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Next-to-leading order corrections to deeply virtual production of pseudoscalar mesons

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ABSTRACT

We complete the perturbative next-to-leading order corrections to the hard scattering amplitudes of deeply virtual meson leptoproduction processes at leading twist-two level by presenting the results for the production of flavor singlet pseudoscalar mesons. The new results are given in the common momentum fraction representation and in terms of conformal moments. We also comment on the flavor singlet results for deeply virtual vector meson production.

1. Much experimental effort has been spent during the last decade and will be spent in future by the JLAB and COMPASS collaborations to measure exclusive leptoproduction processes in the deeply virtual regime in which the virtuality of the exchanged photon is considered as large. The phenomenological goal of such measurements is to access generalized parton distributions (GPDs) [1–3], which encode partonic information that are complementary to parton distribution functions or hadronic distribution amplitudes, see, e.g., Refs. [4,5]. These process independent (universal) quantities are related to observables by convolution formulae where the hard-scattering amplitude is perturbatively calculable in leading twist-two approximation. Examples of such observables are the transverse cross section of deeply virtual Compton scattering (DVCS) and the longitudinal cross sections for the deeply virtual meson production (DVMP) of pseudo scalar and longitudinally polarized vector mesons. They are experimentally accessible in exclusive lepton–nucleon reaction $l(k)N(P_1) \to l(k')N(P_2)M(q_2)$ in which the virtual one-photon exchange contribution with four momentum $q_1 = k - k' = P_2 + q_2 - P_1$ is the dominant one. To utilize the factorization theorem [6], it is required to address the longitudinally polarized differential cross section [7–9], e.g., in the notation of Ref. [10] it is given as transition form factors (TFFs) that appear in a form factor decomposition of the amplitude. For example, in the case of pseudo scalar meson production

$$\epsilon_1^{\mu}(0)\langle MN|j_{\mu}|N\rangle = e\,\overline{u}(P_2,s_2)\left[\not{m}\gamma_5\,\widetilde{\mathcal{H}}_M + \gamma_5\frac{m\cdot(P_2-P_1)}{2M_N}\,\widetilde{\mathcal{E}}_M\right]u(P_1,s_1)\,, \tag{1}$$

where the vector m^{μ} might be equated to $(q_1+q_2)^{\mu}/(P_1+P_2)\cdot(q_1+q_2)$ and e is the unit electrical charge. The TFFs, generally denoted as $\mathcal{F}_M(x_B,t,\mathcal{Q}^2)$, depend on the Bjorken variable $x_B = \mathcal{Q}^2/2P_1\cdot q_1$, the momentum transfer square $t = (P_2-P_1)^2$, and the photon virtuality square $\mathcal{Q}^2 = -q_1^2$. The leading order formalism for different channels of such processes, depicted in Fig. 1, were worked out for some time [11–13,8,7,9,14–16].

For setting up a robust GPD phenomenology there is necessity to address perturbative higher-order as well as higher-twist corrections. The former ones can be calculated according to the state of the art while the evaluation of higher twist corrections is a problematic task, pioneered for DVCS by V. Braun and A. Manashov [17,18]. Note that a fixed order calculation induces a residual scale dependence that is maximal in the leading-order (LO) approximation. To reduce this dependence it is necessary to take higher order corrections into account. DVMP for flavor non-singlet pseudo-scalar mesons and longitudinally polarized vector mesons were already worked out at next-to-leading order (NLO) level in Refs. [19] and [20], respectively. The NLO corrections of the former ones might be obtained by analytic continuation

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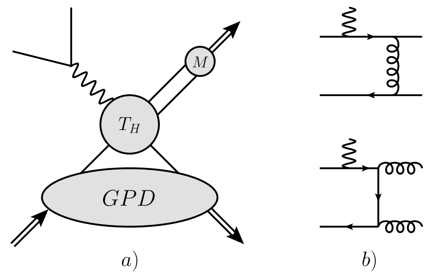

Fig. 1. a) Factorization of the DVMP amplitude for a longitudinally polarized photon exchange in GPD, meson distribution amplitude, and hard scattering part T_H. In b) representative LO diagrams for the hard scattering amplitude are shown for the quark–quark (up) and quark–gluon (down) channel.

from the existing result of the pion form factor, see e.g. Ref. [21], while the latter one requires a diagrammatic calculation of hard partonic processes.

In this study we address the NLO corrections for DVMP of flavor singlet pseudoscalar mesons. We calculate NLO corrections to the corresponding partonic processes in the quark–quark channel $\gamma_L^* q \to (q\bar{q})q$ and in the quark–gluon channel $\gamma_L^* q \to (gg)q$, which was found to vanish at LO [22,23]. That completes the compendium of NLO results for DVMP at twist-two level. We present our new results also in terms of conformal moments, which allow to set up efficient GPD models and numerical code for the analysis of experimental data. In presenting our results we follow closely the notation of our previous work [10] and refer there for common definitions.

2. According to the flavor content of the meson, the TFFs (1) might be decomposed in partonic TFFs. In particular, for the flavor octet and singlet components of the η meson,

$$|\eta^{(8)}\rangle = \frac{1}{\sqrt{6}}\Big(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle\Big), \quad |\eta^{(0)}\rangle = \frac{1}{\sqrt{3}}\Big(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\Big), \tag{2}$$

we utilize the decompositions

$$\mathcal{F}_{\eta^{(8)}} = \frac{2}{3\sqrt{6}}\mathcal{F}^{u^{(-)}}_{\eta^{(8)}} - \frac{1}{3\sqrt{6}}\mathcal{F}^{d^{(-)}}_{\eta^{(8)}} + \frac{2}{3\sqrt{6}}\mathcal{F}^{s^{(-)}}_{\eta^{(8)}}, \quad \mathcal{F}_{\eta^{(0)}} = \frac{2}{3\sqrt{3}}\mathcal{F}^{u^{(-)}}_{\eta^{(0)}} - \frac{1}{3\sqrt{3}}\mathcal{F}^{d^{(-)}}_{\eta^{(0)}} - \frac{1}{3\sqrt{3}}\mathcal{F}^{s^{(-)}}_{\eta^{(0)}} \tag{3}$$

where $\mathcal{F} \in \{\widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$ introduced in (1), and the charge factors are included in (3). These TFFs allow to address the corresponding charge odd quark GPDs

$$F^{q^{(-)}}(x,\eta,t,\mu^2) = F^q(x,\eta,t,\mu^2) - F^q(-x,\eta,t,\mu^2) \quad \text{for} \quad F \in \{\widetilde{H}, \widetilde{E}\}, \tag{4}$$

which depend on the momentum fraction x, the skewness η, t, and the factorization scale μ. They are antisymmetric in x and are thus assigned with a signature factor $\sigma = +1$ ($F^{q,(\sigma)}(-x,\eta,t) = -\sigma F^{q,(\sigma)}(x,\eta,t)$). Our definitions, see, e.g., appendix A1 of Ref. [10], are such that in the forward limit $\widetilde{H}^{q^{(-)}}$ reduces to the difference of standard polarized quark (Δq) and anti-quark ($\Delta\bar{q}$) distributions: $\widetilde{H}^{q^{(-)}}(x,\eta=0,t=0,\mu^2) = \Delta q(x,\mu^2) - \Delta\bar{q}(x,\mu^2)$ for $x > 0$. The $\widetilde{H}^{q^{(-)}}$ and $\widetilde{E}^{q^{(-)}}$ GPDs satisfy the evolution equation

$$\mu^2 \frac{d}{d\mu^2} F^{q^{(-)}}(x,\xi,t,\mu^2) = \int_{-1}^{1} \frac{dy}{2\xi}\, {}^{+}V\left(\frac{x+\xi}{2\xi}, \frac{y+\xi}{2\xi}, \alpha_s(\mu)\right) F^{q^{(-)}}(y,\xi,t,\mu^2) \text{ for } F \in \{\widetilde{H}, \widetilde{E}\}. \tag{5}$$

The kernel ${}^{+}V = \frac{\alpha_s}{2\pi} V^{(0)} + \frac{\alpha_s^2}{(2\pi)^2}\, {}^{+}V^{(1)} + O(\alpha_s^3)$ is in LO approximation given by

$$V^{(0)}(u,v) = C_F\,\theta\left(1 - \frac{u}{v}\right)\theta\left(\frac{u}{v}\right)\text{sign}(v)\frac{u}{v}\left[1 + \frac{1}{(v-u)_+}\right] + \frac{3C_F}{2}\delta(u-v) + \left\{\begin{matrix} u \to \bar{u} \\ v \to \bar{v} \end{matrix}\right\}, \tag{6}$$

where $C_F = 4/3$, $\bar{u} = 1-u$, and $\bar{v} = 1-v$. The NLO kernel can be found in Eq. (177) of Ref. [24], denoted there as ${}^{QQ}V^{(1)+}$.

The formation of the meson is described by a distribution amplitude (DA), see Fig. 1. In the DV$\eta^{(0)}$P process it belongs to the flavor singlet sector and might be presented by a vector

$$\boldsymbol{\varphi}_{\eta^{(0)}}(v,\mu^2) = \begin{pmatrix} \varphi^{\Sigma}_{\eta^{(0)}}(v,\mu^2) \\ \varphi^{G}_{\eta^{(0)}}(v,\mu^2) \end{pmatrix}, \qquad \varphi^{\Sigma}_{\eta^{(0)}}(\bar{v}) = \varphi^{\Sigma}_{\eta^{(0)}}(v), \quad \varphi^{G}_{\eta^{(0)}}(\bar{v}) = -\varphi^{G}_{\eta^{(0)}}(v) \tag{7}$$

that contains the quark and gluon component, depending on the momentum fraction v and the factorization scale μ. The quark component is normalized as $\int_0^1 dv \varphi^{\Sigma}_{\eta^{(0)}}(v,\mu^2) = 1$. More precisely, the entries of the flavor singlet meson DA (7) are defined by the following expectation values

$$i f_{\eta^{(0)}} \varphi^{\Sigma}_{\eta^{(0)}}(v,\mu^2) = \int \frac{d\kappa}{\pi} e^{i(v-\bar{v})(p\cdot n)\kappa} \sum_{q=u,d,s} \langle 0|\bar{q}(-\kappa n) n\cdot\gamma\gamma^5 q(\kappa n)|\eta^{(0)}(p)\rangle_{(\mu^2)} \tag{8}$$

$$i f_{\eta^{(0)}} \varphi^{G}_{\eta^{(0)}}(v,\mu^2) = \frac{2}{p\cdot n}\int \frac{d\kappa}{\pi} e^{i(v-\bar{v})(p\cdot n)\kappa} \langle 0|G^{+\mu}(-\kappa n) i\epsilon^{\perp}_{\mu\nu} G^{\nu+}(\kappa n)|\eta^{(0)}(p)\rangle_{(\mu^2)}, \tag{9}$$

where $f_{\eta^{(0)}}$ is the decay constant. Here $\epsilon^{\perp}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} n^{*\alpha} n^{\beta}$ with $\epsilon^{0123} = 1$ and n^{μ} and $n^{*\mu}$ being light-like vectors satisfying $n \cdot n^* = 1$ and $a^+ \equiv a \cdot n$. The evolution of the DA is governed by the equation

$$\mu^2 \frac{d}{d\mu^2} \boldsymbol{\varphi}_{\eta^{(0)}}(u, \mu^2) = \boldsymbol{V}(u, v|\alpha_s(\mu)) \overset{v}{\otimes} \boldsymbol{\varphi}_{\eta^{(0)}}(v, \mu^2)\,, \tag{10}$$

where the matrix valued LO expression of the flavor singlet kernel is [22]

$$\boldsymbol{V}(u, v|\alpha_s) = \frac{\alpha_s}{2\pi} \begin{pmatrix} {}^{\Sigma\Sigma}V^{(0)} & {}^{\Sigma\mathrm{G}}V^{(0)}/2 \\ 2\,{}^{\mathrm{G}\Sigma}V^{(0)} & {}^{\mathrm{GG}}V^{(0)} \end{pmatrix}(u, v) + O(\alpha_s^2)\,, \tag{11a}$$

$${}^{\mathrm{AB}}V^{(0)}(u, v) = \theta(v-u)\, {}^{\mathrm{AB}}v^{(0)}(u, v) \pm \begin{Bmatrix} u \to \bar{u} \\ v \to \bar{v} \end{Bmatrix} \text{ for } \begin{cases} \mathrm{A} = \mathrm{B} \\ \mathrm{A} \neq \mathrm{B}. \end{cases}$$

The quark–quark entry ${}^{\Sigma\Sigma}V^{(0)}$ is given by the non-singlet kernel (6) and the remaining entries are

$${}^{\Sigma\mathrm{G}}v^{(0)}(u, v) = -n_f \frac{u}{v^2}\,, \qquad {}^{\mathrm{G}\Sigma}v^{(0)}(u, v) = C_{\mathrm{F}} \frac{u^2}{v}\,, \tag{11b}$$

$${}^{\mathrm{GG}}v^{(0)}(u, v) = C_{\mathrm{A}} \frac{u^2}{v^2}\left[2 + \frac{1}{(v-u)_+}\right] - \frac{\beta_0}{2}\delta(u-v)\,, \tag{11c}$$

where $\beta_0 = 2/3 n_f - 11 C_{\mathrm{A}}/3$ and $C_{\mathrm{A}} = 3$, and n_f is the number of active quarks. The NLO corrections to the evolution kernels are presented in Eqs. (177)–(181) of Ref. [25].

The partonic TFFs (3) are predicted to leading twist-two accuracy by the convolution formula

$$\mathcal{F}^{q^{(-)}}_{\eta^{(0)}}(x_{\mathrm{B}}, t, \mathcal{Q}^2) \overset{\text{tw}-2}{=} \frac{4\pi C_F f_{\eta^{(0)}}}{N_c \mathcal{Q}} \int_{-1}^{1} \frac{dx}{2\xi} \int_{0}^{1} dv\, F^{q^{(-)}}(x, \xi, t, \mu_{\mathrm{F}}^2) \times \boldsymbol{T}\left(\frac{\xi + x - i\epsilon}{2(\xi - i\epsilon)}, v \middle| \alpha_s(\mu_{\mathrm{R}}), \frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2}, \frac{\mathcal{Q}^2}{\mu_{\varphi}^2}, \frac{\mathcal{Q}^2}{\mu_{\mathrm{R}}^2}\right) \boldsymbol{\varphi}_{\eta^{(0)}}(v, \mu_{\varphi}^2), \tag{12}$$

where $\xi \simeq x_{\mathrm{B}}/(2 - x_{\mathrm{B}})$, the number of colors is $N_c = 3$, and $\boldsymbol{T}(u, v|\cdots) = \alpha_s\left(\frac{1}{\bar{u}\,\bar{v}}, 0\right) + O(\alpha_s^2)$, i.e., the gluonic component vanishes in LO approximation. Note that the factor 4π in the overall normalization was reshuffled in [10]. Here μ_F and μ_φ represent the scales at which the collinear singularities and, hence, soft and hard physics, are factorized. Although it is often taken $\mu_{\mathrm{F}}^2 = \mu_{\varphi}^2$ and even equal to Q^2, we choose to present the full expression, and to distinguish the scale at which the GPD and the meson DA are factorized in order to be able to keep track of the different factorization logarithms. The scale μ_R represents the renormalization scale of the coupling constant connected to the renormalization of the UV singularities. The truncation of the perturbative series in $\alpha_s(\mu_R)$ at finite order introduces the residual dependence of $\boldsymbol{T}$ and thus $\mathcal{F}^{q^{(-)}}_{\eta^{(0)}}$ on μ_R. As mentioned in the introduction, this dependence is the strongest at LO, while at NLO the additional renormalization logarithms stabilize this dependence. Still various scale settings can be used that employ particular physical or just mathematical properties of the expansion and these were investigated for the related process (same parton subprocesses in collinear limit), i.e., meson electromagnetic form factor (see [21] and references therein), as well as for DV vector meson production in [10].[1]

Let us add that the results for DV$\eta^{(8)}$P TFFs formally follows from (12) by reduction to the flavor non-singlet case, i.e., we set $\boldsymbol{\varphi}_{\eta^{(0)}}(v, \mu_{\varphi}^2) \to \varphi_{\eta^{(8)}}(v, \mu_{\varphi}^2)$ and $\boldsymbol{T} \to {}^{+}T$, where

$${}^{+}T(u, v|\cdots) = \alpha_s(\mu_{\mathrm{R}}) T^{(0)}(u, v) + \frac{\alpha_s^2(\mu_{\mathrm{R}})}{2\pi} {}^{+}T^{(1)}\left(u, v \middle| \frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2}, \frac{\mathcal{Q}^2}{\mu_{\varphi}^2}, \frac{\mathcal{Q}^2}{\mu_{\mathrm{R}}^2}\right) + O(\alpha_s^3)\,, \tag{13}$$

with $T^{(0)}(u, v) = 1/\bar{u}\,\bar{v}$. The NLO expression for ${}^{+}T^{(1)}$ is presented in Eqs. (4.39) and (4.41) of Ref. [10], where the signature factor is $\sigma = +1$.

3. The hard scattering amplitude of the partonic processes $\gamma_L^* q(p_1) \to [q(v)\bar{q}(\bar{v})]q(p_2)$ and $\gamma_L^* q(p_1) \to [g(v)g(\bar{v})]q(p_2)$ are calculated in the collinear approximation, where the incoming [outgoing] quark GPD momentum is $p_1 = (x+\xi)P/2$ $[p_2 = (x-\xi)P/2]$ with $P = P_1 + P_2$ and the quark [anti-quark] momentum of the meson is vq_2 $[\bar{v}q_2]$. In the calculation we employed dimensional regularization together with the γ^5-prescription of 't Hooft–Veltman, equivalent to Breitenlohner–Maison prescription [26,27]. In this HVBM scheme one renders a mathematically consistent result. Based on the one-loop Feynman integral reduction formalism [29], the regularized hard scattering amplitude in D dimensional space

$$\overline{\boldsymbol{T}}(u, v|\alpha_s, \cdots) = \alpha_s \overline{\boldsymbol{T}}^{(0)}(u, v) + \frac{\alpha_s^2}{2\pi} \overline{\boldsymbol{T}}^{(1)}(u, v|\cdots) \quad \text{with} \quad \overline{\boldsymbol{T}}^{(0)} = \left(\frac{6-D}{2}\frac{1}{\bar{u}\,\bar{v}}, 0\right) \tag{14}$$

[1] The thorough analysis of the scale dependence for this process we postpone for future phenomenological analysis.

was calculated and cross checked at one loop level by two independently written codes. In one Feynman diagrams were implemented by hand and in the other generated with the FeynArt program [30]. The collinear singularities were regularized by taking $D = 4 + 2\epsilon$ and they were absorbed in the dressed meson DA and GPD via the *modified* minimal subtraction scheme. Note that due to vanishing LO, the NLO gluon and pure singlet quark contributions are ultraviolet finite. The dressed hard scattering amplitude is finally obtained by taking the limit

$$\boldsymbol{T}(u,v|\alpha_s,\cdots) = \lim_{D\to 4}\int_0^1 du' \int_0^1 dv'\, Z(u',u|\alpha_s)\overline{\boldsymbol{T}}(u',v'|\alpha_s,\cdots)\boldsymbol{Z}(v',v|\alpha_s)\,,$$

where the Z-factors to one loop order accuracy, expressed by the kernels (6) and (11), are

$$Z(u,v) = \delta(u-v) + \frac{2\left(4\pi e^{-\gamma_E}\right)^{\frac{4-D}{2}}}{4-D}\frac{\alpha_s}{2\pi}V(u,v) + O(\alpha_s^2)\,, \tag{15a}$$

$$\boldsymbol{Z}(u,v) = \begin{pmatrix} \delta(u-v) & 0 \\ 0 & \delta(u-v)\end{pmatrix} + \frac{2\left(4\pi e^{-\gamma_E}\right)^{\frac{4-D}{2}}}{4-D}\frac{\alpha_s}{2\pi}\boldsymbol{V}^{(0)}(u,v) + O(\alpha_s^2)\,, \tag{15b}$$

with renormalized α_s and $\gamma_E = 0.5772\ldots$ is the Euler–Mascheroni constant.

To transform from the HVBM scheme to the common adopted one, requiring that the spin independent and spin dependent evolution kernels in the flavor non-singlet case are the same, in addition to the minimal subtraction a finite subtraction should be performed with the z-factor

$$\boldsymbol{z}^{\rm HVBM}(u,v) = \begin{pmatrix} \delta(u-v) & 0 \\ 0 & \delta(u-v)\end{pmatrix} + \frac{\alpha_s}{2\pi}\begin{pmatrix} 4C_F V^a(u,v) & 0 \\ 0 & 0\end{pmatrix} + O(\alpha_s^2)\,, \tag{16}$$

where $V^a(u,v) = \theta(v-u)\frac{u}{v} + \theta(u-v)\frac{\overline{u}}{\overline{v}}$. This scheme transformation does not affect the quark–gluon channel and contributes to the flavor non-singlet part, which is already known [28]. Note that this is entirely in agreement with the definition used in deep inelastic scattering, see, e.g., Eqs. (33)–(39) and (40) in Ref. [31], where the correspondence $4C_F V^a(u,v) \leftrightarrow 4C_F(1-z)$ holds.

The NLO corrections to the hard scattering amplitude of DV$\eta^{(0)}$P,

$$\boldsymbol{T}(u,v|\cdots) = \left({}^{\Sigma}T(u,v|\cdots), \frac{n_f}{C_F}{}^{G}T(u,v|\cdots)\right), \; {}^{\Sigma}T(\cdots) = T(\cdots) + n_f{}^{pS}T(\cdots), \tag{17a}$$

contain besides T, see Eq. (13), the pure singlet (pS) quark and the gluonic (G) entries,

$${}^{pS}T(u,v|\cdots) = \frac{\alpha_s^2(\mu_R)}{2\pi}{}^{pS}T^{(1)}(u,v) + O(\alpha_s^3)\,, \tag{17b}$$

$${}^{G}T(u,v|\cdots) = \frac{\alpha_s^2(\mu_R)}{2\pi}\left[C_F{}^{G}T^{(1,F)}\left(u,v\middle|\frac{\mathcal{Q}^2}{\mu_\varphi^2}\right) + C_A{}^{G}T^{(1,A)}(u,v)\right] + O(\alpha_s^3)\,. \tag{17c}$$

Here, we exploit symmetry so that our NLO expressions have only poles at $u = 1$ and $[1,\infty]$ cuts on the positive real axis in the complex u-plane:

$${}^{pS}T^{(1)} = \frac{\mathrm{Li}_2(v)-\zeta_2}{\overline{u}\,\overline{v}} - \frac{\ln\overline{v} + \mathrm{Li}_2(v)}{\overline{u}\,v} - \left[\frac{\vec{\partial}}{\partial v}v - 2\right]\left[\frac{L(u,v)}{u(u-v)}\right]^{\rm sub} - \left[\frac{L(u,v)}{u(u-v)\overline{v}}\right]^{\rm sub} \tag{18a}$$

$${}^{G}T^{(1),F} = \frac{\ln\overline{v}}{2\overline{u}\,v^2}\left[\ln\frac{\mathcal{Q}^2}{\mu_\varphi^2} - \frac{3}{2} + \frac{1}{2}\ln\overline{v}\right] - \frac{\ln\overline{u} - u}{2u\overline{u}}\frac{\ln\overline{v}}{v\overline{v}} - \frac{\mathrm{Li}_2(u)}{2u\overline{v}} - \frac{\mathrm{Li}_2(u)-\zeta_2}{2\overline{u}\,\overline{v}} + \frac{1}{4}\left[\frac{\vec{\partial}^2}{\partial v^2}v\overline{v} + 2\right]\left[\frac{L(u,v)}{u(u-v)\overline{v}}\right]^{\rm sub}, \tag{18b}$$

$${}^{G}T^{(1),A} = \frac{\ln\overline{u}}{4u\overline{u}}\frac{\ln\overline{v}}{v^2\overline{v}} + \frac{\mathrm{Li}_2(u)}{2u\overline{u}\,\overline{v}} - \frac{\overline{v}-v}{4\overline{u}\,v\overline{v}}\frac{\ln\overline{v}}{v} + \frac{(\overline{v}-v)\left[\mathrm{Li}_2(v)-\zeta_2\right]}{4\overline{u}\,\overline{v}^2} - \frac{(\overline{v}-v)\mathrm{Li}_2(v)}{4\overline{u}\,v^2} - \frac{\overline{v}-v}{4}\frac{\vec{\partial}}{\partial v}\left[\frac{L(u,v)}{u(u-v)\overline{v}}\right]^{\rm sub}, \tag{18c}$$

where $\zeta_2 = \pi^2/6$. The non-separable terms are expressed by end-point subtracted building blocks

$$\left[\frac{L(u,v)}{u(u-v)}\right]^{\rm sub} \equiv \frac{L(u,v)}{u(u-v)} + \frac{L(u=0,v)}{uv}\,, \tag{19a}$$

$$\left[\frac{L(u,v)}{u(u-v)\overline{v}}\right]^{\rm sub} \equiv \frac{L(u,v)}{u(u-v)\overline{v}} + \frac{L(u,v=1)}{u\overline{u}\,\overline{v}} + \frac{L(u=0,v)}{uv\overline{v}} - \frac{L(u=0,v=1)}{u\overline{v}}\,, \tag{19b}$$

with $L(u,v) = \mathrm{Li}_2(u) - \mathrm{Li}_2(v) + \ln\overline{u}\ln v - \ln\overline{v}\ln v$.

The subtraction of end-point singularities in the non-separable terms (19) ensures that they provide numerically small contributions. In the pure singlet quark result the most singular contribution is given by the pole $1/\overline{u}$ at $u = 1$. Its residue is a rather harmless function in v that contain no end-point singularities. Thus, these perturbative corrections are relatively small. Contrarily, in the quark–gluon channel the most singular term $(\ln\overline{u})/\overline{u}\,(\ln\overline{v})/\overline{v}$, contained in the second and first term of Eq. (18b) and (18c), respectively, can potentially provide large corrections. Using $C_A/2 - C_F = 1/(2N_c)$, one realizes that the most singular term is numerically suppressed in the large N_c limit.

Nevertheless, besides a $(\ln\overline{u})/\overline{u}(\ln\overline{v})/2v^2 \sim (\ln\overline{u})/2\overline{u}\,\overline{v}$ term, the net result has also $1/\overline{u}$ pole contributions. The most singular terms can be collected into

$$\frac{{}^{\mathrm{G}}T^{(1)}}{C_{\mathrm{F}}} \sim \frac{\alpha_s^2}{2\pi}\left[\ln\overline{u}+\ln\overline{v}+2\zeta_2-\frac{1}{2}+\ln\frac{\mathcal{Q}^2}{\mu_\varphi^2}+\frac{1}{N_c^2-1}\left\{\ln\overline{u}\,\ln\overline{v}+\ln\overline{u}+1+\zeta_2\right\}\right]\frac{1}{2\overline{u}\,\overline{v}}$$

and might provide in dependence on the gluonic $\eta^{(0)}$ DA a moderate or sizeable correction.

We also calculated the flavor singlet hard scattering amplitude for longitudinal vector meson production, e.g., for $\mathrm{DV}\rho_L^{(0)}\mathrm{P}$. The results from Ref. [20] are obtained making an average over two transverse gluon polarization states. However, it is standard PDF convention to take an average over $D-2$ transverse polarizations available to gluons in D dimensions. Thus, the dimensional regularized LO hard scattering amplitude changes:

$$\overline{\boldsymbol{T}}^{(0)}=\left(\frac{D-2}{2}\frac{1}{n_f}\frac{1}{\overline{u}\,\overline{v}},\frac{D-2}{2}\frac{1}{C_{\mathrm{F}}\xi}\frac{1}{\overline{u}\,\overline{v}}\right)\Rightarrow\overline{\boldsymbol{T}}^{(0)}=\left(\frac{D-2}{2}\frac{1}{n_f}\frac{1}{\overline{u}\,\overline{v}},\frac{1}{C_{\mathrm{F}}\xi}\frac{1}{\overline{u}\,\overline{v}}\right)$$

and by the same overall factor $2/(D-2)$ in the gluon entry at NLO (and beyond). To ensure that the forward limit of the gluon GPD provides the common definition of the PDF, used in the phenomenology of (semi-)inclusive measurements, the original results [20] should be corrected in the pure quark singlet [32] and the gluon sector by an additional NLO term:

$$\boldsymbol{T}^{(1)}(u,v|\cdots)\Rightarrow\boldsymbol{T}^{(1)}(u,v|\cdots)+\frac{1}{\overline{v}}\int_0^1\frac{du'}{\overline{u}'}\left(\frac{2}{C_{\mathrm{F}}}{}^{\mathrm{G}\Sigma}V^{(0)}(u',u),-\frac{1}{2n_f\xi}{}^{\Sigma\mathrm{G}}V^{(0)}(u',u)\right). \tag{20}$$

This change can be easily taken into account in the formula set of Ref. [10] by the replacement

$$\ln\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2}\Rightarrow\ln\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2}+1\quad\text{and}\quad\ln\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2}\Rightarrow\ln\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2}-1$$

in ${}^{\mathrm{pS}}T^{(1)}$ [see Eqs. (4.46a), (4.47a), and (4.48a) of Ref. [10]] and in ${}^{\mathrm{G}}T^{(1,F)}$ [see Eqs. (4.51b), (4.52b), and (4.53b) of Ref. [10]], respectively. A more detailed account of here summarized NLO calculations, as well as their application to other channels is in preparation [34].

4. For the GPDs we might employ a Mellin–Barnes integral representation (for further details see Sec. 3.3 of Ref. [10]) and for the $\eta^{(0)}$ DA an integral conformal partial wave expansion. In such an expansion the evolution can be explicitly included in the TFFs (12), which read now as

$$\mathcal{F}_{\eta^{(0)}}^{q^{(-)}}(x_{\mathrm{B}},t,\mathcal{Q}^2)\overset{\mathrm{tw-2}}{=}\frac{4\pi C_F f_{\eta^{(0)}}}{N_c\mathcal{Q}}\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dj\,\xi^{-j-1}\left[i+\tan\left(\frac{\pi j}{2}\right)\right]\left[\sum_{\substack{k=0\\\text{even}}}^{\infty}\mathrm{T}_{jk}\left(\mathcal{Q}^2,\mathcal{Q}_0^2\right)\boldsymbol{\varphi}_{\eta^{(0)},k}(\mathcal{Q}_0^2)\right]F_j^{q^{(-)}}(\xi,t,\mathcal{Q}_0^2). \tag{21}$$

The conformal GPD moments $F_j^{q^{(-)}}(\xi,t,\mathcal{Q}_0^2)$ at the input scale $\mathcal{Q}_0$ coincide for integer $j=n$ with

$$F_n^{q^{(-)}}(\eta,t,\mathcal{Q}_0^2)=\frac{\Gamma\left(\frac{3}{2}\right)\Gamma(n+1)}{2^n\Gamma\left(n+\frac{3}{2}\right)}\frac{1}{2}\int_{-1}^{1}dx\,\eta^n C_n^{3/2}\left(\frac{x}{\eta}\right)F^{q^{(-)}}(x,\eta,t,\mathcal{Q}_0^2), \tag{22}$$

and those of the $\eta^{(0)}$-DA (7) are collected in the vector

$$\boldsymbol{\varphi}_{\eta^{(0)},k}(\mathcal{Q}_0^2)=\begin{pmatrix}\varphi^{\Sigma}_{\eta^{(0)},k}(\mathcal{Q}_0^2)\\ \varphi^{\mathrm{G}}_{\eta^{(0)},k}(\mathcal{Q}_0^2)\end{pmatrix}=\int_0^1 dv\begin{pmatrix}\frac{2(2k+3)}{3(k+1)_2}C_k^{3/2}(v-\overline{v})\varphi^{\Sigma}_{\eta^{(0)}}(v,\mathcal{Q}_0^2)\\ \frac{4(2k+3)}{(k)_4}C_{k-1}^{5/2}(v-\overline{v})\varphi^{\mathrm{G}}_{\eta^{(0)}}(v,\mathcal{Q}_0^2)\end{pmatrix}, \tag{23}$$

where $(k)_m=k\cdots(k+m-1)$ is the Pochhammer symbol and C_k^{ν} are the Gegenbauer polynomials of order k and index ν. The zeroth moments are given by $\varphi^{\Sigma}_{\eta^{(0)},0}=1$ and $\varphi^{\mathrm{G}}_{\eta^{(0)},0}=0$ and, thus, the sum in the gluonic component always starts from $k=2$.

The vector valued amplitude T_{jk} consists of the hard scattering one that is convoluted with the evolution operators

$$\mathrm{T}_{jk}(\mathcal{Q}^2,\mathcal{Q}_0^2)=\sum_{\substack{l=0\\\text{even}}}^{\infty}\sum_{\substack{m=0\\\text{even}}}^{\infty}\boldsymbol{T}_{j+m,k+l}\left(\alpha_s(\mu_{\mathrm{R}}),\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2},\frac{\mathcal{Q}^2}{\mu_\varphi^2},\frac{\mathcal{Q}^2}{\mu_{\mathrm{R}}^2}\right)\boldsymbol{E}_{k+l,k}(\mu_\varphi,\mathcal{Q}_0)\,{}^{+}E_{j+m,j}(\mu_{\mathrm{F}},\mathcal{Q}_0). \tag{24}$$

The evolution operator for the GPD moments, formally written as path ordered exponential

$${}^{+}E_{jm}(\mu,\mu_0)=\mathcal{P}\exp\left\{-\int_{\mu0}^{\mu}\frac{d\mu'}{\mu'}{}^{+}\gamma_{jm}(\alpha_s(\mu'))\right\}, \tag{25}$$

is expressed by the $\sigma=+1$ anomalous dimensions ${}^{+}\gamma_{jm}=\frac{\alpha_s}{2\pi}\gamma_j^{(0)}\delta_{jm}+\frac{\alpha_s^2}{(2\pi)^2}{}^{+}\gamma_{jm}^{(1)}+O(\alpha_s^3)$ with

$$\gamma_j^{(0)} = C_{\rm F}\left(4S_1(j+1) - 3 - \frac{2}{(j+1)(j+2)}\right), \tag{26}$$

where $S_1(n) = \sum_{m=1}^{n} \frac{1}{m}$ is the harmonic sum of order one. The evolution operator,

$$\boldsymbol{E}_{km}(\mu, \mu_0) = \mathcal{P}\exp\left\{-\int_{\mu 0}^{\mu} \frac{d\mu'}{\mu'} \boldsymbol{\gamma}_{km}(\alpha_s(\mu'))\right\}, \tag{27}$$

for the $\eta^{(0)}$ DA is expressed by the anomalous dimension matrix of conformal operators,

$$\boldsymbol{\gamma}_{km} = \frac{(2k+3)(m+1)_2}{(2m+3)(k+1)_2}\begin{pmatrix} {}^{\Sigma\Sigma}\gamma_{km} & \frac{m(m+3)}{12}\,{}^{\Sigma G}\gamma_{km} \\ \frac{12}{k(k+3)}\,{}^{G\Sigma}\gamma_{km} & \frac{m(m+3)}{k(k+3)}\,{}^{GG}\gamma_{km}\end{pmatrix}, \tag{28}$$

where ${}^{AB}\gamma_{km} = \frac{\alpha_s}{2\pi}\,{}^{AB}\gamma_k^{(0)}\delta_{km} + \frac{\alpha_s^2}{(2\pi)^2}\,{}^{AB}\gamma_{km}^{(1)} + O(\alpha_s^3)$. To LO accuracy the quark–quark entry is given in (26) and the three remaining entries read

$${}^{\Sigma G}\gamma_k^{(0)} = -\frac{12n_f}{(k+1)(k+2)}, \qquad {}^{G\Sigma}\gamma_k^{(0)} = -C_{\rm F}\frac{k(k+3)}{3(k+1)(k+2)}, \tag{29a}$$

$${}^{GG}\gamma_k^{(0)} = C_{\rm A}\left(4S_1(k+1) - \frac{8}{(k+1)(k+2)}\right) + \beta_0 . \tag{29b}$$

The evolution operators are specified to NLO accuracy in Sec. 4.3 of Ref. [33], where, however, the anomalous dimension matrix (28) must be used.

The conformal moments of the hard scattering amplitude (17) read

$$\boldsymbol{T}_{jk}(\cdots) = \frac{2^{j+1}\,\Gamma\left(j+\frac{5}{2}\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma(j+3)}\left(3\,{}^{\Sigma}c_{jk}(\cdots),\ \frac{3n_f}{C_{\rm F}}\,{}^{G}c_{jk}(\cdots)\right), \qquad {}^{\Sigma}c_{jk} = c_{jk} + n_f\,{}^{\rm pS}c_{jk}\,. \tag{30}$$

The integral values of the c_{jk} coefficients are normalized as following

$${}^{A}c_{nk} = \int_0^1 du \int_0^1 dv\, 2u\overline{u}\, C_n^{3/2}(u-\overline{u})\,{}^{A}T(u,v|\cdots) 2v\overline{v}\, C_k^{3/2}(v-\overline{v}), \tag{31a}$$

$${}^{G}c_{nk} = \int_0^1 du \int_0^1 dv\, 2u\overline{u}\, C_n^{3/2}(u-\overline{u})\,{}^{G}T(u,v|\cdots) 12v^2\overline{v}^2 C_{k-1}^{5/2}(v-\overline{v}) \tag{31b}$$

for the quark–quark channel $A \in \{q, \Sigma, {\rm pS}\}$ and the quark–gluon channel, respectively.

The perturbative expansion of these moments is analogous to those of the hard scattering amplitude (17), replace there ${}^{\cdots}T^{(1\ldots)}(u,v|\cdots)$ by ${}^{\cdots}c_{jk}^{(1\ldots)}(\cdots)$, where $c_{jk}^{(0)} = 1$. The NLO expressions $c_{jk}^{(1)}$ for the quark–quark channel can be read off from Eq. (4.44) in Ref. [10], where the signature is $\sigma = +1$. Utilizing the method and results presented in Sec. 4.1 of Ref. [10], we find the remaining coefficients from the hard scattering amplitudes (18):

$${}^{\rm pS}c_{jk}^{(1)} = -\frac{(k+1)_2+2}{[(k+1)_2]^2} + \frac{\Delta S_2\left(\frac{k+1}{2}\right) + \Delta S_2\left(\frac{j+1}{2}\right)}{2} + \frac{(k+1)_4}{2k+3}\frac{\Delta S_2\left(\frac{1+j}{2}, \frac{k+2}{2}\right)}{2} - \frac{(k-1)_4}{2k+3}\frac{\Delta S_2\left(\frac{1+j}{2}, \frac{k}{2}\right)}{2} \tag{32}$$

for the pure singlet quark part and

$$\begin{aligned}{}^{G}c_{jk}^{(1,F)} = {}& -2S_1(j+1)\,[S_1(k+1)-1] + \frac{k(k+3)}{2(k+1)_2}\left[\ln\frac{\mathcal{Q}^2}{\mu_\varphi^2} - \frac{1}{2} - 2S_1(j+1) - 2S_1(k+1)\right. \\ & \left. + \frac{1}{(k+1)_2}\right] - \frac{(k)_4}{2k+3}\,\frac{(k+1)(k+4)\Delta S_2\left(\frac{j+1}{2}, \frac{k+2}{2}\right) - (k-1)(k+2)\Delta S_2\left(\frac{1+j}{2}, \frac{k}{2}\right)}{8}\end{aligned} \tag{33}$$

$$\begin{aligned}{}^{G}c_{jk}^{(1,A)} = {}& S_1(j+1)\,[S_1(k+1)-1] + \frac{\zeta_2+1}{2} - \frac{2(k+1)_2+2}{[(k+1)_2]^2} - \frac{\Delta S_2\left(\frac{j+1}{2}\right)}{4} - \frac{(k+1)_2-4}{2} \\ & \times \frac{\Delta S_2\left(\frac{j+1}{2}\right) + \Delta S_2\left(\frac{k+1}{2}\right)}{4} - \frac{(k)_4}{2k+3}\,\frac{(k+4)\Delta S_2\left(\frac{j+1}{2}, \frac{k+2}{2}\right) + (k-1)\Delta S_2\left(\frac{j+1}{2}, \frac{k}{2}\right)}{4}\end{aligned} \tag{34}$$

for the quark–gluon channel. Here, $S_i(n) = \Sigma_{m=1}^{n} m^{-i}$ are the harmonic sums of order i and

$$\Delta S_2(n,m) = \frac{\Delta S_2(n) - \Delta S_2(m)}{4(n-m)(1+2m+2n)}, \qquad \Delta S_2(n,n) = -\frac{\Delta S_3(n)}{2(1+4n)}$$

with $\Delta S_i(n) = S_i(n) - S_i(n-1/2)$.

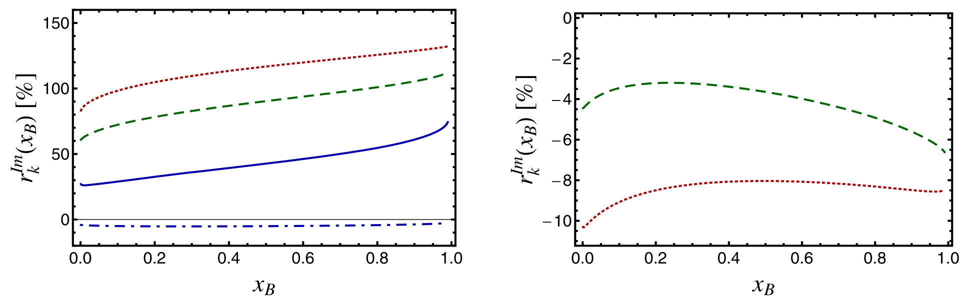

Fig. 2. Relative NLO corrections (36) to the imaginary part of the TFF (21) versus $x_{\rm B}$ for the $k=0$ (solid), $k=2$ (dashed), $k=4$ (dotted) partial waves arising from the quark–quark channel (left panel) and quark–gluon channel (right panel). The pure singlet quark contribution for $k=0$ is shown as dash-dotted line in the left panel.

To quantify the NLO corrections we take a simple model for the charge odd quark GPDs,

$$F_j^{q^{(-)}}(\xi, t=0, \mathcal{Q}_0^2) = n^{q^{(-)}} \frac{6\Gamma\left(j+\frac{1}{2}\right)}{\Gamma\left(j+\frac{9}{2}\right)} \left(\frac{\xi}{2}\right)^{j+1} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(j+2)}{\Gamma\left(j+\frac{3}{2}\right)}\, {}_2F_1\left(\left.{-j-1,\, j+2 \atop 1}\right| \frac{-1+\xi}{2\xi}\right), \tag{35}$$

which in the forward limit reduce to the PDF $F^{q^{(-)}}(x, \xi=0, t=0, \mathcal{Q}_0^2) = n^{q^{(-)}} x^{-1/2}(1-x)^3$. Setting $\alpha_s(\mathcal{Q}_0 \sim 1.6~\text{GeV}) = 0.1\pi$, in Fig. 2 we show the relative NLO corrections

$$\boldsymbol{r}_k^{\Im \mathrm{m}}(x_{\rm B}, \mathcal{Q}_0^2) = \frac{\frac{\alpha_s^2(\mathcal{Q}_0)}{2\pi} \Im\mathrm{m} \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i+\tan\left(\frac{\pi j}{2}\right)\right] \boldsymbol{T}_{jk}^{(1)}\left(\mathcal{Q}_0^2, \mathcal{Q}_0^2\right) F_j^{q^{(-)}}(\xi, t=0, \mathcal{Q}_0^2)}{\Im\mathrm{m} \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i+\tan\left(\frac{\pi j}{2}\right)\right] \boldsymbol{T}_{jk=0}\left(\mathcal{Q}_0^2, \mathcal{Q}_0^2\right) \binom{1}{0} F_j^{q^{(-)}}(\xi, t=0, \mathcal{Q}_0^2)}, \tag{36}$$

of the imaginary part for the first three $k \in \{0(\text{solid}), 2(\text{dashed}), 4(\text{dotted})\}$ partial waves of the DA, which are normalized to the full NLO result for $k=0$. The corrections are very large in the quark–quark channel (left panel) and they grow with increasing k. Thereby, the pure singlet quark part reduces the $k=0$ partial wave by few percents, see dash-dotted curve. Furthermore, Eq. (32) tells us that the pure singlet quark part behaves as $\propto 1/k^4$ for large k and, thus, it becomes strongly suppressed for higher partial waves. Consequently, the NLO corrections in the quark–quark channel essentially arise from the flavor nonsinglet part, which are for instance analyzed in Sec. 5.3.1 of Ref. [10]. As one realizes in the left panel of Fig. 2, these corrections grow with increasing k, which is caused by a logarithmical enhancement,

$${}^{+}c_{jk}^{(1)} \sim 2C_{\rm F}\left[S_1(k+1)\right]^2 + \left[4C_{\rm F}\left(S_1(j+1) - \frac{5}{4} - \frac{1}{2(j+1)_2}\right) - \beta_0\right] S_1(k+1) + \cdots \text{ for large } k\,,$$

where $S_1(k) = \ln k + \gamma_E + O(1/k)$ for large k. Both of these terms are related to collinear singularities, where the squared one is a reminder of the soft double poles that cancel in the net result (see Table I of Ref. [21]). In the $x_{\rm B} \to 1$ limit the NLO corrections are governed by the analogous logarithmical corrections in the large j limit (${}^{+}c_{kj}^{(1)} = {}^{+}c_{jk}^{(1)}$) and they might be also analytically calculated, see Sec. 5.3.1 of Ref. [10].

As discussed above in momentum fraction representation, the gluonic contributions (right panel) are moderate, however, they are negative and their sizes logarithmically grow with increasing k, see the right panel of Fig. 2 as well as Eqs. (33) and (34). Note that finally the NLO corrections depend on the non-perturbative input $\varphi^{\Sigma}_{\eta^{(0)},k}(\mathcal{Q}_0^2)$ and $\varphi^{\rm G}_{\eta^{(0)},k}(\mathcal{Q}_0^2)$, too. From the photon-to-meson transition form factor information on the first Gegenbauer moment $k=2$ has been obtained [22,35], i.e., $\varphi^{\Sigma}_{\eta^{(0)},2}(\mathcal{Q}_0^2 = 1~\text{GeV}^2) \sim -0.1$ and $\varphi^{\rm G}_{\eta^{(0)},2}(\mathcal{Q}_0^2 = 1~\text{GeV}^2) \sim 0.5$. Since for this DA model the NLO corrections to the imaginary part stemming from the $k=2$ partial waves are negative, the net result are smaller than from the zeroth partial wave, shown as solid line.

Finally, let us summarize. We employed an efficient and straightforward method to calculate the NLO corrections to DVMP for the flavor singlet sector in the momentum fraction representation. The results were mapped into the space of conformal moments which allow in future to employ the Mellin–Barnes integral representation in phenomenology. We found that the NLO corrections to the pure singlet quark part are small while the quark–gluon channel might imply moderate corrections. The main corrections are large and arise from the quark–quark channel.

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