O}(\epsilon^3) \tag{14}$$

- Assume a perturbative ansatz

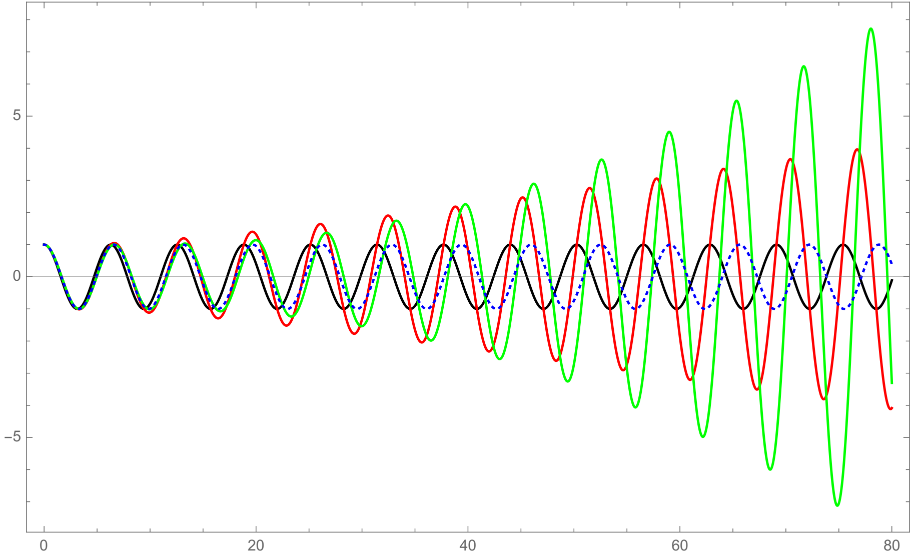
$$X = X_0 + \epsilon X_1 + \epsilon^2 X_2 + \dots \tag{15}$$

and organize the equations in powers of ϵ

$$\ddot{X}_0 + \omega^2 X_0 = 0\,, \tag{16}$$

$$\ddot{X}_1 + \omega^2 X_1 = X_0^{(4)} = \omega^4 X_0\,, \tag{17}$$

$$\ddot{X}_2 + \omega^2 X_2 = X_1^{(4)} = \omega^4 X_1\,, \tag{18}$$


5
0
−5
0
20
40
60
80

Time-dependent perturbation theory

- Spurious secular effects in time-dependent perturbation theory
- Another dimensionful scale in the problem: time
- Validity of approximation: $\epsilon\omega t \ll 1$

- Problem independent from low energy effective theory restriction!

Resummations 1

- Beyond perturbation theory: resummation
- Resummation: write down the truncated equation, but solve it *exactly*
- Cannot solve higher derivative equation (runaway solutions worse than perturbation theory)

  $\Rightarrow$ Need to reduce the derivative order

$$\ddot{X} + \omega^2 X = \epsilon X^{(4)} + \mathcal{O}(\epsilon^2) \tag{19}$$

- Find equation that produces the same perturbative solutions as H. D. one to leading order in ϵ,

$$\ddot{X} + \omega^2 X = -\omega^2 \epsilon \ddot{X} + \mathcal{O}(\epsilon^2) \tag{20}$$

Resummations 2

- Not unique (α can be anything)

$$\ddot{X} + \omega^2 X = -\alpha\omega^2\epsilon\ddot{X} + (1-\alpha)\omega^4\epsilon X + \mathcal{O}(\epsilon^2) \qquad (21)$$

- Error after resummation: $\epsilon^2\omega t$ – better than $\epsilon\omega t$ before
- In principle a systematic way to improve approximation
- Here resummation with any α equally good.

Resummations 3: anharmonic oscillator

- Consider interacting theory

$$S[X] = \int dt \left[\frac{1}{2}\dot{X}^2 - U(X) + \epsilon \ddot{X}^2 \right] \quad (22)$$

$$\Rightarrow \qquad \ddot{X} + U' = \epsilon X^{(4)} \quad (23)$$

- Reducing the derivative order on the equation of motion

$$\ddot{X} + U' = \epsilon \dot{X}^2 U''' - \alpha\epsilon \ddot{X} U'' + \epsilon(1-\alpha)U'U'' + \mathcal{O}(\epsilon^2) \quad (24)$$

- Not every choice of α is good (enough)

Resummations 4: anharmonic oscillator

- Only $\alpha = 1$ corresponds to equation derivable from action principle

$$S_{\rm red}[X] = \int dt \left[\frac{1}{2}\left[1 + 4\epsilon U''\right]\dot{X}^2 - U - \epsilon(U')^2 + \mathcal{O}(\epsilon^2)\right] \quad (25)$$

- Only $\alpha = 1$ admits energy conservation!

$\Rightarrow$ Reduce derivative order at the level of action

Resummations 5: scalar field theory

- Consider a Lorentz-invariant scalar field theory

$$S_{\rm red}[\phi] = \int d^4x \left[-\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - U(\phi) + \epsilon(\Box\Box\phi)(\Box\Box\phi) \right] \tag{26}$$

$$\Rightarrow \qquad \Box\phi - U' + 2\epsilon\Box\Box\Box\Box\phi = 0 \tag{27}$$

- Equation of motion cannot be reduced while preserving Lorentz invariance

$$\Box\phi - U' + 8\epsilon \Big[(\text{lower der.}) + 2U^{(5)}(\partial_\mu\partial_\nu\partial_\rho\phi)(\partial^\mu\phi)(\partial^\nu\phi)(\partial^\rho\phi)$$
$$+ 6U^{(4)}(\partial_\mu\partial_\nu\partial_\rho\phi)(\partial^\mu\partial^\nu\phi)(\partial^\rho\phi) + U'''(\partial_\mu\partial_\nu\partial_\rho\phi)(\partial^\mu\partial^\nu\partial^\rho\phi) \Big] = 0 \tag{28}$$

Derivative reduction of action

- Reducing derivative order in the action allows control over symmetries (conservation laws)
- Methods for derivative reduction of actions:

  - Perturbative constraints (canonical method; does not require field redefinitions)

  - Perturbative field redefinitions

Field redefinitions: scalar field

- Consider the action with higher derivative corrections

$$S[\phi] = \int d^4x \left[-\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - U + \epsilon(\partial_\mu\phi)(\partial^\mu\phi)(\partial_\nu\partial_\rho\phi)(\partial^\nu\partial^\rho\phi) \right] \quad (29)$$

- Field redefinition:

$$\phi \to \phi - \epsilon(\partial_\mu\phi)(\partial^\mu\phi)(\Box\phi) \quad (30)$$

transforms the action:

$$S[\phi] = \int d^4x \Big\{ -\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - U + \epsilon U'(\partial_\mu\phi)(\partial^\mu\phi)(\Box\phi) - \epsilon(\partial_\alpha\phi)(\partial^\alpha\phi)\big[(\Box\phi)(\Box\phi) - (\partial_\mu\partial_\nu\phi)(\partial^\mu\partial^\nu\phi)\big] + \mathcal{O}(\epsilon^2) \Big\} \quad (31)$$

Perturbative constraints 1

- Consider a system with the action containing higher derivative terms that come with small control parameter ϵ,

$$S[X_i] = \int dt \left[L_0(X_i, \dot{X}_i) + \epsilon L_1(X_i, \dot{X}_i, \ddot{X}_i) + \mathcal{O}(\epsilon^2) \right] \quad (32)$$

- Require all quantities take values in the space of polynomials in ϵ
- Define the canonical action maintaining perturbativity in ϵ

$\Rightarrow$ Perturbativity in ϵ leads to second-class constraints eliminating DOFs associated with HDs

$$\mathscr{S}_{\text{red}}[X_i, P_i] = \int dt \left[f_i(X,P)\dot{X}_i + g_i(X,P)\dot{P}_i - H_{\text{red}}(X,P) \right] \quad (33)$$

Perturbative constraints 2

- Generally no reduced Lagrangian formulation without perturbative field redefinitions
- Method always reduces derivative order
- Still unclear what happens to symmetries (difficult in Hamiltonian formalism)
- Non-canonical Poisson brackets resulting from second-class constraints ($\{X_i, X_j\} \neq 0$)
- Complicated to implement

Scalar-tensor EFT 1

- Horndeski theories: can they be seen to generically arise as EFT corrections?

$$S^{(0)} = \frac{1}{\kappa^2}\int d^Dx\,\sqrt{-g}\left\{F(\varphi)R - \frac{1}{2}Z(\varphi)g^{\mu\nu}(\nabla_\nu\varphi)(\nabla_\nu\varphi) - U(\varphi)\right\} \tag{34}$$

- Write down all the possible dimension 4 corrections (4-derivative corrections)
- Use perturbative field redefinitions to remove H.D. terms

Scalar-tensor EFT 2

- All independent 4-derivative corrections

$$S_1^{(1)} = \int d^D x \sqrt{-g}\, f_1(\varphi)\, (\nabla_\mu \varphi)(\nabla^\mu \varphi)(\nabla_\nu \varphi)(\nabla^\nu \varphi)\,, \tag{35}$$

$$S_2^{(1)} = \int d^D x \sqrt{-g}\, f_2(\varphi)\, (\nabla_\mu \varphi)(\nabla^\mu \varphi)(\Box \varphi)\,, \tag{36}$$

$$S_3^{(1)} = \int d^D x \sqrt{-g}\, f_3(\varphi)\, (\Box \varphi)(\Box \varphi)\,, \tag{37}$$

$$S_4^{(1)} = \int d^D x \sqrt{-g}\, f_4(\varphi)\, (\nabla_\mu \varphi)(\nabla^\mu \varphi) R\,, \tag{38}$$

$$S_5^{(1)} = \int d^D x \sqrt{-g}\, f_5(\varphi)\, (\nabla_\mu \varphi)(\nabla_\nu \varphi) R^{\mu\nu}\,, \tag{39}$$

$$S_6^{(1)} = \int d^D x \sqrt{-g}\, f_6(\varphi)\, (\Box \varphi) R\,, \tag{40}$$

$$S_7^{(1)} = \int d^D x \sqrt{-g}\, f_7(\varphi)\, R^2\,, \tag{41}$$

$$S_8^{(1)} = \int d^D x \sqrt{-g}\, f_8(\varphi)\, R_{\mu\nu} R^{\mu\nu}\,, \tag{42}$$

$$S_9^{(1)} = \int d^D x \sqrt{-g}\, f_9(\varphi)\, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}\,, \tag{43}$$

Scalar-tensor EFT 3

- Field redefinitions $\varphi \to \varphi + \kappa^2 \delta\varphi$, $g_{\mu\nu} \to g_{\mu\nu} + \kappa^2 \delta g_{\mu\nu}$ of type

$$\delta\varphi = C_1 \times (\Box\varphi) + C_2 \times (\nabla_\mu\varphi)(\nabla^\mu\varphi) + C_3 \times R \,, \tag{44}$$

$$\begin{aligned} \delta g_{\mu\nu} = {} & C_4 \times g_{\mu\nu} R + C_5 \times g_{\mu\nu}(\Box\varphi) + C_7 \times (\nabla_\mu\nabla_\nu\varphi) \\ & + C_8 \times g_{\mu\nu}(\nabla_\rho\varphi)(\nabla^\rho\varphi) + C_9 \times (\nabla_\mu\varphi)(\nabla_\nu\varphi) \,, \end{aligned} \tag{45}$$

  preserves covariance at each order
- Coefficients can be chosen so that

$$\begin{aligned} S = \int d^D x \Big\{ & \frac{1}{\kappa^2}\Big[F_1 R - \frac{1}{2}F_2(\nabla_\mu\varphi)(\nabla^\mu\varphi) - F_3\Big] \\ & + F_4(\nabla_\mu\varphi)(\nabla^\mu\varphi)(\nabla_\nu\varphi)(\nabla^\nu\varphi) + F_5(\nabla_\mu\varphi)(\nabla^\mu\varphi)(\Box\varphi) \\ & + F_6 G^{\mu\nu}(\nabla_\mu\varphi)(\nabla_\nu\varphi) + F_7\big(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\big)\Big\} \end{aligned} \tag{46}$$

Scalar-tensor EFT 4

- All the 4-derivative corrections are healthy
  – all fall under Horndeski theories
- Is this true in general? – NO
- at 6-derivative order one $(R_{\mu\nu\rho\sigma})^3$ cannot be removed
  (pure quantum gravity is divergent at two loops)
- Solomon and Trodden [2017] examined 6-derivative corrections with shift-symmetry:
  there are 5 genuinely H.D. terms that cannot be removed by field redefinitions
  (definitely less than 5, I would say 2 [work in progress])

Non-removable higher derivatives

- Generally not all H.D. EFT corrections cannot be removed by field redefinitions that preserve local/global symmetries
- What is the physical information in them and how to quantify it?

  - Hamiltonian formulation via perturbative constraints

  - Give up locality of EFT expansion

  - Non-local field redefinitions [Krasnov 2008]

- Might lead to formulation of new theories (phenomenology?)