The Frustration of being Odd: Universal Area Law violation in local systems

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Our Claim

- Mix:
  - AFM weakly (non-extensively) frustrated chain
  - Quantum interactions with discrete symmetry
- To obtain:

A new quantum phase of matter

- Which is
  - gapless but non-relativistic
  - with peculiar very long-range correlations
  - area law violating, yet not diverging
  - extended, robust & universal
- All in a very simple, natural setting (Ising chain!)

Outline

1. Basics on Frustration
2. Our results for this weakly frustrated phase:
   - *Local* and *Quasi-Local* correlation functions
   - The entanglement entropy:
     - ✓ violates Area law, but does not diverge
     - ✓ collapses on a Universal curve
3. The analytical case of the Ising Chain
4. Conclusions

Basics on Frustration

- Frustration:

  competing interactions favoring different orders

$\Rightarrow$ impossible to minimize all energy contributions

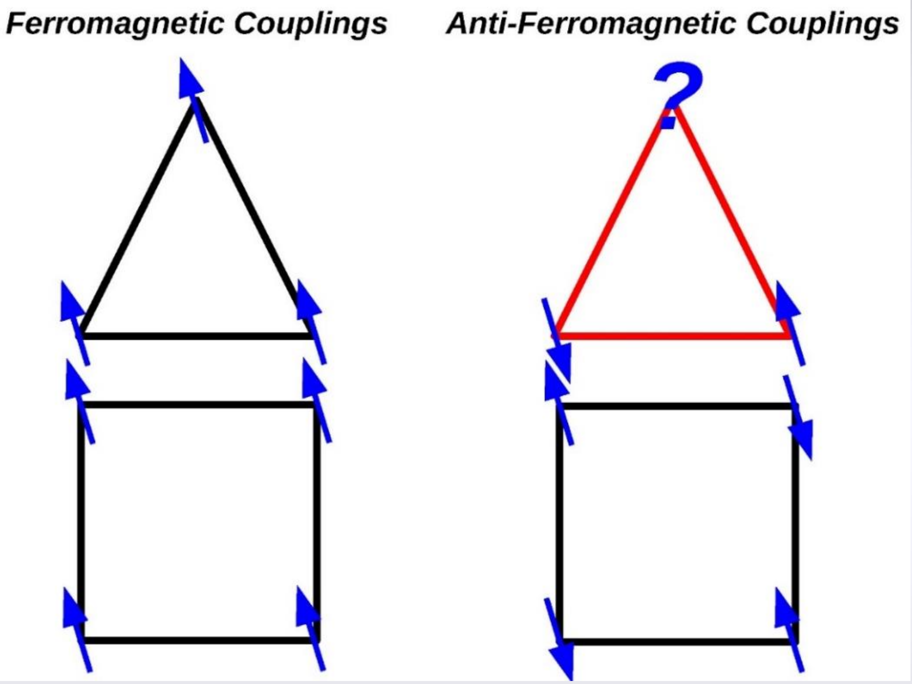
- Remark: all genuine quantum phases are frustrated (non-commuting terms promote diff. arrangements)

- E.g. Ising Chain: $H_{\text{Ising}} = \sum_{l=1}^{N} \left(\sigma_l^x \sigma_{l+1}^x - h\, \sigma_l^z \right)$

$\left[\sigma_l^x \sigma_{l+1}^x, \sigma_l^z \right] \neq 0$: ground state as a trade-off

Geometrical Frustration

- Originally, frustration in classical systems:
  - ➢ Arise from geometry
  - ➢ Toulouse Criterion: a classical systems is frustrated if there is a close loop for which $-1^{\mathcal{N}_{AFM}} = -1$
  - ➢ More loops ⇒ more frustration



- Remark: adding one site changes GS degeneracy from 2 to 2N and vice versa (challenges perturbative picture)

Frustrated Systems

- Certain degree of frustration is very common
- In any dimension, due to closed AFM loops
- Typically: extensive frustration (# loops scale with system size)
  - Ordered (ANNNI model, spin-ice…)
  - Disordered (Sherrington-Kirkpatrick model, spin glasses…)
- Peculiar physics: residual entropy, local zero-modes, algebraic decay, artificial EM, monopoles, Dirac strings…

Frustration & Quantum Interactions

- In quantum systems, subtle interplay between geometrical frustration and quantum interactions
- Hard Problem
- We consider the simplest setting:
  - ➢ 1 loop (1D chain): non-extensive frustration
  - ➢ Most generic n.n. AFM quantum chain ($N = 2M + 1$)

$$H = \frac{1}{2} \sum_{l=1}^{N} \left[\left(\frac{1+\gamma}{2} \right) \sigma_l^x \sigma_{l+1}^x + \left(\frac{1-\gamma}{2} \right) \sigma_l^y \sigma_{l+1}^y \right] + \frac{\Delta}{2} \sum_{l=1}^{N} \sigma_l^z \sigma_{l+1}^z - \sum_{l=1}^{N} h \, \sigma_l^z$$

with periodic boundary conditions: $\sigma_{l+N}^{\alpha} = \sigma_l^{\alpha}$

1D Spin Chain

$$H = \frac{J}{2}\sum_{l=1}^{N}\left[\left(\frac{1+\gamma}{2}\right)\sigma_l^x\sigma_{l+1}^x + \left(\frac{1-\gamma}{2}\right)\sigma_l^y\sigma_{l+1}^y\right] + \frac{\Delta}{2}\sum_{l=1}^{N}\sigma_l^z\sigma_{l+1}^z - \sum_{l=1}^{N} h\,\sigma_l^z$$

- $J=-1$: Easy-Plane Ferromagnet
- $J=1$: Easy-Plane Anti-Ferromagnet (AFM)
- $\Delta=0$: XY Chain (Free Fermions)
- $h=0$: Integrable XYZ chain
- $\gamma=0$: U(1) global symmetry (Integrable XXZ chain)
- $\gamma\neq 0$: Z_2 global residual symmetry

Parity Operator: $P \equiv \prod_{l=1}^{N}\sigma_l^z$, $[H,P]=0$

Weakly frustrated Quantum Chain

$$H = \frac{J}{2}\sum_{l=1}^{N}\left[\left(\frac{1+\gamma}{2}\right)\sigma_l^x\sigma_{l+1}^x + \left(\frac{1-\gamma}{2}\right)\sigma_l^y\sigma_{l+1}^y\right] + \frac{\Delta}{2}\sum_{l=1}^{N}\sigma_l^z\sigma_{l+1}^z - \sum_{l=1}^{N} h\,\sigma_l^z$$

with PBC: $\sigma_{l+N}^{\alpha} = \sigma_l^{\alpha}$

- $J = -1$: Ferromagnet → no geometrical frustration
- $J = 1$: AFM → $\begin{cases} N = 2M & \text{: no geometrical frustration} \\ N = 2M+1 & \text{: weak frustration} \end{cases}$
- Not much in literature on frustrated case
  - $\gamma = 0 \rightarrow$ U(1) : trivial effect of frustration ($S^Z = \pm\frac{1}{2}$)
  - $\gamma \neq 0 \rightarrow$ Z_2 : only now, very non-trivial

Weakly frustrated XY Chain

$$H = \frac{1}{2} \sum_{l=1}^{N} \left[\left(\frac{1+\gamma}{2} \right) \sigma_l^x \sigma_{l+1}^x + \left(\frac{1-\gamma}{2} \right) \sigma_l^y \sigma_{l+1}^y \right] - \sum_{l=1}^{N} h \, \sigma_l^z$$

with PBC: $\sigma_{l+N}^{\alpha} = \sigma_l^{\alpha}$. For $|h| < 1$:

- $N = 2M$: No frustration $\Rightarrow$ SSB of Z_2 symmetry
  - Gapped
  - Doubly degenerate GS $\rightarrow$ Spontaneous magnetization
  - Exponential decay of correlations
- $N = 2M + 1$: Weak frustration + Z_2 quantum symmetry $\Rightarrow$
  - Gapless, but not relativistic (Galilean)
  - Non-degenerate GS $\rightarrow$ No order parameter
  - Mixture of exponential and algebraic correlations

Local and Quasi-Local Correlators

- Ising Chain: $H_{\text{Ising}} = \frac{1}{2} \sum_{l=1}^{2M+1} \left(\sigma_l^x \sigma_{l+1}^x - h\, \sigma_l^z \right)$
- Mappable to free fermions (more later):
  unique GS, gapless (with quadratic dispersion) for |h|<1
- "*Local*" correlators:

$$C^{zz}(R) \equiv \langle \sigma_l^z \sigma_{l+R}^z \rangle = m_z^2 - \frac{c_1^z(h)}{R^2} \left(\frac{h^2}{J^2} \right)^R + \frac{4 m_z}{N} \left[1 + c_2^z(h) (-1)^R \left| \frac{h}{J} \right|^R \right]$$

- "*Quasi-Local*" correlators (Dong et al. JSTAT '16)

$$C^{xx}(R) \equiv \langle \sigma_l^x \sigma_{l+R}^x \rangle = (-1)^R \left(1 - \frac{h^2}{J^2} \right)^{1/4} \left[1 + \frac{c^x(h)}{R^2} \left(\frac{h^2}{J^2} \right)^R \right] \left(1 - \frac{2R}{N} \right)$$

- Depending on the number of fermions appearing after Jordan-Wigner transformation (more later)

Correlation Functions

- "*Local*" correlators:

$$C^{zz}(R) \equiv \langle \sigma_l^z \sigma_{l+R}^z \rangle = m_z^2 - \frac{c_1^z(h)}{R^2}\left(\frac{h^2}{J^2}\right)^R + \frac{4m_z}{N}\left[1 + c_2^z(h)(-1)^R \left|\frac{h}{J}\right|^R\right]$$

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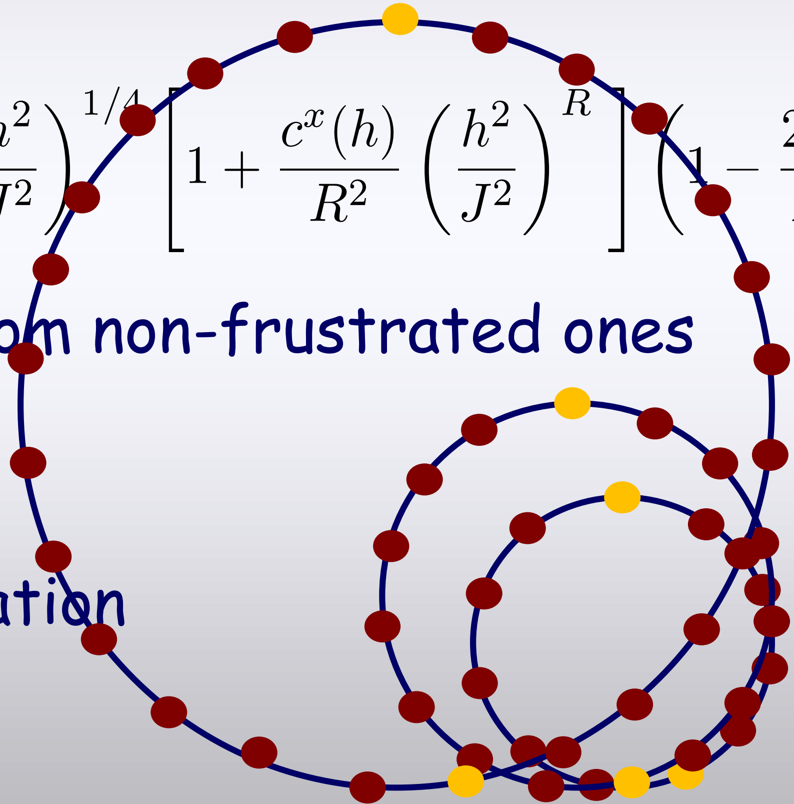
- Locally: indistinguishable from non-frustrated ones

- Order parameter/
  Spontaneous Magnetization



Correlation Functions

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- Locally: indistinguishable from non-frustrated ones

- Order parameter: $$\langle \sigma^x \rangle = \lim_{N \to \infty} \sqrt{C^{xx} \left(\frac{N-1}{2} \right)} = 0$$

- Inconsistent with thermodynamic Limit ($N \to \infty$)

Order Parameter

$$H_{\text{Ising}} = \frac{1}{2} \sum_{l=1}^{2M+1} \left(\sigma_l^x \sigma_{l+1}^x - h\, \sigma_l^z \right) \qquad P \equiv \prod_{l=1}^{N} \sigma_l^z \,, [H, P] = 0$$

- Parity eigenstates have vanishing order parameter:
  $\langle \sigma^x \rangle = \langle \sigma^+ + \sigma^- \rangle = 0$
- Non-zero magnetization only for degenerate GS of mixed parities: impossible at finite N
- Spontaneous Symmetry breaking by
  - Symmetry breaking field (not possible for gapless phases)
  - Long-range order in 2-point function: $C^{xx}(R) \equiv \langle \sigma_l^x \sigma_{l+R}^x \rangle$

Improved Thermodynamic Limit

$$C^{xx}(R) \equiv \langle \sigma_l^x \sigma_{l+R}^x \rangle = (-1)^R \left(1 - \frac{h^2}{J^2}\right)^{1/4} \left[1 + \frac{c^x(h)}{R^2} \left(\frac{h^2}{J^2}\right)^R\right] \left(1 - \frac{2R}{N}\right)$$

$$\langle \sigma^x \rangle = \lim_{N \to \infty} \sqrt{C^{xx}\left(\frac{N-1}{2}\right)} = 0$$

- Finite sub-region behavior cannot depend on even/oddness of large chain lengths
- Yet, the order parameter does
- Traditional therm. limit restores finite order parameter
- To account for the frustrated behavior need to use an Improved Thermodynamic Limit: $N \to \infty$ as $r \equiv \frac{R}{N} = \text{const}$

A New Frustrated Phase?

$$\langle \sigma^x \rangle = \lim_{N\to\infty} \sqrt{C^{xx}\left(\frac{N-1}{2}\right)} = 0$$

- So far, Ising model and 2-points correlation functions
- Technically: GS + 1 quasi-particle excitation
- Is this new phase robust to interactions?
- If local information not very sensitive to the new behavior, let us consider a non-local correlation function:

Entanglement entropy

in the

Improved Thermodynamic Limit

$N \to \infty$ as

$$r \equiv \frac{R}{N} = \text{const}$$

Entanglement Entropy

- Measures genuine quantum correlation
- Consider a pure (ground-state) and bi-partition (A|B)
- If system wave-function:

$$|\Psi^{A,B}\rangle = |\Psi^{A}\rangle \otimes |\Psi^{B}\rangle$$ → No Entanglement

$$|\Psi^{A,B}\rangle = \sum_{j=1}^{\mathcal{D}} \sqrt{\lambda_j} |\Psi_j^{A}\rangle \otimes |\Psi_j^{B}\rangle$$ → Entangled

(with $\mathcal{D} > 1$, $|\Psi_j^{A}\rangle$ & $|\Psi_j^{B}\rangle$ linearly independent):

- Entangled: Measurements on B affect A

Von Neumann Entropy

- To quantify entanglement with a number:

$$|\Psi^{A,B}\rangle = \sum_{j=1}^{\mathcal{D}} \sqrt{\lambda_j} |\Psi_j^A\rangle \otimes |\Psi_j^B\rangle$$

$$\begin{aligned} \rho_A &= \mathrm{tr}_B |\Psi^{A,B}\rangle\langle\Psi^{A,B}| \\ &= \sum \lambda_j |\Psi_j^A\rangle \, \langle\Psi_j^A| \end{aligned}$$

➢ Reduced Density Matrix

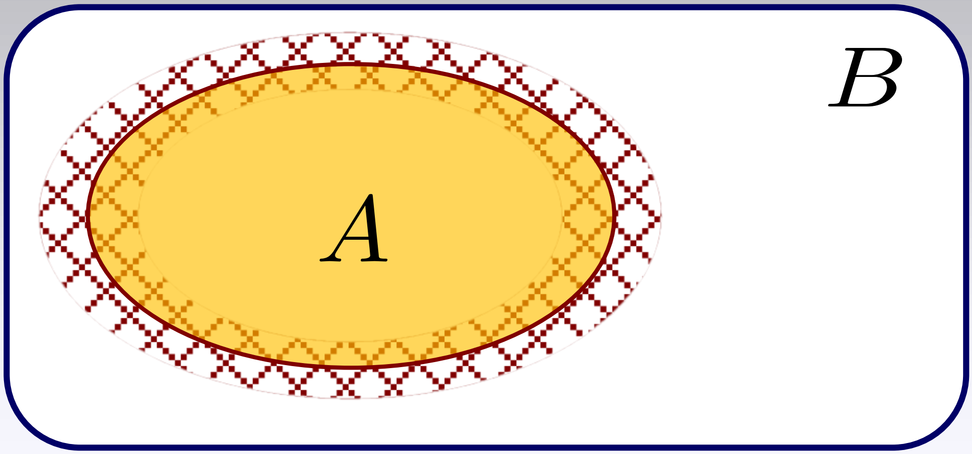
- Von Neumann (Quantum analog of Shannon Entropy):

$$S = -\mathrm{tr}_A \left(\rho_A \log \rho_A\right) = -\sum \lambda_j \log \lambda_j$$

- Non-local, but accounts for all different correlations!
- Universality

Area Law

- For GS, correlations between two systems localized around the boundary dividing them

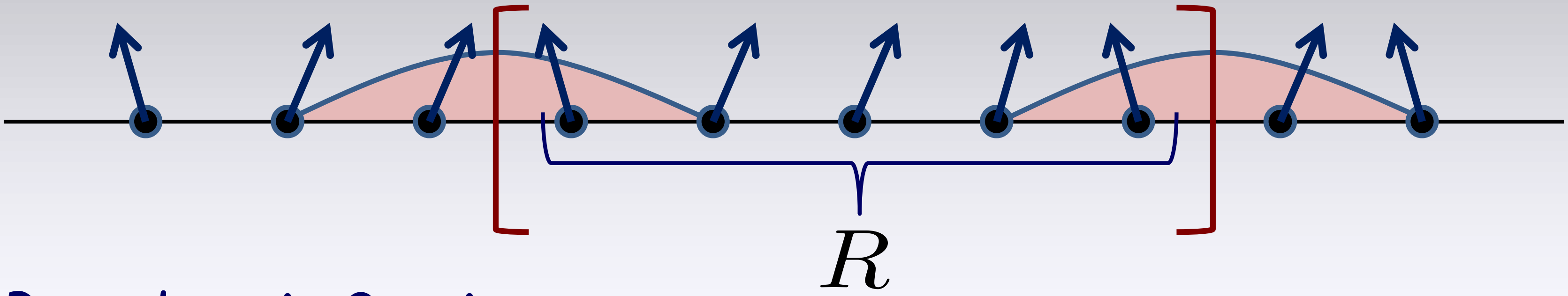


- Gapped systems:
  correlations decay within few correlation length
  $\Rightarrow$ EE proportional to Area
- Critical systems:
  correlation decay algebraically
  $\Rightarrow$ corrections to area law
  (logarithmic with Fermi surfaces)

> - $\rho_A = \mathrm{Tr}_B |GS\rangle\langle GS|$
>   Reduced Density Matrix
> - $S_A = -\mathrm{Tr}_A \left[\rho_A \ln \rho_A\right]$
>   Von Neumann Entropy

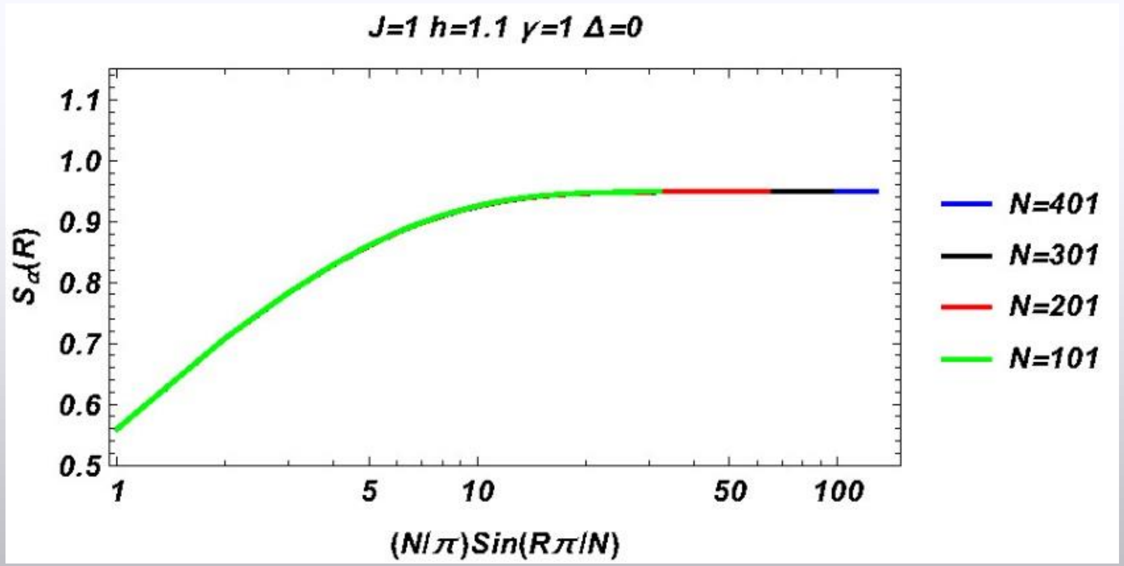
Area law in 1D

- Divide system into two connected sets of R and N-R sites



- Boundary is 2 points
- Gapped: area law

  → Saturation



Area law in 1D

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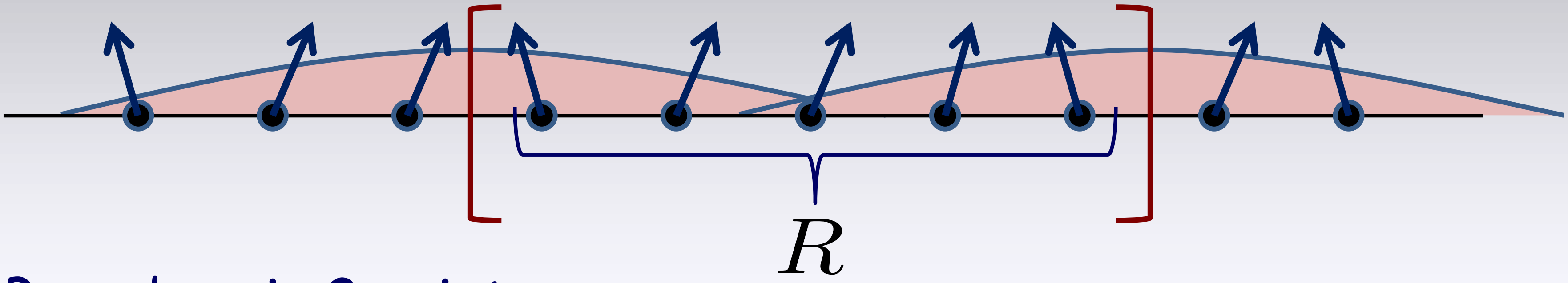


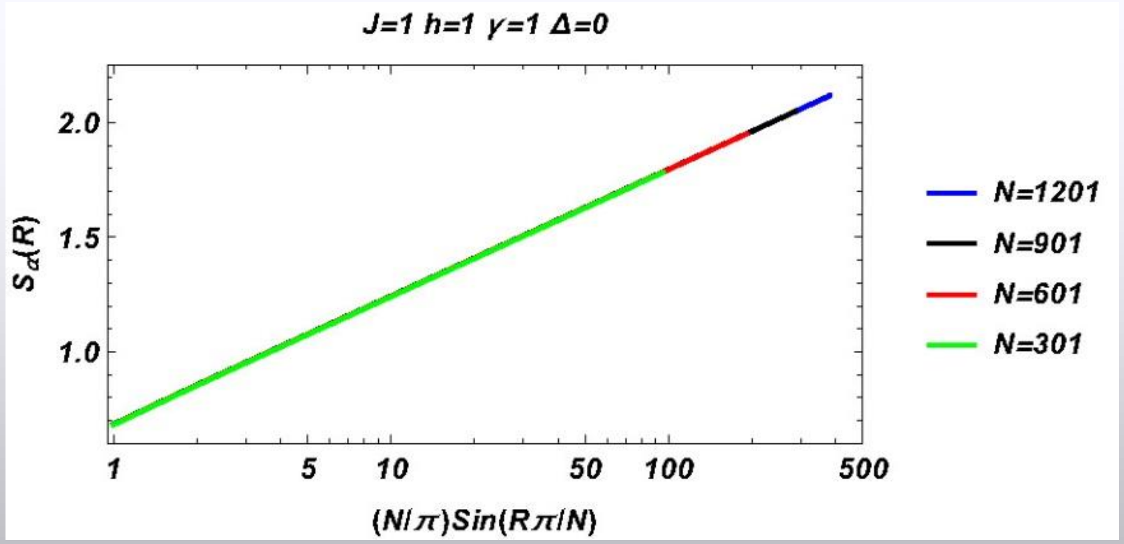
- Boundary is 2 points

- Critical (CFT)

  → Log-divergence

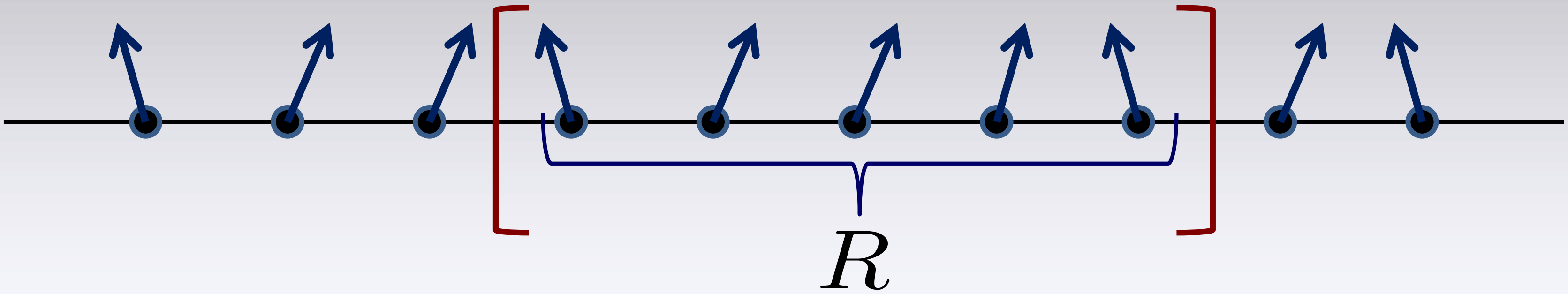
$$S(R) = \frac{c}{6} \ln \frac{N}{\pi} \sin \frac{\pi R}{N}$$

c: central charge



Area law Violation in 1D

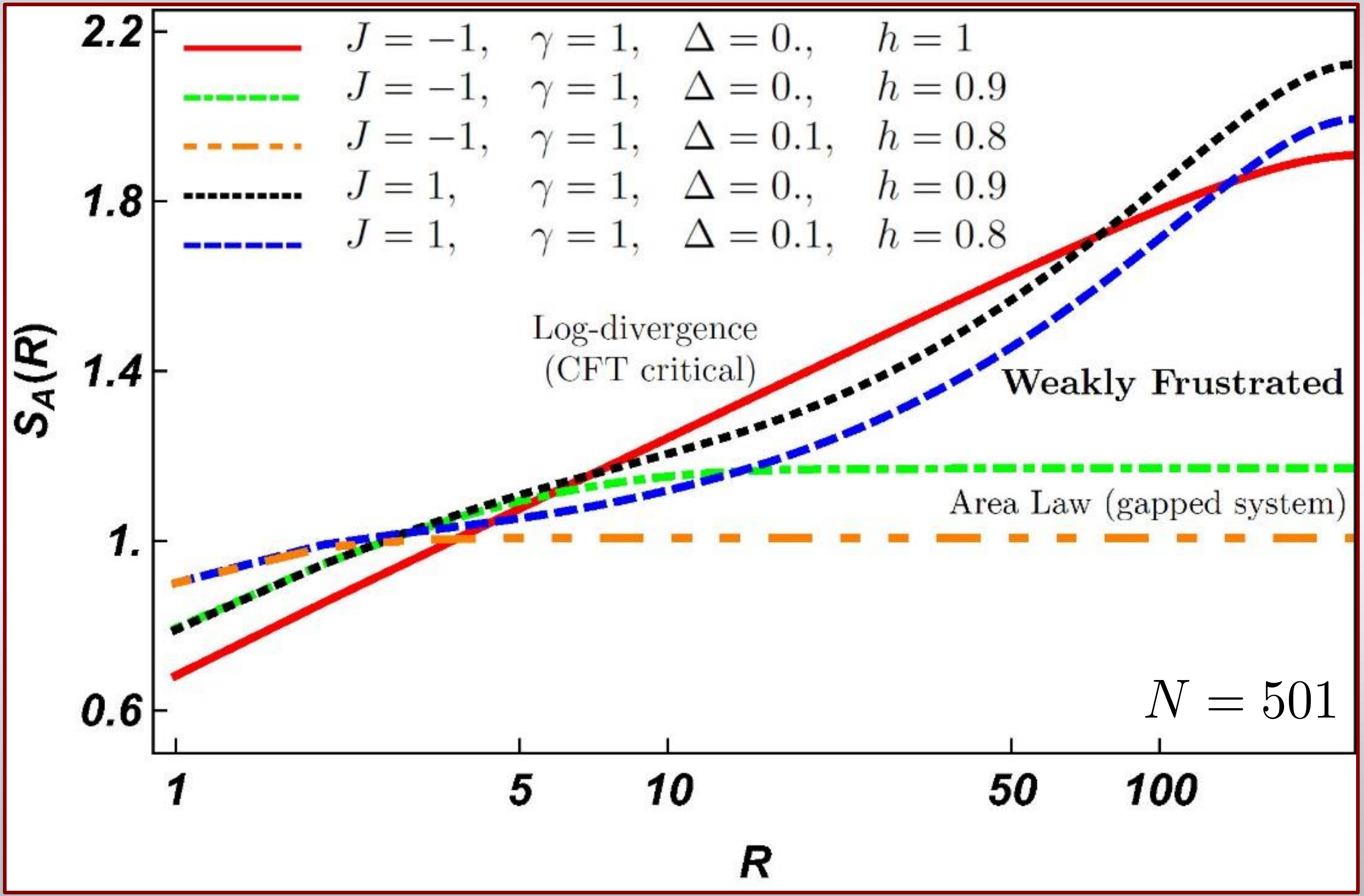
- Divide system into two connected sets of R and N-R sites



- Area law violation for
  - Breaking of translational invariance (Vitagliano et al. NJP '10) (Ramirez et al. JSTAT '14)
  - Long-range/non-local interactions (complex Fermi surface) (Vodola et al. PRL '14) (Gori et al. PRB '15)
  - Frustration Free systems with massive degeneracy (Motzkin & Fredkin chains) (Bravyi et al. PRL '12) (Dell'Anna et al. PRB '16)

EE in weakly frustrated phase

- Novel behavior: area law violation!



EE in weakly frustrated phase

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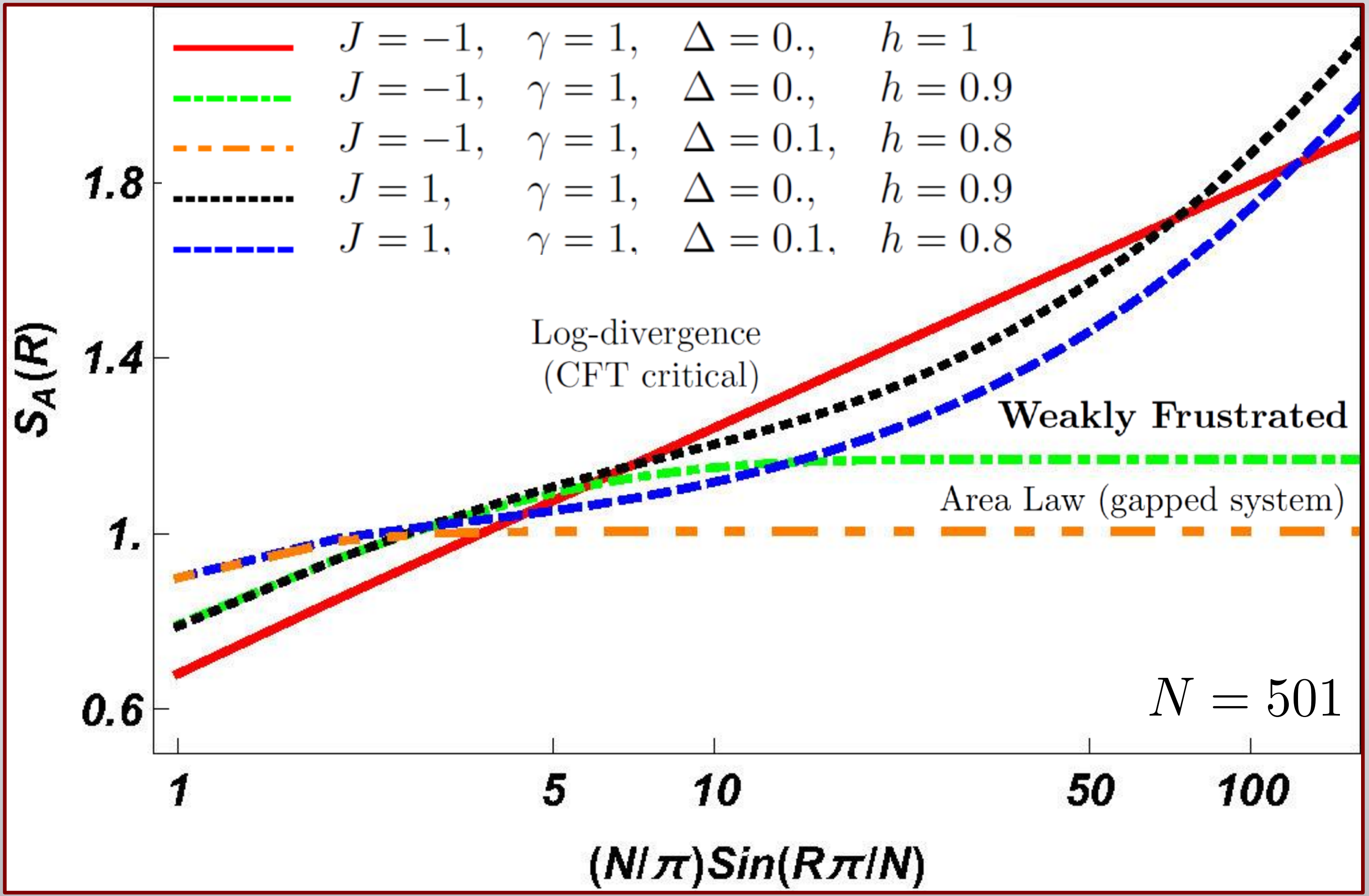
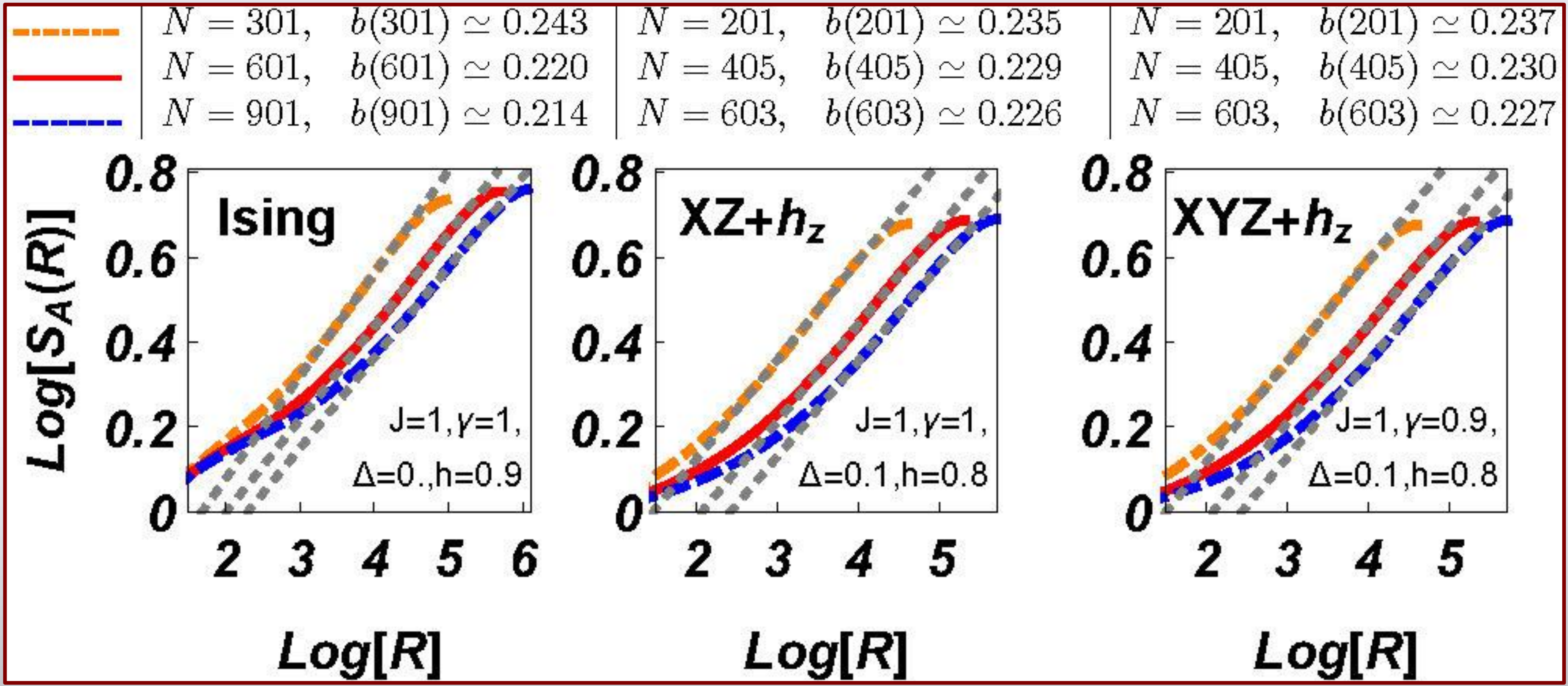


Algebraic Area Law Violation

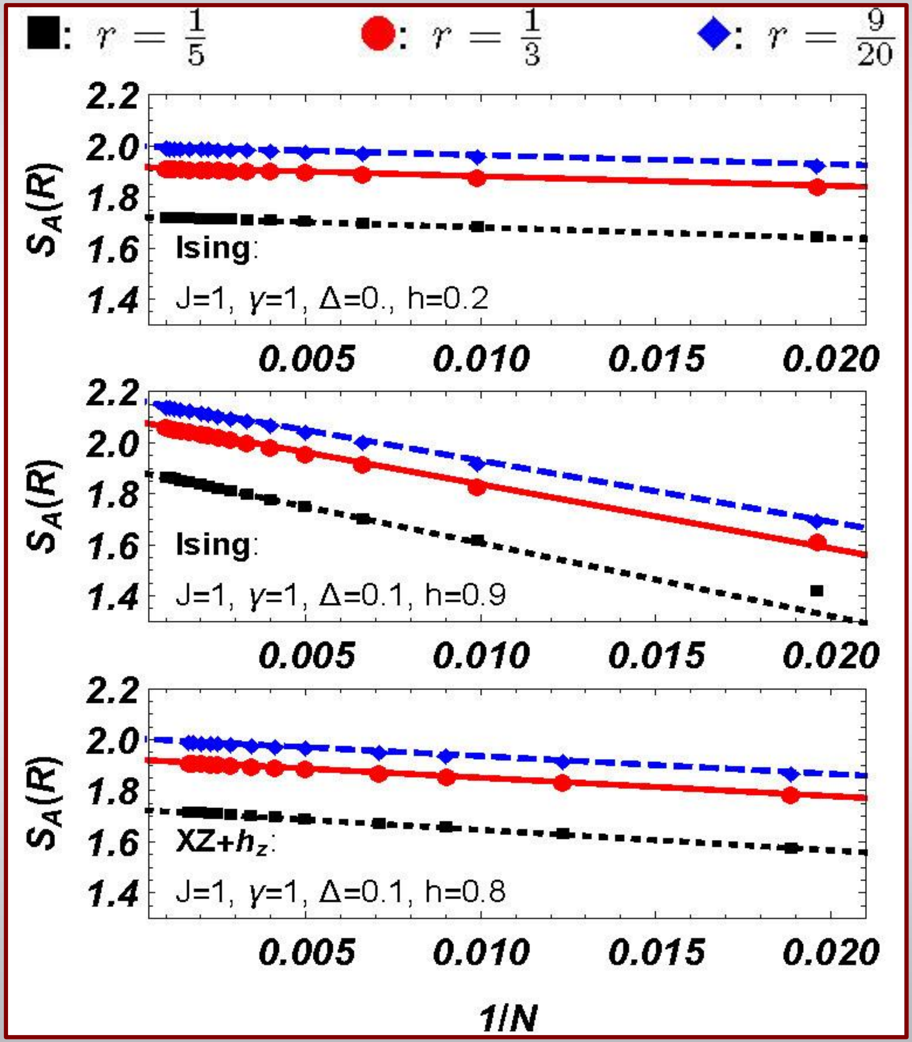
- Log-Log plots show power-law behavior in the bulk:

  (grey dashed lines: best fit with $S_A(R) \simeq a(N) R^{b(N)}$)



And yet it saturates

- Despite area law violation, the EE does not diverge



(lines are best fit with

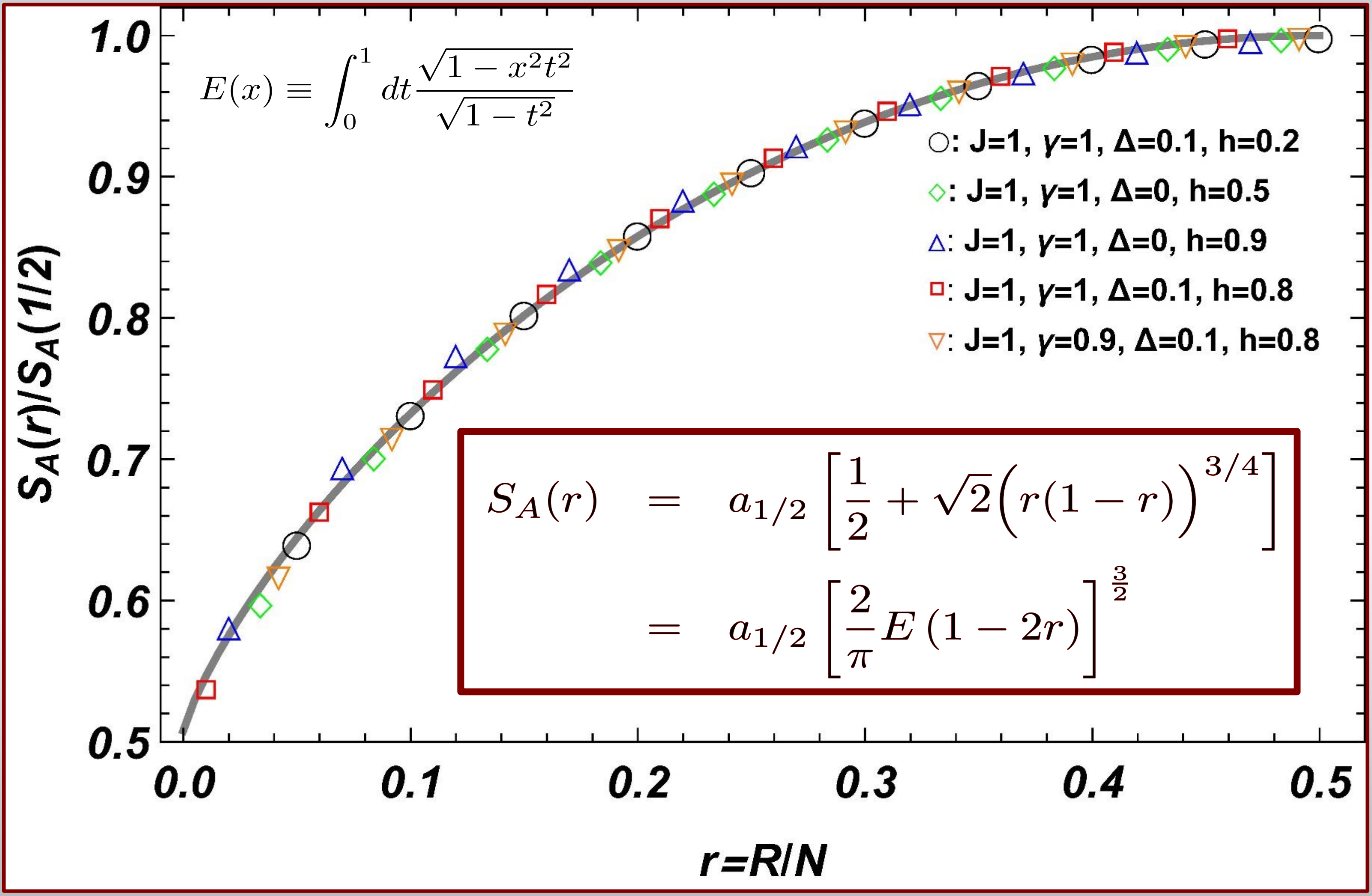
$$a_r + \frac{b_r}{N}:$$

for $N \to \infty$ EE saturates at finite a_r)

- Improved Therm. Limit provides limiting EE value to compare behavior in parameter space

Universal Behavior

- Every saturation point collapses on the same curve!

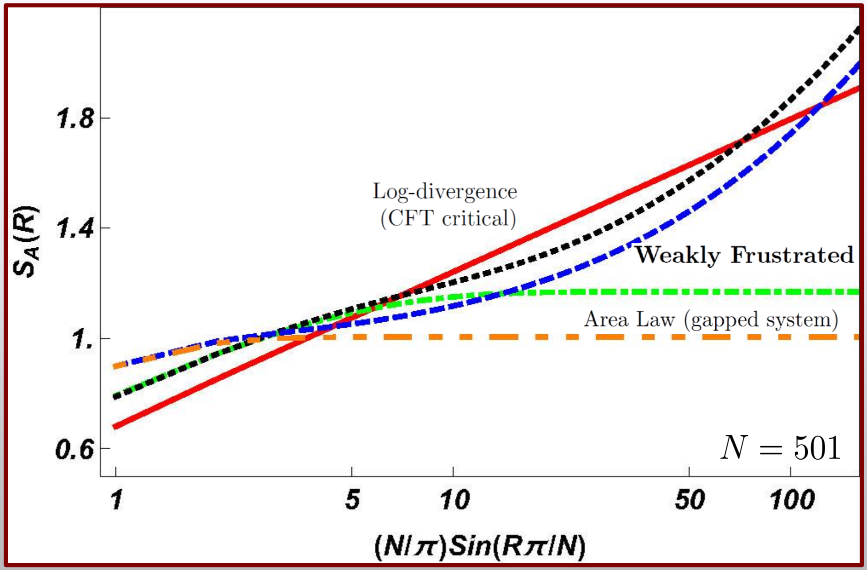


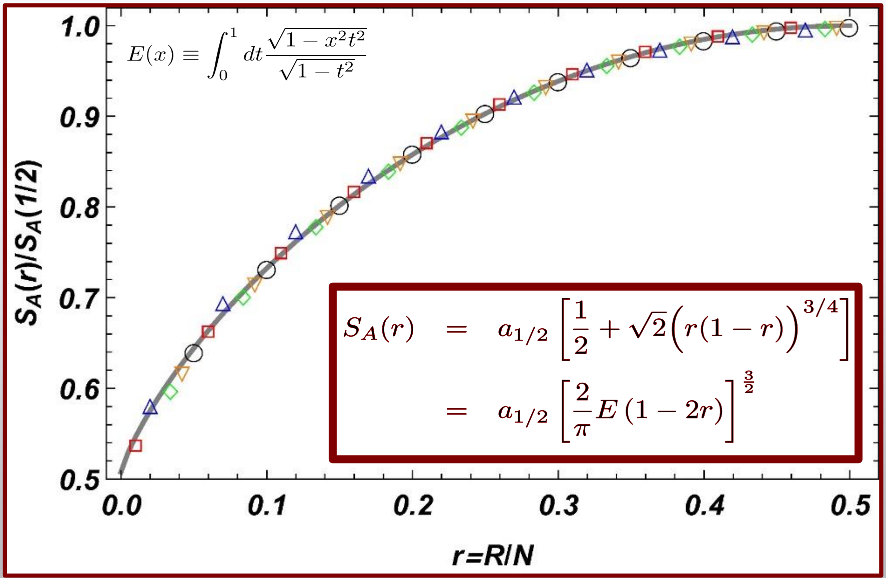
$$S_A(r) = a_{1/2}\left[\frac{1}{2}+\sqrt{2}\Big(r(1-r)\Big)^{3/4}\right] = a_{1/2}\left[\frac{2}{\pi}E\left(1-2r\right)\right]^{\frac{3}{2}}$$

Interpretation of the results

- Frustrated system has a "finite amount" of entanglement (like in a gapped case, it does not diverge)
- Correlations span the whole systems (like a critical phase, but not power-law correlation): technological applications?

$$C^{xx}(R) \equiv \langle \sigma_l^x \sigma_{l+R}^x \rangle = (-1)^R \left(1 - \frac{h^2}{J^2}\right)^{1/4} \left[1 + \frac{c^x(h)}{R^2}\left(\frac{h^2}{J^2}\right)^R\right]\left(1 - \frac{2R}{N}\right)$$





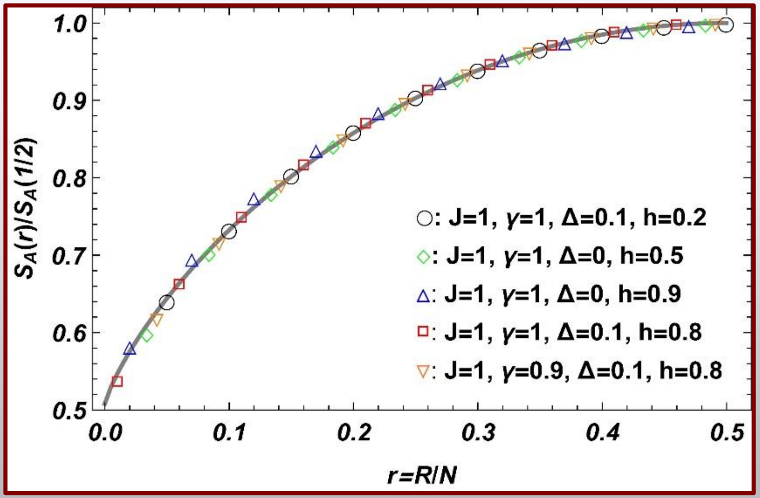
Conclusions

- Weak frustration + discrete quantum symmetry

  ⇒ New critical quantum phase

arXiv:1807.07055

- Both exponential and algebraic (not power-law!) correlations
- Entanglement Entropy to study phase properties
- Algebraic area law violation, but no divergence
- Robust against perturbation: very natural systems!
- Universal collapse with improved thermodynamic limit/scaling



Frustrated Ising Chain

$$H_{\text{Ising}} = \frac{1}{2} \sum_{l=1}^{2M+1} \left(\sigma_l^x \sigma_{l+1}^x - h\, \sigma_l^z \right)$$

- Jordan-Winger transformation turns spins into spinless fermions:

$$\sigma_l^+ = e^{i\pi \sum_{j<l} \psi_j^\dagger \psi_j} \psi_l \,, \qquad \sigma_l^z = 1 - 2\psi_l^\dagger \psi_l$$

- Separate Hilbert space according to parity:

$$H = \frac{1+P}{2} H^+ + \frac{1-P}{2} H^- \qquad P \equiv \prod_{l=1}^{N} \sigma_l^z \,, [H,P] = 0$$: Parity

- Rotation in Fourier space (Bogoliubov rotation) to get:

$$H^\pm = \sum_{q \in \Gamma_\pm} \varepsilon\left(\tfrac{2\pi}{N} q\right) \left\{ \chi_q^\dagger \chi_q - \frac{1}{2} \right\} , \qquad \Gamma_P = \left\{ n + \frac{1+P}{4} \right\}_{n=0}^{N-1}$$

$$\varepsilon(\alpha) \equiv \sqrt{(h+\cos\alpha)^2 + \sin^2\alpha} \,, \quad \varepsilon(0) = h+1 \,, \quad \varepsilon(\pi) = h-1$$

Even Parity

- Absolute GS in even parity sector (P=1): $\chi_q |GS\rangle = 0, \forall q \in \mathbb{N} + \frac{1}{2}$
- GS never degenerate!
- For h<1, occupation of π-mode lowers the energy

$$|GS\rangle \to E_0 = -\frac{1}{2} \sum_{q=0}^{2M} \varepsilon \left[\tfrac{2\pi}{N} \left(q + \tfrac{1}{2} \right) \right] + 1 - h$$

excited states with P=1 lie arbitrarily close in energy to GS, forming a band with quadratic dispersion:

$$\chi^\dagger_{M+1/2} \chi^\dagger_{p+1/2} |GS\rangle \to E_p = -\frac{1}{2} \sum_{q=0}^{2M} \varepsilon \left[\tfrac{2\pi}{N} \left(q + \tfrac{1}{2} \right) \right] + \varepsilon \left[\tfrac{2\pi}{N} \left(p + \tfrac{1}{2} \right) \right]$$

$$E(k) \simeq E_0 + \frac{1}{2} \left(\frac{h}{1-h} \right) (k - \pi)^2 + \dots$$

Odd Parity

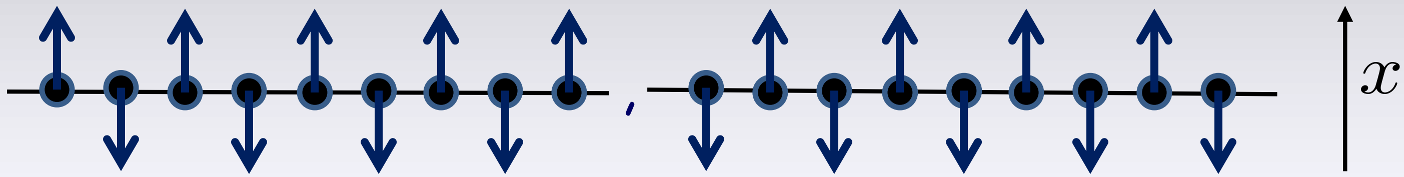
- Vacuum does not belong to odd parity sector (P=-1):
  $\chi_q |0'\rangle = 0, \forall q \in \mathbb{N}$
- Low energy states have one excitation: $\chi_p^\dagger |0'\rangle$
- Lowest energy state(s) for p=M/M+1:

$$\chi^\dagger_{M,M+1}|0'\rangle = |GS'\rangle \to E'_0 = -\frac{1}{2}\sum_{q=0}^{2M} \varepsilon\left[\tfrac{2\pi}{N}\left(q+\tfrac{1}{2}\right)\right] + \varepsilon\left(\pi \pm \frac{\pi}{N}\right)$$

  which is bigger than E_0, closing in polynomially!
- Low energy states also form a band above |GS'> with quadratic dispersion, intertwining with that of the even parity sector
- In total: Even + Odd produce a gapless band of 2N states

Physical picture for frustrated spectrum

- $H_{\text{Ising}} = \frac{1}{2}\sum_{l=1}^{2M+1}\left(\sigma_l^x\sigma_{l+1}^x - h\,\sigma_l^z\right)$: start from h=0
- GS (Neel):



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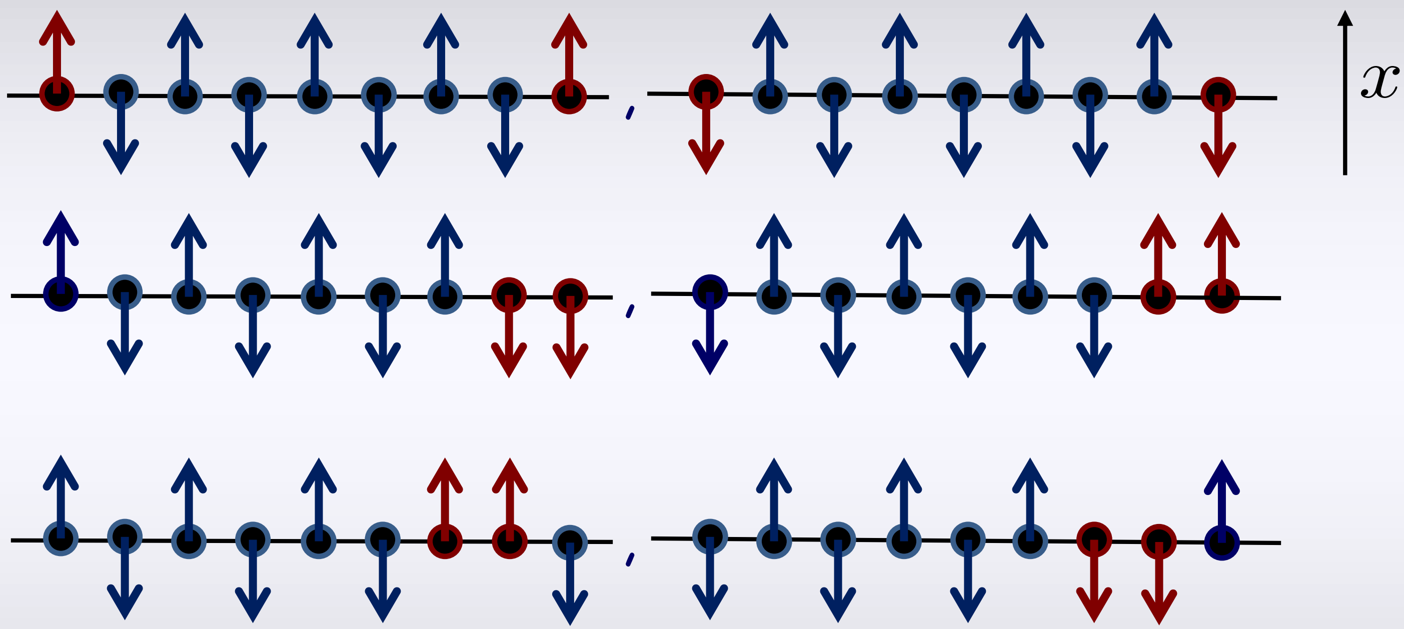

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2N States!

Physical picture for frustrated spectrum

$$H_{\rm Ising} = \frac{1}{2} \sum_{l=1}^{2M+1} \left(\sigma_l^x \sigma_{l+1}^x - h \, \sigma_l^z \right)$$

- At h=0: 2N-degenerate GS (2 x Neel with 1 domain wall) (compare to 2-degenerate in non-frustrated)
- Turn on h>0: it does not open a gap proportional to h !

  (because of Z_2 symmetry)
- Low-energy eigenstates are in continuity with plane wave superposition of domain walls

Correlation functions

$$H_{\text{Ising}} = \frac{1}{2} \sum_{l=1}^{2M+1} \left(\sigma_l^x \sigma_{l+1}^x - h\, \sigma_l^z \right)$$

$$\begin{aligned} \sigma_l^+ &= e^{i\pi \sum_{j<l} \psi_j^\dagger \psi_j}\; \psi_l \\ \sigma_l^z &= 1 - 2\psi_l^\dagger \psi_l \end{aligned}$$

- Correlation functions can be calculated starting from FF picture
- Introduce Majorana Fermions: $A_l \equiv \psi_l^\dagger + \psi_l \, , B_l \equiv i(\psi_l - \psi_l^\dagger)$

$$\langle A_l A_m \rangle = \langle B_l B_m \rangle = \delta_{l,m} \; ,$$

$$\langle A_{l+R} B_l \rangle = iG(R, J, h) \; ,$$

$$\nu(h,R) = \begin{cases} (-1)^R & h > 0 \\ -1 & h < 0 \end{cases}$$

$$G(R,1,h) = -G(R,-1,-h) + \frac{2}{N} \nu(h,R)$$

- Compared to the standard case, the frustrated GS correlators have 1 additional contribution as for 1 (π-)mode excited state

Local and Quasi-Local Correlators

$$\langle A_{l+R} B_l \rangle = iG(R,J,h)\,, \quad G(R,1,h) = -G(R,-1,-h) + \frac{2}{N}\nu(h,R)$$

- Local Correlation functions have a finite number of Majoranas

$$\langle \sigma^z_{l+R}\, \sigma^z_l \rangle = \langle A_{l+R} B_{l+R} A_l B_l \rangle$$
$$= m_z^2 - \frac{c_1^z(h)}{R^2}\left(\frac{h^2}{J^2}\right)^R + \frac{4m_z}{N}\left[1 + c_2^z(h)(-1)^R \left|\frac{h}{J}\right|^R\right]$$

- Quasi-local ones have # of Majorana growing with distances

$$\langle \sigma^x_{l+R}\, \sigma^x_l \rangle = \langle B_{l+R} A_{l+R-1} B_{l+R-1} \ldots A_{l-1} B_{l-1} A_l \rangle$$
$$= (-1)^R \left(1 - \frac{h^2}{J^2}\right)^{1/4}\left[1 + \frac{c^x(h)}{R^2}\left(\frac{h^2}{J^2}\right)^R\right]\left(1 - \frac{2R}{N}\right)$$

- The $\frac{1}{N}$ contributions add up to be finite in improved therm. limit
- Same for Entanglement Entropy

Conclusions & Outlook

- Weak frustration + discrete quantum symmetry

  ⇒ New critical quantum phase   **arXiv:1807.07055**

- Both exponential and algebraic (not power-law!) correlations
- Entanglement Entropy to study phase properties
- Algebraic area law violation, but no divergence
- Robust against perturbation: very natural systems!
- Universal collapse with improved thermodynamic limit/scaling
- Other symmetries? Stronger frustration? Higher D?
- Spontaneous breaking of translational symmetry?
- Origin/description of universality?

Thank you!

Quantifying Frustration

- First quantify "quantum" frustration:
  - Write Hamiltonian as sum of local terms
  - Find GS of H and of all the H_j separately and construct projectors

$$H = \sum_j H_j \longrightarrow \begin{cases} H \to \Pi \equiv |GS\rangle\langle GS| \\ H_j \to \Pi_j \equiv \sum_\alpha |GS_j^\alpha\rangle\langle GS_j^\alpha| \end{cases}$$

  - Measure Hilbert-Schmidt distance between them

$$F_j \equiv Tr\left(\Pi_j \Pi\right)$$

  - If translational invariance: $F \equiv F_j$

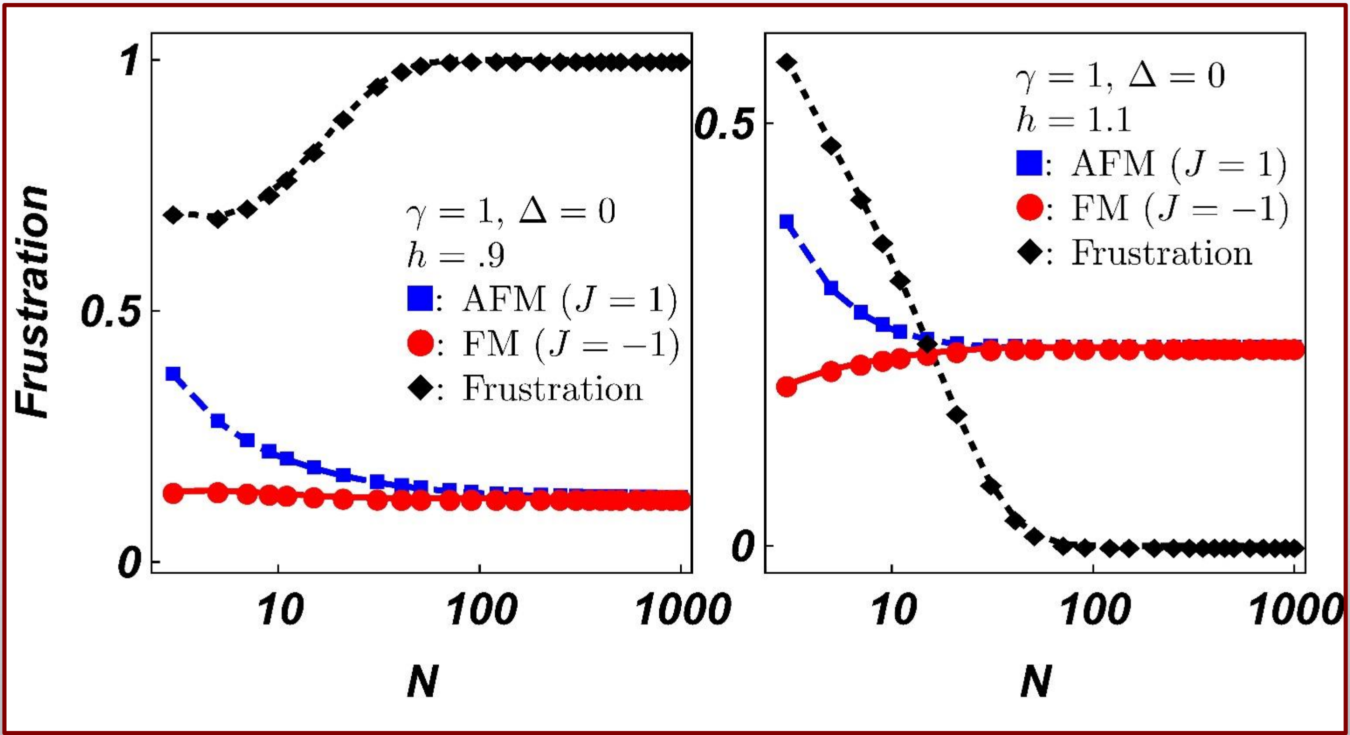
(Giampaolo et al. PRL '11)

Quantifying Frustration

- Consider frustration of Ferromagnetic (J=-1) $F(J=-1)$

  and AFM system (J=1) $F(J=1)$

- Geometrical frustration: $g_F = \sum_{j=1}^{N} \left[F(J=1) - F(J-1) \right]$



Frustration does not increase with the system's length and vanishes in non-frustrated phase

Approaching h→1

- CFT behavior up to (non-frustrated) correlation length scale & deviation beyond it