



# Spacetime deformation effect on the early universe and the PTOLEMY experiment



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## ABSTRACT

Using a fully-fledged formulation of gauge field theory deformed by the spacetime noncommutativity, we study its impact on relic neutrino direct detection, as proposed recently by the PTOLEMY experiment. The noncommutative background tends to influence the propagating neutrinos by providing them with a tree-level vector-like coupling to photons, enabling thus otherwise sterile right-handed (RH) neutrinos to be thermally produced in the early universe. Such a new component in the universe's background radiation has been switched today to the almost fully active sea of non-relativistic neutrinos, exerting consequently some impact on the capture on tritium at PTOLEMY. The peculiarities of our nonperturbative approach tend to reflect in the cosmology as well, upon the appearances of the coupling temperature, above which RH neutrinos stay permanently decoupled from thermal environment. This entails the maximal scale of noncommutativity as well, being of order of  $10^{-4}M_{Pl}$ , above which there is no impact whatsoever on the capture rates at PTOLEMY. The latter represents an exceptional upper bound on the scale of noncommutativity coming from phenomenology.

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Out of the three pillars of the standard Big Bang model, Big Bang nucleosynthesis (BBN) [1] relates directly to neutrinos and provides us with useful (but somewhat indirect) information about the universe when it was just about 1 minute old. Another pillar of the Big Bang, the cosmic microwave background radiation (CMBR), the relic radiation left over from the moment the universe cooled off and became transparent, allows us to see directly into cosmos when it was 380000 years old. It was measured recently so precisely that this has deepened our understanding of the early universe to a hitherto undreamed-of scale [2]. A related prediction of the standard theory is the undisputed existence of a relic neutrino background, whose direct detection would enable to see what the universe was doing when it was only about one second old.

Given the fact that neutrinos interact only feebly with ordinary matter, the relic neutrino background turns out to be composed altogether of neutrinos which are nonrelativistic today, making them consequently very difficult to directly detect in the laboratory. This

also turns out to be the only primary source of nonrelativistic neutrinos in the universe at present.

A first promising proposal to detect such a cold sea of neutrinos at the temperature of around 2 Kelvin, was to use the inverse beta decay of Tritium nucleus,  $\nu_e + {}^3\text{H} \rightarrow {}^3\text{He} + e^-$  [3]. The possibility of detecting such a background experimentally, using this process, was investigated in [4]. Earlier attempts to detect relic neutrino sea were precisely compiled, but also strongly criticized in [5]. With the recently proposed PTOLEMY experiment, with an energy resolution  $\Delta \sim 0.15$  eV and implementing a 100 gram sample of Tritium, the detection of relic neutrino background might soon become a dream come true [6].

For a long time, BBN has proven as one of the most powerful available probes of physics beyond the standard model (SM), giving many interesting constraints on particle properties. The BBN has played a central role in constraining particle properties since the seminal paper of Steigman, Schramm and Gunn [7], in which the observation-based determination of the primordial abundance of  ${}^4\text{He}$  was used for the first time to constrain the number of light neutrino species. Later, with the inclusion of other light element abundances (D,  ${}^3\text{He}$  and  ${}^7\text{Li}$ ) and their successful agreement with the theoretically predicted abundances, many aspects of physics

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beyond SM have been further constrained [8]. One usually parameterizes the energy density of new relativistic particles in the early universe in terms of the effective additional number of neutrino species,  $\Delta N_{eff}$ . After decades in which  $\Delta N_{eff}$  remained poorly constrained, a combination of *Planck* observations (*Planck* 2015 results [2]) with other astrophysical data has recently strongly constrained the neutrino sector of the theory, giving  $(\Delta N_{eff})^{max} = 0.33$ . Since the data favour  $N_{eff} = 3.15 \pm 0.23$  [2], one finds this consistent with the standard model value  $N_{eff} = 3.046$  itself.

Entertaining the possibility to thermally produce right-handed (RH) neutrinos  $\nu_R$  in some extension of the standard model, we note that the energy density of 3 light RH neutrinos is equivalent to the effective number  $\Delta N_{eff}$  of additional doublet neutrinos

$$\Delta N_\nu = 3 \left( \frac{T_{\nu_R}}{T_{\nu_L}} \right)^4, \quad (1)$$

where  $T_{\nu_L}$  is the temperature of the SM neutrinos, being the same as that of photons down to  $T \sim 1$  MeV. Hence we have

$$3 \left( \frac{T_{\nu_R}}{T_{\nu_L}} \right)^4 \lesssim (\Delta N_{eff})^{max}. \quad (2)$$

In the following we take the latest *Planck* result,  $(\Delta N_{eff})^{max} = 0.33$ .

How the temperature of  $\nu_R$ 's, which decoupled at  $T_{dec}$ , relates to the temperature of still interacting  $\nu_L$ 's below  $T_{dec}$ , stems easily from the fact that the entropy in the decoupled species and the entropy in the still interacting ones are separately conserved. The ratio of the temperatures is a function of  $T_{dec}$  and is given by [9, 10]

$$\frac{T_{\nu_R}}{T_{\nu_L}} = \left[ \frac{g_{*\nu_R}(T_{dec}) g_{*S}(T_{\nu_L})}{g_{*\nu_R}(T_{\nu_L}) g_{*S}(T_{dec})} \right]^{1/3}, \quad (3)$$

where  $g_{*\nu_R}$  and  $g_{*S}$  are the degrees of freedom specifying the entropy of the decoupled and of the interacting species, respectively [1, 11]. Since in our case we ignore the possibility that the decoupled particles may subsequently annihilate into other non-interacting species,  $g_{*\nu_R}$  stays constant after decoupling and therefore, for all practical purposes, the first ratio in (3) equals unity.

Now, combining (2) with (3) and noting that at the time of BBN  $g_{*S}(T_{\nu_L} \sim \text{MeV}) = 10.56$  [12], one arrives at

$$g_{*S}(T_{dec}) \gtrsim \frac{24.1}{(\Delta N_\nu^{max})^{3/4}}. \quad (4)$$

With the latest bound  $\Delta N_\nu^{max} = 0.33$ , (4) implies  $g_{*S}(T_{dec}) > 55.3$  which, given the temperature dependence of  $g_{*S}$  [12], can be seen to enforce  $T_{dec} \gtrsim T_C$ , where  $T_C$  is the critical temperature for the deconfinement restoration phase transition,  $T_C \sim 200$  MeV.

Since background neutrinos are ultra-relativistic at freeze out, the left-handed neutrinos  $\nu_L$  almost exactly coincide with the left-helical neutrinos  $\nu_l$  (similarly for anti-neutrinos), which means that in the standard theory the right-helical neutrinos  $\nu_r$  are (practically) not populated at all. If, by some mechanism, the right-handed neutrinos  $\nu_R$  were thermally produced in the early universe, they again almost exactly coincide with the right-helical neutrinos  $\nu_r$  (and similarly for anti-neutrinos). Since for free-streaming neutrinos it is their helicity that is conserved [13], and the relic neutrino background is non-relativistic today (for neutrino masses  $m_\nu \gtrsim 10^{-3}$  eV), one finds that non-relativistic right-helical neutrinos  $\nu_r$  are no longer inert, in fact, they can (almost) equally be captured in the  $\nu_e + {}^3\text{H} \rightarrow {}^3\text{He} + e^-$  process as their left-helical partners  $\nu_l$ 's do.

As calculated in detail in [14] (for earlier calculations see also [15]) the total capture rate boils down to a simple expression

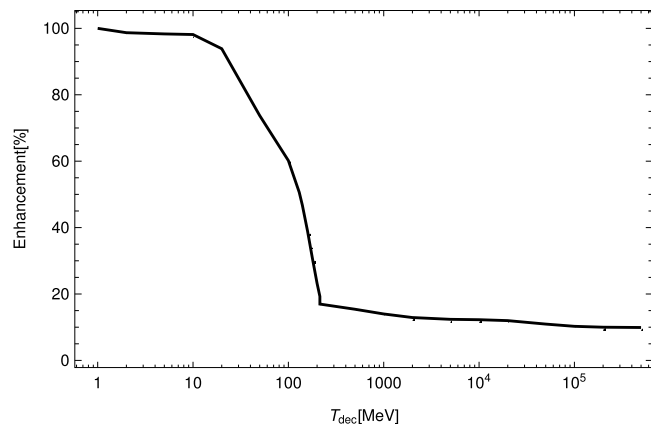


Fig. 1. PTOLEMY capture rate enhancement (%) versus  $T_{dec}$ , based on the temperature dependence of  $g_{*S}$  given in [12].

$$\Gamma = \bar{\sigma} [n(\nu_l) + n(\nu_r)] N_{trit}, \quad (5)$$

where  $\bar{\sigma} \approx 4 \times 10^{-45}$  cm<sup>2</sup>,  $N_{trit}$  is the number of tritium nuclei and  $n(\nu_l)$  and  $n(\nu_r)$  are the number densities of left- and right-helical neutrinos per degree of freedom. In the standard theory, both active degrees of freedom for the massive Majorana case equally contribute to the process, while in the Dirac case only one active (out of four) degrees of freedom does so. Hence, the capture rate in the Majorana case is twice of that in the Dirac case [14].

Note that the thermal production of right-handed Dirac neutrinos in the early Universe has been discussed before in the literature and the cosmological bound on the extra effective number of neutrino species can be satisfied [16]. A possible way to discriminate between thermal and non-thermal cosmic relic neutrinos was proposed in [17].

When the right-handed neutrinos are produced by some non-standard mechanism in the early universe, their relative contribution in (5) is given by the ratio of the temperatures cubed ( $n_\nu \sim T_\nu^3$ ), as given by (3). This is because the ratio (3) remains constant below  $T \sim \text{MeV}$ , as both  $\nu_l$  and  $\nu_r$  are then decoupled. This implies around 20% magnification of the capture rate at PTOLEMY if  $T_{dec} \sim T_C$ , and around 10% magnification if  $T_{dec} \sim T_{EW}$ , where  $T_{EW}$  is the critical temperature for electroweak phase transition,  $T_{EW} \approx 200$  GeV. We plot the capture rate enhancement (percentage) versus decoupling temperature in Fig. 1.

As a working example to realize a thermal production of right-handed neutrinos  $\nu_R$  in the early universe, via plasmon decay into neutrino pairs [18–20], we propose a fully-fledged Seiberg–Witten (SW) map based [21, 22]  $\theta$ -exact formulation of noncommutative (NC) gauge field theory. This model further preserves unitarity [23], has a correct commutative limit [24–29], and for which it has been shown that a nice UV/IR behavior at the quantum level could in fact be achieved, especially when supersymmetry is included [30–36].

Alluding to the model above, we now introduce an effective coupling involving neutrinos and photons on NC spaces which can result in thermal production of right-handed neutrinos in the early universe, giving consequently a nonzero right-helicity component in the cosmic neutrino background. Such an additional component would result in an enhancement to the Tritium capture rate in the PTOLEMY experiment, which, if observed and assumed to be due to the space-time noncommutativity [37–40], could potentially probe its associated scale.

In the presence of space-time noncommutativity it is possible to directly couple neutrinos to Abelian gauge bosons (photons) via a star( $\star$ )-commutator in the NC covariant derivative of the action [39]:

$$S = \int -\frac{1}{4} \widehat{F}^{\mu\nu} \widehat{F}_{\mu\nu} + i \widehat{\Psi} (\widehat{D} - m) \widehat{\Psi}, \quad (6)$$

$$\widehat{D}^\mu = \partial^\mu - ie\kappa [\widehat{A}^\mu \star, ], \quad (7)$$

with  $\widehat{A}^\mu, \widehat{\Psi}$  being noncommutative fields on the Moyal space and a coupling constant  $e\kappa$  corresponds to a multiple (or fraction)  $\kappa$  of the positron charge  $e$ . The  $\star$ -product above is associative but, in general, not commutative – otherwise the proposed coupling to the noncommutative photon field  $\widehat{A}_\mu$  would of course be zero.

In view of the NC covariant derivative  $\widehat{D}^\mu$  (7) one may think of the noncommutative neutrino field  $\widehat{\Psi}$  as having left charge  $+e\kappa$ , right charge  $-e\kappa$  and total charge zero. From the perspective of non-Abelian gauge theory, one could also say that the neutrino field is charged in a noncommutative analogue of the adjoint representation with the matrix multiplication replaced by the  $\star$ -product. From a geometric point of view, the interaction is seen as a modified photon- $\theta$  background throughout which neutrinos tend to propagate.

All NC fields in the action are composite functions of commutative fields (denoted as  $A^\mu, \Psi$ ) expanded/expressed in powers of ordinary gauge field via the  $\theta$ -exact SW maps

$$\begin{aligned} \widehat{A}_\mu &= A_\mu - \frac{e\kappa}{2} \theta^{ij} A_i \star_2 (\partial_j A_\mu + F_{j\mu}) + \mathcal{O}(A^3), \\ \widehat{\Psi} &= \Psi - e\kappa \theta^{ij} A_i \star_2 \partial_j \Psi + \mathcal{O}(A^2) \Psi. \end{aligned} \quad (8)$$

This makes theory exact with respect to the noncommutative parameter  $\theta$ . Here  $\Psi$  means commutative  $\Psi_{\binom{L}{R}}$ , i.e. left/right Dirac-type<sup>1</sup> massive neutrino field, and the  $\star_2$ -product is defined as follows:

$$f(x) \star_2 g(x) = f(x) \frac{\sin \frac{\partial_x \Delta \partial_y}{2}}{\frac{\partial_x \Delta \partial_y}{2}} g(y) \Big|_{y=x}. \quad (9)$$

Expanding the action in terms of the commutative gauge fields, and isolating cubic terms up to the  $A_\mu$  first order in Lagrangian, one obtains the relevant  $\theta$ -exact Feynman rules:

$$\Gamma^\mu = ie\kappa \left[ (\not{p} - m)(\theta q)^\mu - (p\theta q)\gamma^\mu - (\theta p)^\mu \not{q} \right] F(q, p), \quad (10)$$

involving the following function  $F(q, p)$ ,

$$F(q, p) = \frac{\sin \frac{q\theta p}{2}}{\frac{q\theta p}{2}}, \quad q\theta p \equiv q_i \theta^{ij} p_j. \quad (11)$$

As described in details in [25], the coupling (10) with one arbitrary  $\kappa$  can be included into neutrino-mass extended noncommutative standard model(s). It is also demonstrated in the section 2 of [25] that different  $\kappa$  values (or, more generally, different left/right charge combinations) can be assigned to different generations of matter fields which are minimally coupled to a U(1) gauge field via NC covariant derivative(s). On the other hand, following the analysis in the later sections of [25] one can easily notice that such generation dependence of  $\kappa$  is, within the context of neutrino-mass extended noncommutative standard model(s), constrained by the gauge invariance of NC mass and/or Yukawa terms: Gauge invariance of a mass term  $\bar{N}_1 \star M_{12} \star N_2$  or a Yukawa term  $\bar{N}_1 \star H_{12} \star N_2$  requires the very left gauge transformation of  $\bar{N}_1$  and the very right transformation of  $N_2$  to cancel each other via the cyclicity of the Moyal star product [25], which forces  $N_1$  and  $N_2$  to share the same  $\kappa$ -value for gauge invariance. From this viewpoint

<sup>1</sup> Note that instead of SW map of Dirac neutrinos  $\Psi$  one may consider a chiral SW map, which is compatible with grand unified models having chiral fermion multiplets [41].

a universal  $\kappa$ -value across all flavor generations as in [25] actually allows most general neutrino mixing in the gauge invariant mass/Yukawa term constructions. Hence in the rest of the paper we will deal with universal but otherwise arbitrary  $\kappa$  parameter.

Using standard techniques the plasmon decay rate into neutrinos (per generation) can be calculated to be [24]

$$\Gamma_{\text{NC}}(\gamma_{pl.} \rightarrow \bar{\nu}_{\binom{L}{R}} \nu_{\binom{L}{R}}) = \kappa^2 \frac{\alpha}{2} \omega_{pl} \left( 1 - \frac{\sin X}{X} \right) \quad (12)$$

with  $\alpha$  being the fine structure constant. For the light-like noncommutativity preserving unitarity<sup>2</sup> the full noncommutative effect will be still exhibited through  $X = \omega_{pl}^2 / (2\Lambda_{\text{NC}}^2)$ .

It is important to note that the plasma frequency  $\omega_{pl}$  is determined as the frequency of plasmons at  $|\vec{q}| = 0$ . In the very high temperature regime, where the mass of background electrons is irrelevant and can be put to zero, the dispersion relation for transverse and longitudinal waves can be calculated analytically, giving [20]

$$\omega_{pl}^2 = \mathcal{R}e \Pi_{(T/L)}(q_0, |\vec{q}| = 0) = \frac{e^2 T^2}{9}, \quad (13)$$

where  $\mathcal{R}e \Pi_{(T/L)}$  is the transverse/longitudinal part of the one-loop contribution to the photon self-energy at finite temperature/density.

Now we continue with the investigation of the cosmic neutrino background in NC spacetimes. The RH neutrino is commonly considered to decouple at the temperature  $T_{dec}$  when the condition

$$\Gamma(\gamma_{pl.} \rightarrow \bar{\nu}_R \nu_R) \simeq H(T_{dec}), \quad (14)$$

is satisfied. In this case the plasma frequency reads

$$\omega_{pl} = \frac{e T_{dec}}{3} \sqrt{g_*^{\text{ch}}}, \quad (15)$$

where  $g_*^{\text{ch}}$  counts all (effectively massless) charged-matter loops in  $\Pi_{T/L}$ . On the other hand, the Hubble parameter is given by

$$H(T_{dec}) = \left( \frac{8\pi^3}{90} g_*(T_{dec}) \right)^{1/2} \frac{T_{dec}^2}{M_{pl}}, \quad (16)$$

and  $g_*$  counts the total number of effectively massless degree of freedom. Further on, we stick with parameters  $g_*$  and  $g_*^{\text{ch}}$  fixed at their SM values,  $g_* \simeq g_*^{\text{ch}} \simeq 100$ .

Computing the decoupling temperature  $T_{dec}$  based on the assumption that the decay rate (14) is solely due to NC effects and comparing with lower bounds on  $T_{dec}$  that can be inferred from observational data, we are now in position to determine lower bounds on the scale of noncommutativity  $\Lambda_{\text{NC}}$ . Proceeding in this spirit, one finds that BBN provides the following relation between the decoupling temperature  $T_{dec}$  and the NC scale  $\Lambda_{\text{NC}}$ :

$$\begin{aligned} T_{dec} &\simeq \frac{\kappa^2}{2\pi} \sqrt{5\alpha^3 \frac{g_*^{\text{ch}}}{g_*}} M_{pl} \left( 1 - \frac{\sin X}{X} \right), \\ X &= \frac{2\pi\alpha g_*^{\text{ch}} T_{dec}^2}{9\Lambda_{\text{NC}}^2}. \end{aligned} \quad (17)$$

Note that with fixed  $g_*$  and  $g_*^{\text{ch}}$  one cannot simply dial down  $\Delta N_V^{\text{max}}$  to arbitrary precision to accommodate  $T_{dec}$  being proportional  $M_{pl}$ , as given by (17). On the other hand, sensitivity to PTOLEMY requires small  $T_{dec}$ , which one can only achieve for

<sup>2</sup> The light-like case [23] with notations  $\theta^2 = (\theta^2)_\mu^\mu = \theta_{\mu\nu} \theta^{\nu\mu} = 2(\vec{E}_\theta^2 - \vec{B}_\theta^2)$  specified in [29], corresponds to  $|\vec{E}_\theta| = |\vec{B}_\theta| = 1/(2\Lambda_{\text{NC}}^2)$ , and  $\vec{E}_\theta \cdot \vec{B}_\theta = 0$ .

$(1 - \frac{\sin X}{X}) \ll 1$ . This only occurs when  $X \ll 1$ , so in this limit we can use the leading order term in the expansion in  $X$  to obtain:

$$\Lambda_{\text{NC}}^4 \simeq \frac{\kappa^2 \pi}{243} \sqrt{5\alpha^7 (g_*^{\text{ch}})^5 g_*^{-1}} M_{\text{pl}} T_{\text{dec}}^3. \quad (18)$$

Now setting  $g_* = g_*^{\text{ch}} = 100$ ,  $M_{\text{pl}} = 1.221 \times 10^{19}$  GeV and  $T_{\text{dec}} \gtrsim 200$  MeV (quark-hadron phase transition), a lower bound on  $\Lambda_{\text{NC}}$  can be obtained as

$$\Lambda_{\text{NC}} \gtrsim 0.98 \sqrt{\kappa} \text{ TeV}. \quad (19)$$

For  $T_{\text{dec}} \gtrsim 200$  GeV (EW phase transition), we have

$$\Lambda_{\text{NC}} \gtrsim 175 \sqrt{\kappa} \text{ TeV}. \quad (20)$$

This bounds appear to be relatively mild in comparison with other similar bounds [24,25,28,29]. Also, as shown below and in contrast with those lower bounds on  $\Lambda_{\text{NC}}$ , the full numerical solution to (17) will feature a maximal allowable NC scale  $\Lambda_{\text{NC}}^{\text{max}}$ , above which the RH neutrinos can never stay in the thermal equilibrium via the NC coupling to photon and thus have no impact on PTOLEMY capture rate.

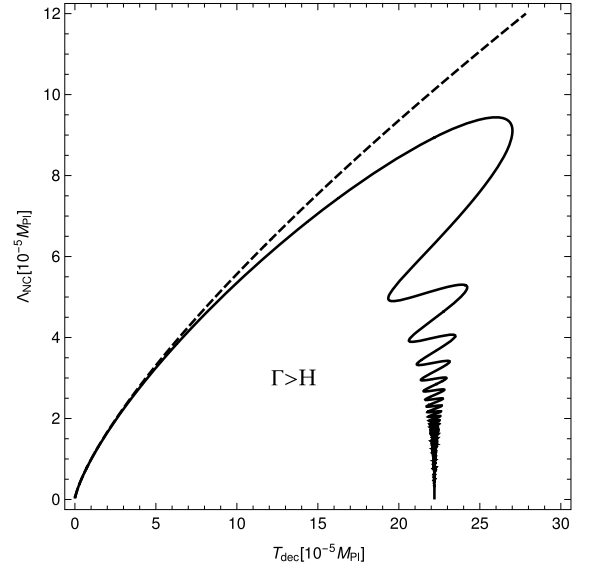
Since the equation (17) is exact with respect to the scale of noncommutativity and decoupling temperature, it is interesting to extend our investigation to a temperature range well beyond the validity of the  $\theta$ -first order approximation (18), which is done by numerical evaluation and shown in the Fig. 2. We find, surprisingly, that due to the switch in the behavior of the plasmon decay rate from  $T^5$  at low temperatures to  $T$  at very high temperatures the solution curve actually drops down at a temperature range roughly independent from the NC scale and singles out a closed region on the scale of noncommutativity  $\Lambda_{\text{NC}}$  versus decoupling temperature  $T_{\text{dec}}$ . Within this region surrounded by the solid curve, the Hubble expansion rate (16) is always smaller than the NC plasmon decay rate (12). Therefore the higher temperature solution at each given noncommutative scale, sitting on the right-hand side of the solid curve, may be interpreted as the *coupling* temperature, i.e. the temperature where the NC plasmon decay rate first time catches (or it may be the reheating temperature, whichever is lower) the Hubble rate during cooling of the universe after the Big Bang.

The appearance of a closed region where  $\Gamma > H$  implies that the NC scale can be bounded from above at  $\Lambda_{\text{NC}}^{\text{max}} \simeq 0.95 \times 10^{-4} M_{\text{pl}}$ . For NC scales  $\Lambda_{\text{NC}} > \Lambda_{\text{NC}}^{\text{max}}$  RH neutrinos stay out of thermal equilibrium at any temperature. For each NC scale  $\Lambda_{\text{NC}} < \Lambda_{\text{NC}}^{\text{max}}$ , there exist two temperature scales, namely a, lower, decoupling temperature  $T_{\text{dec}}$  and a higher, coupling temperature  $T_{\text{couple}}$ . As a consequence, RH neutrinos can only stay in effective thermal contact with the rest of the universe for the temperature range  $T_{\text{dec}} \leq T \leq T_{\text{couple}}$ .<sup>3</sup> In other words, during cooling of the universe, RH neutrinos first time enter thermal equilibrium when temperature reaches  $T_{\text{couple}}$ . As the temperature decreases further, the decay rate, starting at  $T_{\text{dec}}$ , drops once again below the Hubble rate and sterile neutrinos finally decouple.

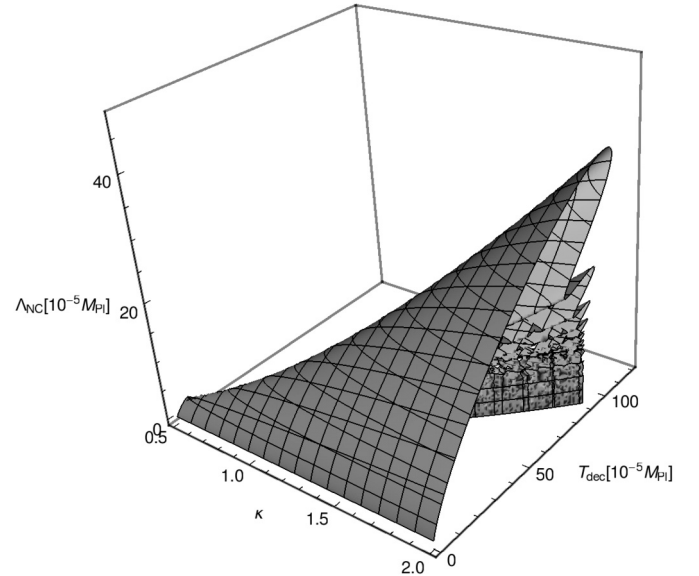
The equation (17) allows us to estimate the bound on  $T_{\text{couple}}$  analytically: In any case it has to be smaller than a fixed temperature scale

$$T_0 = \frac{\kappa^2}{2\pi} \sqrt{5\alpha^3 \frac{g_*^{\text{ch}}}{g_*}} M_{\text{pl}} \simeq 2.22 \times 10^{-4} \kappa^2 M_{\text{pl}}, \quad (21)$$

<sup>3</sup> Actually neutrinos could enter and exit thermal equilibrium a few times beneath the high temperature boundary  $T_{\text{couple}}$  due to the oscillatory nature of the NC production rate.



**Fig. 2.** Numerical plot of the scale of noncommutativity  $\Lambda_{\text{NC}}$  versus decoupling temperature  $T_{\text{dec}}$  according to the eq. (17) (solid curve), and its  $\theta$ -first order approximation (18) (dashed curve). In this plot we are using  $\kappa = 1$ , and  $g_* = g_*^{\text{ch}} = 100$ , respectively.



**Fig. 3.** 3D plot of the decoupling relation (17) with respect to temperature, NC scale  $\Lambda_{\text{NC}}$ , and coupling ratio  $\kappa$  range [0.5, 2] for degrees of freedoms  $g_* = g_*^{\text{ch}} = 100$ . The high temperature boundary of the plot decays very quickly due to the  $\kappa^2$  dependence in  $T_0$  (21).

multiplying the maximum value ( $\simeq 1.217$ ) of the  $(1 - \sin X/X)$  term sitting in the parenthesis, while for sufficiently small NC scales  $T_{\text{couple}}$  converges to  $T_0$ . These facts provide an estimation for  $T_{\text{couple}}$ 's maximal value  $T^{\text{max}} \simeq 1.22 T_0 \simeq 2.7 \times 10^{-4} \kappa^2 M_{\text{pl}}$ . Via  $T_0$ ,  $T^{\text{max}}$  depends on the quadratic power of the parameter  $\kappa$  and gets suppressed rather quickly when  $\kappa$  decreases, as illustrated in 3D Fig. 3.

The existence of  $T_{\text{couple}}$ , bounded from above by  $T^{\text{max}}|_{\kappa=1} \simeq 2.7 \times 10^{-4} M_{\text{pl}}$ , and an upper bound on the scale of noncommutativity  $\Lambda_{\text{NC}}^{\text{max}} \simeq 0.95 \times 10^{-4} M_{\text{pl}}$  for RH neutrinos to reach thermal equilibrium via NC coupling to photon from Fig. 2 represent additional results of our work. Note that decoupling of the production rate for two branches of  $T_{\text{dec}}$  ( $T_{\text{dec}}$  and  $T_{\text{couple}}$ ) exhibits certain



similarity to the UV/IR mixing in the radiative corrections of the NC theories. Both phenomena share the same origin from the exact/nonperturbative treatment of the NC parameter  $\theta$  in the quantum theory as well.

In total we have shown that the PTOLEMY total capture rate in the Dirac neutrino case may be enhanced in the present scenario up to 20% (10%) if the scale of noncommutativity  $\Lambda_{\text{NC}} \gtrsim \mathcal{O}(1)$  TeV ( $\gtrsim \mathcal{O}(100)$ ) TeV. This is still consistent with the bunch of constraints on the scale of noncommutativity obtained from particle physics phenomenology [38,42–47]. If, however, one adopts a more “natural” value for  $\Lambda_{\text{NC}}$ , which is closer to the string (or even Planck [48]) scale, then the total capture stays as predicted by the standard theory. Hence, the results of the PTOLEMY experiment could not only be used as a test of noncommutative gauge field theories, but also could provide an independent constraint on the scale of noncommutative deformation of spacetime as well.

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