

Revisiting the decoupling effects in the running of the Cosmological Constant

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Abstract We revisit the decoupling effects associated with heavy particles in the renormalization group running of the vacuum energy in a mass-dependent renormalization scheme. We find the running of the vacuum energy stemming from the Higgs condensate in the entire energy range and show that it behaves as expected from the simple dimensional arguments meaning that it exhibits the quadratic sensitivity to the mass of the heavy particles in the infrared regime. The consequence of such a running to the fine-tuning problem with the measured value of the Cosmological Constant is analyzed and the constraint on the mass spectrum of a given model is derived. We show that in the Standard Model (SM) this fine-tuning constraint is not satisfied while in the massless theories this constraint formally coincides with the well known Veltman condition. We also provide a remarkably simple extension of the SM where saturation of this constraint enables us to predict the radiative Higgs mass correctly. Generalization to constant curvature spaces is also given.

1 Introduction

It is widely accepted that our today's universe is undergoing the phase of the accelerated expansion which is commonly explained by the presence of the Cosmological Constant (CC) Λ . However, the value of Λ required by experiment is in a contradiction with the values emerging from the physics scales associated with known phase transitions in the universe so that severe fine-tuning has to be applied which is at heart of the CC problem. To recall the main aspects of this problem we begin with the Standard Model (SM) formulated on the classical curved background. In order to construct a renormalizable gauge theory in an external gravitational field

one starts from the classical action (with φ as the Higgs doublet field)

$$S_{\text{vac}} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \xi \varphi^\dagger \varphi R + a_1 R^2_{\mu\nu\alpha\beta} + a_2 R^2_{\mu\nu} + a_3 R^2 + a_4 \square R - \frac{1}{16\pi G_{\text{vac}}} (R + 2\Lambda_{\text{vac}}) \right\}. \quad (1)$$

The renormalization procedure for the theory (1) consists of the renormalization of the SM matter fields, couplings and masses, non-minimal coupling ξ and the gravitational couplings $a_{1,2,3,4}$, G_{vac} and Λ_{vac} . We are going to work in the low energy domain of the gravitational physics and, for that reason, the short distance effects from the higher derivative terms $a_{1,2,3,4}$, in (1) are not important for our considerations, and so we start with the usual bare Hilbert–Einstein action with coupling constants G_{vac} , Λ_{vac} supplemented with non-minimal coupling ξ :

$$S_{HE} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \xi^0 \varphi^\dagger \varphi R - \frac{1}{16\pi G_{\text{vac}}^0} (R + 2\Lambda_{\text{vac}}^0) \right\}. \quad (2)$$

The bare quantities are defined with the superscript “0”. Let us focus on the CC itself which, as we mentioned above, must be renormalized and the connection with experimentally measured value ρ_{phys} is achieved via the renormalization condition (see Eq. (8) below) imposed on the vacuum energy density:

$$(\rho_\Lambda^0)^{\text{vac}} = \frac{\Lambda_{\text{vac}}^0}{8\pi G_{\text{vac}}^0} \quad (3)$$

at some energy scale μ so that $\rho_\Lambda^{\text{vac}}(\mu)$. Moreover, in the presence of the dynamical cosmological background characterized by the time-dependent Hubble parameter $H(t)$, $\rho_\Lambda^{\text{vac}}$ can be dynamical $\rho_\Lambda^{\text{vac}}(\mu, t)$ that will be reflected in the evolution of $\rho_\Lambda^{\text{vac}}$ via:

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$$\frac{\partial \rho_{\Lambda}^{\text{vac}}(t)}{dt} = F(H, \rho_{\text{matter}}, \rho_{\Lambda}^{\text{vac}}, \dots), \quad (4)$$

which is the important ingredient for cosmological evolution. Currently, there is no consensus on whether $\rho_{\Lambda}^{\text{vac}}(\mu, t)$ depends on t . Even if it is time-dependent, to understand the precise form of (4) one may go back to the *time-independent* RG problem:

$$\frac{\partial \rho_{\Lambda}^{\text{vac}}(\mu)}{d\mu} = \beta_{\rho_{\Lambda}^{\text{vac}}}(g_i, m_i, \mu, \dots), \quad (5)$$

where g_i and m_i are the dimensionless couplings and masses respectively. The g_i and m_i are also supplemented with their own Renormalization Group (RG) equations.

Besides $\rho_{\Lambda}^{\text{vac}}$, the physical vacuum energy ρ_{phys} consists of several additional parts. One of these parts is "induced" contribution $\rho_{\text{ind}}(\mu)$ to the vacuum energy density arising from the vacuum condensates. For example, if φ_{vac} is the value of the Higgs field $\varphi(x)$ which minimizes the Higgs potential $V(\varphi)$

$$V(\varphi) = -m^2 \varphi^\dagger \varphi + \frac{\lambda}{2} (\varphi^\dagger \varphi)^2, \quad (6)$$

the Higgs condensate contribution (at the classical level) to the vacuum energy is

$$\rho_{\text{ind}}(\mu) = V(\varphi_{\text{vac}}) = -\frac{m^4(\mu)}{2\lambda(\mu)}. \quad (7)$$

Besides the vacuum and induced terms we may have additional effects from the higher derivative terms in (1) as well as corrections from quantum gravity. Again, these contributions can be classified as coming from purely quantum effects and therefore expected to be μ -dependent and some also time-dependent due to the expanding background, and therefore contributing to (4). All in all, the physical value is measured at the cosmological RG scale μ_c , which is experimentally given by $\mu_c = \mathcal{O}(10^{-3})$ eV, as

$$\rho_{\text{phys}} = \rho_{\Lambda}^{\text{vac}}(\mu_c) + \rho_{\text{ind}}(\mu_c) + \dots = 10^{-47} \text{ GeV}^4. \quad (8)$$

The problem now is that if we use the experimental Higgs mass $M_H = 125$ GeV, then the corresponding value $|\rho_{\text{ind}}| \simeq 10^8 \text{ GeV}^4$. In order to keep the QFT consistent with astronomical observations, one has to demand that the parts contributing to the ρ_{phys} should cancel with the accuracy dictated by the current data. For example, if we neglect all the ... terms in (8), the $\rho_{\Lambda}^{\text{vac}}$ and ρ_{ind} should cancel with the precision of 55 decimal orders. This is the CC fine-tuning problem [1, 2].

To understand deeper this tuning, one has to take into account the decoupling effects due to massive particles. Clearly, we expect that contribution to the RG running from the particle of mass m should change dramatically, as we go from $\mu \gg m$ to $\mu \ll m$ regime. Moreover, requiring the absence (or, at least, reduction) of the tuning may provide

a constraint on the spectrum of the particle physics models. In this paper, we deal with the time-independent classical curved background and will derive the RG evolution of $\rho_{\Lambda}^{\text{vac}}$ and ρ_{ind} of the form (5) taking into account the decoupling effects due to massive particles by using the mass-dependent RG formalism.

The motivation for this work is threefold:

- to derive the leading decoupling effects on the RG running of ρ_{phys} . Along the way, we will comment on the inconsistencies of the similar derivation presented in [11], as we demonstrate the importance of considering the RG running of the total vacuum energy $\rho_{\Lambda}^{\text{vac}} + \rho_{\text{ind}}$ since, although $\rho_{\Lambda}^{\text{vac}}$ and ρ_{ind} run separately, it is only the sum that exhibits behavior consistent with the Appelquist–Carazzone decoupling theorem [4].
- to elucidate the implications of these results on the mass spectrum of the SM as well as its extensions. As an outcome, we present a simple phenomenological extension of the SM *predicting* the Higgs mass correctly.
- to provide the generalization of these results to the constant curvature spaces important for studies of curvature-induced running of the vacuum energy as well as curvature-induced phase transitions.

The paper is structured as follows. In the next Sect. 2 we briefly discuss the RG running of the CC in the simple ϕ^4 -theory highlighting the necessary RG formalism we use later and also discuss the basic issue of decoupling in the RG running. In Sect. 3 we extend the RG approach to the full SM, in both, mass-independent and mass-dependent RG schemes. Section 4 deals with applications of the derived heavy-mass threshold effects within and beyond the SM and Sect. 6 presents our conclusions. In "Appendices" we provide the technical details, as well as generalize the flat spacetime results to the spaces with constant curvature.

2 RG running of the Cosmological Constant

To prepare for the discussion of the RG dependence of the CC and to setup the formalism, let us consider the skeleton Lagrangian for the real scalar:

$$L = \frac{1}{2} m^2 \phi^2 + \frac{1}{8} \lambda \phi^4. \quad (9)$$

The schematic contributions to the one-loop effective potential, up to the four external legs, are shown in Fig. 1.

Correspondingly to this diagrammatic picture and for a general QFT, the renormalized effective potential can be split into two pieces: the ϕ -independent (vacuum) term corresponding to the diagram Fig. 1a and the ϕ -dependent "scalar" term connected with diagrams Fig. 1b, c:

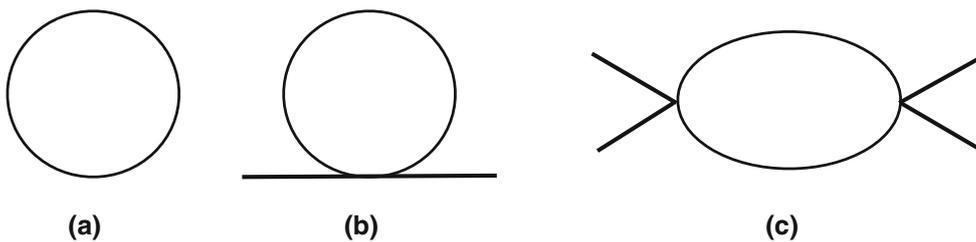


Fig. 1 **a** The one-loop contributions to the vacuum part of CC are just bubbles of matter fields without external legs; **b** the one-loop two-point function contributing to the induced part of the CC. **c** The one-loop four-point function contributing to the induced part of the CC

$$V(\rho_\Lambda^{\text{vac}}, \phi, m^2, \lambda, \mu) = V_{\text{vac}}(\rho_\Lambda^{\text{vac}}, m^2, \lambda, \mu) + V_{\text{scal}}(\phi, m^2, \lambda, \mu), \tag{10}$$

where the parameter $\rho_\Lambda^{\text{vac}} = \rho_\Lambda^{\text{vac}}(\mu)$ depends on the vacuum cosmological constant and will be defined in the next section. In order to understand the origin of this splitting, one introduces the functional called the effective action of the vacuum Γ_{vac} . It is part of the full effective action which is left when the mean scalar field ϕ is set to zero: $\Gamma_{\text{vac}} = \Gamma[\phi = 0]$. Thus, it is a pure quantum object which only depends on the set of parameters $P = m, \lambda, \dots$ of the classical theory. At the functional level, the generating functional W for the vacuum-to-vacuum transition amplitude is

$$W[J = 0] = e^{i\Gamma_{\text{vac}}} = \int \mathcal{D}\phi e^{iS[\phi; J=0]}, \tag{11}$$

where the source J is set to zero. In this way, the functional Γ_{vac} is the generator of the proper vacuum-to-vacuum diagrams.

The RG-invariance of the full renormalized effective potential reads (where $\gamma_m m^2 = \beta_{m^2}$):

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} - \gamma_\phi \phi \frac{\partial}{\partial \phi} + \beta_{\rho_\Lambda^{\text{vac}}} \frac{\partial}{\partial \rho_\Lambda^{\text{vac}}} \right) \times V(\rho_\Lambda^{\text{vac}}, \phi, m^2, \lambda, \mu) = 0. \tag{12}$$

Using (10), we now show that Eq. (12) is, in fact, a sum of two independent RG equations,

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \beta_{\rho_\Lambda^{\text{vac}}} \frac{\partial}{\partial \rho_\Lambda^{\text{vac}}} \right) \times V_{\text{vac}}(\rho_\Lambda^{\text{vac}}, m^2, \lambda, \mu) = 0, \tag{13}$$

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} - \gamma_\phi \phi \frac{\partial}{\partial \phi} \right) \times V_{\text{scal}}(\phi, m^2, \lambda, \mu) = 0. \tag{14}$$

To prove this, notice that from the RG-invariance of the renormalized effective action follows the μ -independence of the renormalized functional Γ_{vac} and, therefore, we arrive at the first identity (13) for the vacuum part of the effective potential, while the second identity is then the result of the subtraction of (13) from (12). We will illustrate this point later.

The net result is that the vacuum and matter parts of the effective potential are overall μ -independent separately and no cancellation between them is expected.

2.1 Vacuum part of the CC

Let us compute the $V_{\text{vac}}(\rho_\Lambda^{\text{vac}}, m^2, \lambda, \mu)$ object at the one-loop level. We start from

$$S_{HE} = -\frac{1}{16\pi G_{\text{vac}}^0} \int d^4x \sqrt{-g} \{ R + 2\Lambda_{\text{vac}}^0 \} + S_{\text{matter}}. \tag{15}$$

As it is well-known the Λ_{vac}^0 -dependent part has exactly the form of the bare vacuum energy density (3)¹:

$$(\rho_\Lambda^0)^{\text{vac}} = \frac{\Lambda_{\text{vac}}^0}{8\pi G_{\text{vac}}^0}. \tag{16}$$

In the standard QFT, the loop-divergent terms in the vacuum density are absorbed by the bare cosmological constant term $(\rho_\Lambda^0)^{\text{vac}}$ of the Hilbert–Einstein action. For this, we split the bare term $(\rho_\Lambda^0)^{\text{vac}}$ as

$$(\rho_\Lambda^0)^{\text{vac}} = \rho_\Lambda^{\text{vac}}(\mu) + \delta\rho_\Lambda^{\text{vac}}, \tag{17}$$

where the counterterm $\delta\rho_\Lambda^{\text{vac}}$ depends on the regularization and the renormalization scheme. Specifically, the one-loop effects encoded in $\bar{V}_{\text{vac}}^{(1)}$ modify this relation as follows:

$$V_{\text{vac}}(\rho_\Lambda^{\text{vac}}, m^2, \lambda, \mu) = (\rho_\Lambda^0)^{\text{vac}} + \bar{V}_{\text{vac}}^{(1)} = \rho_\Lambda^{\text{vac}}(\mu) + \delta\rho_\Lambda^{\text{vac}} + \bar{V}_{\text{vac}}^{(1)}. \tag{18}$$

Now, starting from (11), the variation of vacuum-to-vacuum transition amplitude with mass m leads to the (Euclidean) Green's function at coincident points [3]

¹ Sometimes, in the literature $(\rho_\Lambda^0)^{\text{vac}} = \frac{\Lambda_{\text{vac}}^0}{8\pi G_{\text{vac}}^0} \equiv hm^4$ is used where h is treated as an independent parameter.

$$\begin{aligned} \frac{\partial}{\partial m^2} \log W[J = 0] &= -\frac{1}{2} \int d_E^n x \Delta_E(0) \quad \text{with} \quad \Delta_E(0) \\ &= \int \frac{d_E^n k}{(2\pi)^n} \frac{1}{k^2 + m^2} = \frac{(m^2)^{\frac{n}{2}-1}}{(4\pi)^{n/2}} \Gamma(1 - \frac{n}{2}). \end{aligned} \tag{19}$$

In terms of Feynman diagrams this is just the vacuum bubble shown in Fig. 1a. Integrating this equation and adding $(\rho_\Lambda^0)^{\text{vac}}$ we obtain the vacuum energy density (18) as:

$$\begin{aligned} W[J = 0] &= e^{-\int d_E^n x V_{\text{vac}}} \quad \text{with} \quad V_{\text{vac}} \\ &= (\rho_\Lambda^0)^{\text{vac}} + \bar{V}_{\text{vac}}^{(1)} = (\rho_\Lambda^0)^{\text{vac}} \\ &\quad + \frac{m^n}{(4\pi)^{n/2}} \frac{1}{n} \Gamma\left(1 - \frac{n}{2}\right). \end{aligned} \tag{20}$$

The pole of the Gamma function in four dimensions $\Gamma(1 - \frac{n}{2}) \sim 2/(n - 4)$ so for $n \rightarrow 4$:

$$\bar{V}_{\text{vac}}^{(1)} = \frac{m^4}{64\pi^2} \left(\frac{2}{n - 4} - \ln \frac{4\pi \mu^2}{m^2} + \gamma_E - \frac{3}{2} \right). \tag{21}$$

Equation (21) is divergent and needs a subtraction. If we adopt the \overline{MS} subtraction scheme, the counterterm $\delta\rho_\Lambda^{\text{vac}}$ gets fixed in such a way that the renormalized vacuum energy density at 1-loop is

$$\begin{aligned} V_{\text{vac}}(\rho_\Lambda^{\text{vac}}, m^2, \lambda, \mu) &= \rho_\Lambda^{\text{vac}}(\mu) + \delta\rho_\Lambda^{\text{vac}} + \bar{V}_{\text{vac}}^{(1)} \\ &= \rho_\Lambda^{\text{vac}}(\mu) + \frac{m^4}{64\pi^2} \left(\ln \frac{m^2}{\mu^2} - \frac{3}{2} \right). \end{aligned} \tag{22}$$

This is the result for $V_{\text{vac}}(m^2, \lambda, \rho_\Lambda^{\text{vac}}, \mu)$ at 1-loop. Notice that it is a pure quantum object that (to one-loop order) depends only on the parameter m of the classical Lagrangian and does not depend (to this order) on λ .

It is clear from (22) that the cosmological constant is renormalized according to:

$$(\rho_\Lambda^0)^{\text{vac}} = \mu^{n-4} \left(\frac{m^4}{2(4\pi)^2} \frac{1}{n-4} + \rho_\Lambda^{\text{vac}}(\mu) \right) \tag{23}$$

and

$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} \equiv \beta_{\rho_\Lambda^{\text{vac}}} = \frac{m^4}{32\pi^2}. \tag{24}$$

This is the expression for $\beta_{\rho_\Lambda^{\text{vac}}}$ calculated in the \overline{MS} scheme.

In writing this equation we used the renormalized mass because the RG equations must involve only finite (renormalized) quantities. However, so far, we only computed the vacuum bubble in the free theory, where renormalized and bare masses are the same. In the interacting theory, we have to take care about the renormalization of the mass m itself. The leading mass correction is based on the correction to

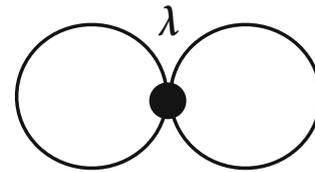


Fig. 2 The two-loop contribution to the cosmological constant in ϕ^4 theory

the scalar propagator shown in Fig. 1b and after the standard calculation we arrive at:

$$m_0^2 = m^2 \left(1 - \frac{3\lambda}{(4\pi)^2} \frac{1}{n-4} \right) \implies \mu \frac{\partial m^2}{\partial \mu} = m^2 \frac{3\lambda}{(4\pi)^2}. \tag{25}$$

The addition of the interactions modifies the renormalization of the cosmological constant according to the two-loop diagram shown in Fig. 2.

which leads to:

$$\begin{aligned} V_{\text{vac}} &= (\rho_\Lambda^0)^{\text{vac}} + \frac{m_0^n}{(4\pi)^{n/2}} \frac{1}{n} \Gamma\left(1 - \frac{n}{2}\right) \\ &\quad + \frac{3\lambda_0}{8} \left[\frac{m_0^{n-2}}{(4\pi)^{n/2}} \Gamma\left(1 - \frac{n}{2}\right) \right]^2. \end{aligned} \tag{26}$$

To put this expression in the renormalized form we have to replace the bare mass by the renormalized one using (25), while we can use renormalized quartic coupling $\lambda_0 = \lambda$ to this order. One obtains:

$$\begin{aligned} V_{\text{vac}} &= (\rho_\Lambda^0)^{\text{vac}} + \frac{m^4}{2(4\pi)^{n/2}} \frac{\mu^{n-4}}{n-4} \left(1 - \frac{3\lambda}{(4\pi)^2} \frac{1}{n-4} \right) \\ &\quad + \text{finite} \end{aligned} \tag{27}$$

where we observe that the leading term is written in terms of the renormalized mass. It is clear from (27) that the cosmological constant is then renormalized according to:

$$\begin{aligned} (\rho_\Lambda^0)^{\text{vac}} &= \mu^{n-4} \left[\frac{m^4}{2(4\pi)^2} \frac{1}{n-4} \left(1 - \frac{3\lambda}{(4\pi)^2} \frac{1}{n-4} \right) + \rho_\Lambda^{\text{vac}}(\mu) \right] \\ \implies \beta_{\rho_\Lambda^{\text{vac}}} &= \frac{m^4}{32\pi^2} \left[1 + \mathcal{O}\left(\frac{\lambda}{16\pi^2}\right) \right]. \end{aligned} \tag{28}$$

Note again that there is no correction to the RG to the leading order in λ . Basically, each of the two bubbles in Fig. 2 acts as a mass correction to the other one and gets reabsorbed into the renormalized mass.

2.2 Decoupling effects

By definition, the RG equation (28) holds in the region $\mu \gg m$ and to go to the opposite regime $\mu \ll m$ would require to take into account: (1) the contribution of heavy particles at the energies near their mass, (2) the residual effects from the heavy particles at energies well below their mass.

It is well-known that the decoupling of heavy particles does not hold in a mass-independent scheme like the \overline{MS} , and for this reason they must be decoupled by hand using the sharp cut-off procedure or some of the mass-dependent schemes. The quantum effects of the massive particles are, in principle, suppressed at low energies by virtue of the Appelquist–Carazzone theorem [4], so that in the region below the mass of the particle its quantum effects become smaller. At this point we need the relation between the IR and the UV regions which would require to extend the Wilson RG for the quantitative description of the threshold effects, and to apply a mass-dependent RG formalism.

On purely dimensional grounds, in the regime $\mu \ll m$ one expects the corrections to the CC of the type $\mu^2 m^2$. These corrections can be seen from the fact that in a mass-dependent subtraction scheme a heavy mass m enters the β -functions through the dimensionless combination μ/m , so that the CC, being a dimension-4 quantity, is expected to have the β -function corrected as follows:

$$\beta\left(m_{\text{light}}, \frac{\mu}{m}\right) = a m_{\text{light}}^4 + b \left(\frac{\mu}{m}\right)^2 m^4 + c \left(\frac{\mu}{m}\right)^4 m^4 + \dots \tag{29}$$

where a, b and c are some coefficients, m_{light} is some light mass $m_{\text{light}} \ll \mu$, and the dots stand for terms suppressed by higher order powers of $\mu/m \ll 1$. Equipped with the necessary RG formalism and expectation of decoupling behavior based on the dimensional analysis, we will show how one can deal with the decoupling effect in the full SM and how to calculate explicitly the coefficients a, b, c for any model.

3 RG running of the Cosmological Constant in the Standard Model

Before discussing the mass-dependent RG schemes relevant for decoupling, let us recall the results in the usual \overline{MS} scheme.

3.1 Mass-independent (\overline{MS}) scheme

The renormalized effective potential of the SM, V , can be written in the 't Hooft–Landau gauge and the \overline{MS} scheme as [5,6]

$$V(\rho_{\Lambda}^{\text{vac}}, \phi, m^2, \lambda_i, \mu) \equiv V_0 + V_1 + \dots, \tag{30}$$

where $\lambda_i \equiv (g, g', \lambda, h_t)$ runs over all dimensionless couplings and V_0, V_1 are the tree level potential and the one-loop correction respectively, namely

$$V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4, \tag{31}$$

Table 1 Contributions to the effective potential (32) from the SM particles W^\pm, Z^0 , top quark t , Higgs ϕ and the Goldstone bosons $\chi_{1,2,3}$

Φ	i	n_i	κ_i	κ_i^m	c_i
W^\pm	1	6	$g^2/4$	0	5/6
Z^0	2	3	$(g^2 + g'^2)/4$	0	5/6
t	3	-12	$y_t^2/2$	0	3/2
ϕ	4	1	$3\lambda/2$	1	3/2
χ_i	5	3	$\lambda/2$	1	3/2

$$V_1 = \sum_{i=1}^5 \frac{n_i}{64\pi^2} M_i^4(\phi) \left[\log \frac{M_i^2(\phi)}{\mu^2} - c_i \right] + \rho_{\Lambda}^{\text{vac}}, \tag{32}$$

with²

$$M_i^2(\phi) = \kappa_i \phi^2 - \kappa_i^m m^2, \tag{33}$$

and coefficients $n_i, \kappa_i, \kappa_i^m$, and c_i defined in Table 1.

$M_i^2(\phi)$ are the tree-level expressions for the background-dependent masses of the particles that enter in the one-loop radiative corrections, namely $M_1 \equiv m_W, M_2 \equiv m_Z, M_3 \equiv m_t, M_4 \equiv m_{\text{Higgs}}, M_5 \equiv m_{\text{Goldstone}}$. The parameter $\rho_{\Lambda}^{\text{vac}}(\mu)$ is the SM analogue of the renormalized cosmological constant $\rho_{\Lambda}^{\text{vac}}(\mu)$ for the real scalar field discussed in the previous section. Repeating the procedure as before, we split the effective potential into two pieces: the ϕ -independent (vacuum) term and the ϕ -dependent ‘‘scalar’’ term

$$V(\rho_{\Lambda}^{\text{vac}}, \phi, m^2, \lambda_i, \mu) = V_{\text{vac}}(\rho_{\Lambda}^{\text{vac}}, m^2, \lambda_i, \mu) + V_{\text{scal}}(\phi, m^2, \lambda_i, \mu). \tag{34}$$

Various pieces satisfy the RG equations (12) and (14, 13) with $\lambda \rightarrow \lambda_i$ and these equations are valid for any value of ϕ .

However, for the extremum value $\phi = \langle \phi \rangle$ defined via $\left. \frac{\partial V_{\text{scal}}(\phi)}{\partial \phi} \right|_{\langle \phi \rangle} = 0$, the term containing anomalous dimension of the Higgs γ_{ϕ} drops out and (14) reads:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_{\lambda_i} \frac{\partial}{\partial \lambda_i} + \gamma_m m^2 \frac{\partial}{\partial m^2} \right) V_{\text{scal}}(\langle \phi \rangle, m^2, \lambda, \mu) = 0. \tag{35}$$

Using the tree-level potential (31), it is useful to define parameter

² There is a logarithmic singularity associated with massless particles. In the SM, it is well-known that in the Landau gauge the Goldstone boson mass, which vanishes at the minimum of the effective potential, presents an infrared logarithmic divergence for the running Higgs mass. However, the physical mass M_{phys} (corrected by the self-energy shift from $p^2 = 0$ to $p^2 = M_{\text{phys}}^2$) is finite since the divergent contribution to the running mass coming from the Goldstone bosons is cancelled by the contribution of the Goldstones to the self-energy [7].

$$\rho_{\text{ind}}(\mu) \equiv V_0(\langle\phi\rangle) = -\frac{m^4(\mu)}{2\lambda(\mu)}. \tag{36}$$

The running of this parameter reads:

$$\beta_{\rho_{\text{ind}}} \equiv \mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} = \rho_{\text{ind}}(\mu) \left(2\gamma_m - \frac{\beta_\lambda}{\lambda} \right). \tag{37}$$

By equating the terms with the different powers of ϕ :

$$\mu \frac{dV}{d\mu} \sim (\phi^4[\dots] + m^2\phi^2[\dots] + m^4[\dots]) = 0, \tag{38}$$

it is straightforward to check that the requirement (12) applied to the full one-loop effective potential (30) leads to

$$\frac{1}{8}\beta_\lambda - \frac{1}{2}\gamma_\phi\lambda = \sum_i \frac{n_i\kappa_i^2}{32\pi^2}, \tag{39}$$

$$\frac{1}{2}\gamma_m - \gamma_\phi = \sum_i \frac{n_i\kappa_i\kappa_i^m}{16\pi^2}, \tag{40}$$

$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} = m^4 \sum_i \frac{n_i(\kappa_i^m)^2}{32\pi^2}, \tag{41}$$

up to two-loop corrections. The first two equations come from $\phi^4[\dots]$ and $m^2\phi^2[\dots]$ terms respectively and they belong to (14) and the last condition came from $m^4[\dots]$ and satisfy (13). These equations show explicitly that the vacuum V_{vac} and scalar V_{scal} parts of the full effective potential satisfy independent RG equations.

Subtracting (39) from (40) appropriately, we reconstruct 37

$$\beta_{\rho_{\text{ind}}} = \rho_{\text{ind}} \left(2\gamma_m - \frac{\beta_\lambda}{\lambda} \right) = m^4 \left(\sum_i \frac{n_i\kappa_i^2}{8\pi^2\lambda^2} - \sum_i \frac{n_i\kappa_i\kappa_i^m}{8\pi^2\lambda} \right). \tag{42}$$

Combining (42) and (41) we finally obtain

$$\boxed{\mu \frac{\partial (\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}})}{\partial \mu} = m^4 \left(\sum_i \frac{n_i\kappa_i^2}{8\pi^2\lambda^2} - \sum_i \frac{n_i\kappa_i\kappa_i^m}{8\pi^2\lambda} + \sum_i \frac{n_i(\kappa_i^m)^2}{32\pi^2} \right) = \sum_i \frac{n_i}{32\pi^2} M_i^4(\langle\phi\rangle),} \tag{43}$$

where we used

$$M_i^2(\langle\phi\rangle) = \kappa_i\langle\phi\rangle^2 - \kappa_i^m m^2 = m^2 \left(\kappa_i \frac{2}{\lambda} - \kappa_i^m \right) \tag{44}$$

and $\langle\phi\rangle^2 = 2m^2/\lambda$. Equation (43) is the central equation valid in the UV regime of massless and massive theories, theories with the spontaneous symmetry breaking (SSB) and without. This equation defines, in a compact form, the total running of the implicit μ -dependences on the l.h.s. by balancing them with the explicit μ -dependences on the r.h.s [8].

3.2 Mass-dependent scheme

Now, following the discussion above, we may generalize approach to the mass-dependent RG scheme. As we discussed above the decoupling of heavy particles does not hold in a mass-independent \overline{MS} scheme and here we recall how to get around this problem.

The basic issue can be seen in the computations of $2 \rightarrow 2$ scattering amplitude in a simple ϕ^4 -theory:

$$V_{\phi^4} = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4 \tag{45}$$

which is just the potential of (31) limited to one real scalar. The exemplary scattering amplitude is shown in Fig. 3, where $p = p_1 + p_2$ is the total incoming momenta. Expanding in terms of the external momentum p , it is only the term $p = 0$ which is divergent since every power of p effectively gives one less power of k for large k .

Computing the logarithmically-divergent integral using, for example, dimensional regularization, we obtain:

$$A(p^2) = -\frac{1}{32\pi^2} \int_0^1 dx \left(\frac{2}{\epsilon} - \gamma_E + \log(4\pi) - \log[m^2 - x(1-x)p^2] \right) \tag{46}$$

where we see explicitly that p^2 -terms are finite.

In the \overline{MS} renormalization scheme, one chooses counterterms (c.t.) in such a way as to remove the divergent $2/\epsilon$ pole and scale independent number $-\gamma_E + \log(4\pi)$ and therefore, by construction, the counterterms are mass-independent. Also one introduces the arbitrary mass parameter $\mu_{\overline{MS}}$ to make equation dimensionally correct so that finally:

$$A(p^2)_{\overline{MS}} = A(p^2) + \text{c.t.} = \frac{1}{32\pi^2} \int_0^1 dx \log \left(\frac{m^2 - x(1-x)p^2}{\mu_{\overline{MS}}^2} \right). \tag{47}$$

From the RG equation applied to the 4-point function:

$$G_{\overline{MS}}^{(4)}(m^2, \lambda, \mu_{\overline{MS}}) \sim -3i\lambda + 9(-i\lambda)^2 [iA(s)_{\overline{MS}} + iA(t)_{\overline{MS}} + iA(u)_{\overline{MS}}] \tag{48}$$

one now derives the β -function of the theory:

$$\left(\mu_{\overline{MS}} \frac{\partial}{\partial \mu_{\overline{MS}}} + \beta_{\lambda}^{\overline{MS}} \frac{\partial}{\partial \lambda} \right) G^{(4)}(m^2, \lambda, \mu_{\overline{MS}}) = 0 \implies \beta_{\lambda}^{\overline{MS}} = \frac{9\lambda^2}{16\pi^2}. \tag{49}$$

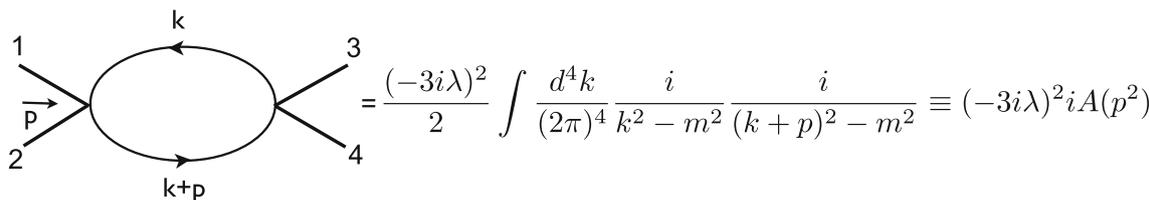


Fig. 3 a The one-loop contributions to 4-point function in ϕ^4 theory

In a mass-dependent renormalization scheme, the counterterms are *mass-dependent* and can be chosen, for example, to subtract from (46), in addition to the divergent pole and scale-independent numbers, also the $\log[m^2 - x(1-x)p^2]$ evaluated at the $p^2 = -\mu^2$ where μ is yet another arbitrary scale. After this additional *finite subtraction*, (46) will be replaced by the corresponding expression in the momentum subtraction scheme (MOM) as

$$A(p^2)_{\text{MOM}} = A(p^2) + \text{c.t.} = \frac{1}{32\pi^2} \int_0^1 dx \log \left(\frac{m^2 - x(1-x)p^2}{m^2 + x(1-x)\mu^2} \right). \tag{50}$$

Again, μ -dependence will determine the beta function of the theory through the RG-equation and we obtain:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda^{\text{MOM}} \frac{\partial}{\partial \lambda} \right) G_{\text{MOM}}^{(4)}(m^2, \lambda, \mu) = 0 \implies \beta_\lambda^{\text{MOM}} = \frac{9\lambda^2}{16\pi^2} \int_0^1 \frac{x(1-x)\mu^2}{m^2 + x(1-x)\mu^2} dx \tag{51}$$

which in the $\mu \ll m$ region, reproduces the decoupling behavior μ^2/m^2 we discussed in (29).

Now, we need to generalize the above derivation in the mass-dependent scheme for the simple ϕ^4 -theory to the full SM including the loops of the W, Z, t and Goldstones. In Appendix A, we show that the appropriate generalization of (39) is given by:

$$\left[\frac{\beta_\lambda}{8} - \frac{\gamma_\phi \lambda}{2} \right]_{\text{MOM}} = \sum_i \frac{n_i \kappa_i^2}{32\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} \tag{52}$$

in agreement with [9]. For the single real scalar case discussed above, we have to 1-loop $\gamma_\phi = 0$ and, from Table 1 we have $n_i = 1$ and $\kappa_i = 3\lambda/2$ so that we reproduce (51). Notice that when performing the sum over Goldstones, the parameter $(M_{\text{phys}}^2)_i$ becomes the physical mass of the vector boson corresponding to the Goldstone of type i .

Similarly, in ‘‘Appendix A’’ we also show that the generalization of (40) is given by:

$$\left[\frac{1}{2} \gamma_m - \gamma_\phi \right]_{\text{MOM}} = \sum_i \frac{n_i \kappa_i \kappa_i^m}{16\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}. \tag{53}$$

We therefore conclude, that in this mass-dependent scheme, the corresponding MOM expression for the \overline{MS} running of ρ_{ind} in (42) takes the following form:

$$\mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} \Big|_{\text{MOM}} = \rho_{\text{ind}}(\mu) \left(2\gamma_m - \frac{\beta_\lambda}{\lambda} \right) = \frac{\langle \phi \rangle^4}{32\pi^2} \sum_i n_i [\kappa_i^2 - \kappa_i \kappa_i^m \lambda] \times \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}. \tag{54}$$

Now it remains to derive the vacuum part, Eqs. (24) and (41), in the mass-dependent scheme. To accomplish that, one starts from the simple observation that the expression for the unrenormalized vacuum density (21) can be brought to the following form:

$$\bar{V}_{\text{vac}}^{(1)} = -\frac{m^4}{64\pi^2} \left(\frac{A_0(m)}{m^2} + \frac{1}{2} \right). \tag{55}$$

In above, $A_0(m)$ is the one-point Passarino–Veltman function with the properties given in the ‘‘Appendix A’’. Now, using the relation

$$\mu \frac{\partial A_0(M)}{\partial \mu} \Big|_{\text{MOM}} = M^2 \mu \frac{\partial B_0(0, M, M)}{\partial \mu} \Big|_{\text{MOM}} = M^2 \mu \frac{\partial B_0(p, M, M)}{\partial \mu} \Big|_{\text{MOM}} = 2M^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{M^2 + x(1-x)\mu^2} \tag{56}$$

and the fact that $V_{\text{vac}}(m^2, \lambda, \rho_\Lambda^{\text{vac}}, \mu)$ satisfies the RG equation (13) we obtain for the running of the vacuum part (41) in the MOM scheme³

³ The ‘‘vacuum bubble’’ in Fig. 1a is independent of the external momentum. In order to have an external momentum probe one needs to consider this ‘‘vacuum bubble’’ with external fields, such as for example the graviton legs. Then, to obtain the beta function for the $\rho_\Lambda^{\text{vac}}$ in the MOM scheme, one has to repeat the same steps as in the ϕ^4 theory above. First, one has to calculate the renormalization of the quantum corrections to the n-point function of gravitons, then make a finite subtraction of the value of this quantity at $p^2 = -\mu^2$ and, finally, calculate the derivative $\mu \partial / \partial \mu$ of the form-factors.

This program was carried out in [10] for the contributions of the loop of massive scalar to the propagator (2-point function) of the gravitational perturbation $h_{\mu\nu}$ on the flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the

$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} \Big|_{\text{MOM}} = m^4 \sum_i \frac{n_i (\kappa_i^m)^2}{32\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} \tag{57}$$

The essence of (56) is to ensure that once the finite subtraction (defining the MOM scheme) was made for $\beta_{\rho_{\text{ind}}}$, the same finite subtraction is made for $\beta_{\rho_\Lambda^{\text{vac}}}$.

Putting everything together, and using again $\langle \phi \rangle^2 = 2m^2/\lambda$, we achieve the generalization of (43)

$$\mu \frac{\partial (\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}})}{\partial \mu} \Big|_{\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M_{\text{phys}}^4)_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} \tag{58}$$

to the mass-dependent scheme. The Eq. (58) is the master equation describing the running of CC in any regime, non-decoupling and decoupling one, which is valid both in the UV and the IR regime. In the UV regime we recover (43), while in the IR regime we obtain the running of $\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$ including the decoupling effects associated with the mass thresholds as

$$\int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} = \begin{cases} 1 & \mu^2 \gg (M_{\text{phys}}^2)_i \\ \frac{\mu^2}{6(M_{\text{phys}}^2)_i} - \frac{\mu^4}{30(M_{\text{phys}}^4)_i} & \mu^2 \ll (M_{\text{phys}}^2)_i \end{cases} \tag{59}$$

which will be used in the following to show the consequences of such running.

In (58) we derived the running of $\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$ valid in the regime of decoupling of heavy particles $\mu^2 \ll M_i^2(\langle \phi \rangle)$ while for the light particles $(m_{\text{light}}^2)_j \ll \mu^2$, we can simply use (43). Working in the region where $(m_{\text{light}}^2)_j \ll \mu^2 \ll M_i^2(\langle \phi \rangle)$, we may combine these asymptotic results to obtain:

$$\begin{aligned} \mu \frac{\partial (\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}})}{\partial \mu} &= \sum_j \frac{n_j (m_{\text{light}}^4)_j}{32\pi^2} + \frac{1}{32\pi^2} \sum_i n_i (M_{\text{phys}}^4)_i \\ &\times \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} \\ &= \sum_j \frac{n_j (m_{\text{light}}^4)_j}{32\pi^2} + \frac{\mu^2}{12(4\pi)^2} \left[-12M_t^2 + 6M_W^2 + 3M_Z^2 + M_H^2 \right] \\ &+ \frac{\mu^4}{30(4\pi)^2} + \mathcal{O} \left(\frac{\mu^6}{(M_{\text{phys}}^2)_i} \right) \end{aligned} \tag{60}$$

The above expression is exactly of the form of (29) and proves the expected decoupling behavior in the effective theories. The light masses m_{light} may be, again, generated by the Higgs vev $m_{\text{light}}(\langle \phi \rangle)$ such as a mass for, say, charm quark, or may

result that in this approach one cannot reveal the beta functions for $\rho_\Lambda^{\text{vac}}$ and the form of the decoupling remained unclear.

be a new mass parameters in the SM Lagrangian related, for example, to the neutrino masses. As the μ -scale slides down the energy, more and more SM masses will migrate from the m_{light}^4 -term to the inside of the brackets in the μ^2 -term.

Notice that, when performing the sum over Goldstones, both beta functions for $\rho_\Lambda^{\text{vac}}$ (57) and ρ_{ind} (54) have the term $\sim M_H^4 (2/M_W^2 + 1/M_Z^2)$ but since it comes with the opposite sign it cancels in (60). This demonstrates the importance of considering the RG running of the total $\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$ parameter rather than RG running of these contributions separately.⁴

The $\mu^2 M_i^2$ term in the running of $\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$ provides the leading RG effect due to the heavy SM particles⁵ and we may demand it to vanish as to reduce the fine-tuning in the physical value of the CC at the $\mu_c = \mathcal{O}(10^{-3})$ eV. This requirement, however, leads to the SM prediction $m_H \approx 550$ GeV, inconsistent within the experimental value of $m_H \approx 125$ GeV.

As discussed in [12], heavy mass terms $\mu^2 M_i^2$ may also affect nucleosynthesis if we choose $\mu \sim T$, because they would induce vacuum energy density $\sim (T^2 M_i^2)/(4\pi)^2$ much bigger than the energy density of radiation ρ_{rad} at the typical energy of nucleosynthesis $T \sim 10^{-4}$ GeV. On the other hand, the m_{light}^4 - and μ^4 -terms obey the constraint $\rho_{\text{rad}} < \rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$ in the energy interval relevant for nucleosynthesis. To avoid the problem, either we have to use alternative choice $\mu \sim H$ or, again, sufficient amount of fine-tuning should be arranged among the various $\mu^2 M_i^2$ terms. Since, as we saw, the heavy SM spectrum does not have this tuning, our results imply that SM has to be extended or $\mu \sim H$ choice is preferred over the $\mu \sim T$ [12].

4 Massless theories

We now apply our result (58) to the massless theories which, as we will see, will give us new insights.

4.1 Massless Standard Model

In the massless limit of (60) $m = 0$ (i.e. all the terms with κ_i^m absent), from (57) we have $\rho_\Lambda^{\text{vac}} = \text{const}$ and only ρ_{ind}

⁴ The μ^2 -term, up to overall numerical factor 1/6, is the result predicted in [11] for the beta function of $\rho_\Lambda^{\text{vac}}$ alone. We also disagree in the μ^4 -term which shows inconsistency of the derivation presented in [11].

⁵ We are tacitly assuming that there is no $\mu^2 M^2$ contribution to the RG running from the particles decoupled at the higher scales such as Grand Unification or Planck scale.

runs with μ . In this case, the μ^2 -term can be related to the Veltman condition as we now show.

In the massless theory at the tree-level $\langle\phi\rangle = 0$, which means that the tree-level mass of the Higgs is zero and the electroweak symmetry needs to be broken radiatively. For this to happen, we need to balance the tree-level potential against the 1-loop contribution, so that for consistent perturbative expansion we have to impose the value of the Higgs quartic couplings at the electroweak scale to be parametrically given as $\lambda \sim (g^4, g'^4, y_t^4)$. This allows us to neglect the λ -terms in (60) associated with the Higgs and Goldstones and we obtain ($i = W, Z, t$ and neglecting the light masses m_{light}):

$$\begin{aligned} \mu \frac{\partial \rho_{\text{ind}}}{\partial \mu} &= \frac{\langle\phi\rangle^4}{32\pi^2} \sum_i n_i \kappa_i^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} \\ &= \frac{\mu^2}{12(4\pi)^2} [-12M_t^2 + 6M_W^2 + 3M_Z^2] + \frac{\mu^4}{20(4\pi)^2} \\ &= \frac{\mu^2 \langle\phi\rangle^2}{12(4\pi)^2} \frac{\partial}{\partial \phi^2} \sum_i n_i M_i^2(\phi) + \frac{\mu^4}{20(4\pi)^2}, \end{aligned} \tag{61}$$

where in the last line we used the fact that in the massless theory with only one background field ϕ , any mass can be written as $M_i^2(\phi) = \kappa_i \phi^2$. Notice that the μ^2 -proportional term is nothing but the generalization of the well known Veltman condition i.e. the requirement of the absence of the quadratic divergence.⁶ for the Higgs mass (cancellation of the prefactor of the ϕ^2 -term). *This means that in the massless case the fine-tuning problem of the Higgs mass is linked to the fine-tuning problem of the Cosmological Constant value⁷*

4.2 Massless Standard Model with extra massless real scalar

Let us now consider the simplest extension of the SM by adding one extra massless real scalar S :

⁶ We are working in the Landau gauge, however the 1-loop quadratically divergent part is gauge invariant in R_ξ gauges [13].

⁷ In the massive version of the SM, where Higgs mass is generated at tree-level via Higgs mechanism, the Veltman condition leads instead to

$$\text{V.C.}^{(\text{SM})} = 3\lambda(\mu) + \frac{9}{4}g^2(\mu) + \frac{3}{4}g'^2(\mu) - 6y_t^2(\mu) = 0 \tag{62}$$

and this is a relation among the dimensionless SM couplings at some energy scale μ . From (44), $M_H^2 = \lambda\langle\phi\rangle^2 = 2m^2$ and using $\mu \approx v_{EW} = 246$ GeV, Eq. (62) leads to:

$$\text{V.C.}^{(\text{SM})} = 3M_H^2 - 12M_t^2 + 6M_W^2 + 3M_Z^2 = 0 \tag{63}$$

predicting $M_H \approx 314$ GeV, which is in conflict with experiment too.

$$V_0 = V_0^{SM} + \lambda_{HS} \Phi^\dagger \Phi S^2 + \frac{\lambda_S}{4} S^4 \tag{64}$$

so that contribution from the Higgs background to the mass of the scalar S is given by $M_S^2(\phi) = \lambda_{HS} \phi^2$. In this model, the solution to the Veltman condition (61) reads⁸

$$\begin{aligned} 12M_t^2 - 6M_W^2 - 3M_Z^2 - M_S^2 = 0 &\implies \lambda_{HS}(\mu) \\ &= 6y_t^2(\mu) - \frac{9}{4}g^2(\mu) - \frac{3}{4}g'^2(\mu) \stackrel{\mu \approx v_{EW}}{\approx} 4.8. \end{aligned}$$

Working in the parameter space of the model where $\langle S \rangle = 0$, see [14] for details, leads to the scalar mass $M_S = \sqrt{\lambda_{HS}} v_{EW} \approx 550$ GeV which we already noticed above in the massive version of the SM where the role of the scalar S was played by the Higgs. Remarkably, with the mass of the scalar S satisfying the Veltman condition, we correctly predict the one-loop induced Higgs mass from the Coleman–Weinberg potential

$$\begin{aligned} M_H^2 &= \frac{3}{8\pi^2} \left[\frac{1}{16}(3g^4 + 2g^2g'^2 + g'^4) - y_t^4 + \frac{1}{3}\lambda_{HS}^2 \right] v_{EW}^2 \\ &\implies M_H \approx 125 \text{ GeV}. \end{aligned} \tag{65}$$

This provides an interesting example of how the demand for the absence of leading RG effects in the running of the ρ_{ind} due to the heavy particles may provide the hints on the possible extensions of the SM. Moreover, in this model there is no problem with nucleosynthesis for either of the choices for the RG scale $\mu \sim T$ or $\mu \sim H$.

5 Standard Model in the constant curvature space

For our final generalization of (58), we work with the full renormalized version of the Hilbert–Einstein action (2) containing additional coupling constants $\kappa = (16\pi G)^{-1} = M_{pl}^2/2$, and non-minimal coupling ξ . We consider the Standard Model in the constant curvature space $R_{\mu\nu} = (R/4)g_{\mu\nu}$ and working in the linear curvature approximation in “Appendix B” we show that appropriate generalization of (43) is given by

$$\mu \frac{\partial(\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}} + \kappa R)}{\partial \mu} = \sum_i \frac{n_i}{32\pi^2} \mathcal{M}_i^4(\langle\phi\rangle) \tag{66}$$

where

$$\mathcal{M}_i^2(\langle\phi\rangle) = \kappa_i \langle\phi\rangle^2 - \kappa_i^m m^2 + \left(\kappa_i^R - \frac{1}{6} \right) R, \tag{67}$$

⁸ Notice that in (61) the value of μ is evaluated at the cosmological RG scale $\mu \approx \mu_c$ which is experimentally given by $\mu_c = \mathcal{O}(10^{-3}) eV$. Naively, one would think that we also need to impose the Veltman condition for the SM dimensionless couplings evaluated at this cosmological scale. However, due to decoupling of the SM degrees of freedom below the electroweak scale, the SM beta functions for the dimensionless couplings will scale as μ^2/m^2 so that coupling values “freeze” quickly at their electroweak scale values and therefore the Veltman condition will remain the same up to μ^2/m^2 effects.

with parameters κ_i^R defined in Table 2 in “Appendix B”. Generalizing to the mass-dependent scheme we obtain (see “Appendix B” for details):

$$\begin{aligned} \mu \frac{\partial(\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}} + \kappa R)}{\partial \mu} &= \sum_i \frac{n_i}{32\pi^2} \mathcal{M}_i^4(\langle\phi\rangle) \\ &\times \int_0^1 \frac{x(1-x)\mu^2 dx}{\mathcal{M}_i^2(\langle\phi\rangle) + x(1-x)\mu^2} \\ &= \sum_j \frac{n_j (\mathcal{M}_{\text{light}}^4)^j}{32\pi^2} + \frac{\mu^2}{12(4\pi)^2} \\ &\times \left[-12\tilde{m}_i^2 + 6\tilde{m}_W^2 + 3\tilde{m}_Z^2 + \tilde{m}_H^2 + \frac{7}{3}R \right] + \frac{\mu^4}{30(4\pi)^2}, \end{aligned} \tag{68}$$

where masses \tilde{m}_i^2 have corrections from the non-minimal Higgs coupling ξ

$$\tilde{m}_i^2 \equiv M_i^2(\langle\phi\rangle) - 2\kappa_i \frac{\xi R}{\lambda} \tag{69}$$

with $M_i^2(\langle\phi\rangle)$ defined in (44). The result (68) generalizes effective theory expansion (29) to the constant curvature space

$$\beta\left(m_{\text{light}}, \frac{\mu}{m}\right) = a_1 m_{\text{light}}^4 + b_1 \mu^2 m^2 + c_1 \mu^4 + d_1 \mu^2 R + \dots \tag{70}$$

which also appears via explicit calculations on the expanding cosmological background where vacuum energy is dynamical [15]. The result (68) also generalizes the flat space result to possibility of, for example, curvature-induced running of the vacuum energy and curvature-induced phase transitions [16–21].

6 Conclusions

We revisited the decoupling effects associated with heavy particles in the RG running of the vacuum energy using the mass-dependent renormalization scheme. We derived the universal one-loop beta function of the vacuum energy $\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$, arising from the Higgs vacuum and the Cosmological Constant term in the entire energy range, valid in the UV and in the IR regime. We have shown that although $\rho_\Lambda^{\text{vac}}$ and ρ_{ind} run separately, it is only the sum $\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$ that exhibits behavior consistent with the decoupling theorem.

At the energy scale lower than the mass of the particle, the leading term in the RG running of $\rho_\Lambda^{\text{vac}} + \rho_{\text{ind}}$ is proportional to the square of the mass of the heavy particle which leads to the enhanced RG running and, consequently, severe

fine-tuning problem with the measured value of the Cosmological Constant. We show that the condition of absence of this leading effect is not satisfied in the SM, while in the massless theories, where Higgs mass is generated radiatively via Coleman–Weinberg mechanism, this constraint formally coincides with Veltman condition. In a simple extension of the SM with addition of one massless real scalar the condition of absence of leading effect in $\beta_{\rho_{\text{ind}}}$ allowed us to predict the radiative Higgs mass correctly.

Finally, we also provided the generalization to the constant curvature space in the linear curvature approximation finding the effective field theory expansion that also appears via explicit calculations on the expanding cosmological background. In view of this, our results also might have impact on the models based on the dynamical cosmological constant which were confronted with new cosmological observations in [22, 23] with the results being still inconclusive [24, 25].

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Appendix A: Mass-dependent scheme derivation of the CC running

In the renormalized perturbation theory, one rewrites the bare parameter θ_0 as

$$\theta_0 = \theta_{os} - \delta\theta_{os} = \theta_{\overline{MS}}(\mu) - \delta\theta_{\overline{MS}} = \theta_{\text{MOM}}(\mu) - \delta\theta_{\text{MOM}} \tag{A1}$$

where we use on-shell (OS), momentum subtraction (MOM) and \overline{MS} schemes with $\theta_{\text{MOM}}(\mu)$, θ_{os} , $\theta_{\overline{MS}}$ as the renormalized MOM, OS, \overline{MS} parameters and $\delta\theta_{\text{MOM}}$, $\delta\theta_{os}$, $\delta\theta_{\overline{MS}}$ as corresponding counterterms. By definition $\delta\theta_{\text{MOM}}$ subtracts, in the dimensional regularization, the usual $\delta\theta_{\overline{MS}}$ structure $1/\epsilon + \gamma - \ln(4\pi)$ and, in addition, makes a finite subtraction of the value of the quantity at $p^2 = -\mu^2$. Concerning the structure of the $1/\epsilon$ poles between any two schemes, one notices that it should be identical once the poles in one scheme are expressed in terms of the quantities of the other scheme. Therefore the difference between the counterterms

in two schemes is finite. Using the fact, that θ_{os} is physical (μ -independent) parameter, this allows to extract the beta function as, for example,

$$\begin{aligned} \theta_{\overline{MS}}(\mu) &= \theta_{os} - (\delta\theta_{os} - \delta\theta_{\overline{MS}})|_{\text{fin}} \implies \\ \beta_{\theta_{\overline{MS}}} &= -\mu \frac{\partial}{\partial \mu} (\delta\theta_{os} - \delta\theta_{\overline{MS}})|_{\text{fin}}. \end{aligned} \tag{A2}$$

Now, to calculate the beta function in the MOM scheme, one calculates the derivative $\mu\partial/\partial\mu$ of the renormalized form-factor ignoring the possible kinematical factors associated with the external momentum. So, our goal is to find the external momentum dependence of the SM parameters $\theta = (\lambda, m)$.

To achieve this, we can use the results of [26], where the finite part of $(\delta\theta_{os} - \delta\theta_{\overline{MS}})|_{\text{fin}}$ was provided in terms of the one- and two-point Passarino–Veltman functions:

$$\begin{aligned} A_0(M) &= M^2 \left(1 - \log \frac{M^2}{\mu_{\overline{MS}}^2} \right), \quad B_0(p; M_1, M_2) \\ &= - \int_0^1 \log \frac{xM_1^2 + (1-x)M_2^2 - x(1-x)p^2}{\mu_{\overline{MS}}^2} dx, \end{aligned} \tag{A3}$$

which are connected as

$$A_0(M) = M^2 [B_0(0, M, M) + 1], \tag{A4}$$

and

$$\mu \frac{\partial A_0(M)}{\partial \mu} = M^2 \mu \frac{\partial B_0(0, M, M)}{\partial \mu}. \tag{A5}$$

In the MOM scheme the following relation is valid

$$\begin{aligned} \mu \frac{\partial A_0(M)}{\partial \mu} \Big|_{\text{MOM}} &= M^2 \mu \frac{\partial B_0(0, M, M)}{\partial \mu} \Big|_{\text{MOM}} \\ &= M^2 \mu \frac{\partial B_0(p, M, M)}{\partial \mu} \Big|_{\text{MOM}}. \end{aligned} \tag{A6}$$

which stems from the fact that

$$\begin{aligned} B_0(p; M_1, M_2) \Big|_{\text{MOM}} \\ = - \int_0^1 \log \frac{xM_1^2 + (1-x)M_2^2 - x(1-x)p^2}{xM_1^2 + (1-x)M_2^2 + x(1-x)\mu^2} dx. \end{aligned} \tag{A7}$$

Moreover, the unrenormalized form of A_0

$$A_0(M) = -M^2 \left[\frac{2}{n-4} + \log \left(\frac{M^2}{4\pi\mu^2} \right) + \gamma_E - 1 \right] \tag{A8}$$

was used in (55).

With the expressions above we can easily reconstruct the external momentum dependence of the renormalized form-factor we are looking for. We use the one-loop result for the quartic coupling [26] (notice that all masses are physical):

$$\begin{aligned} (\delta\lambda_{os} - \delta\lambda_{\overline{MS}})|_{\text{fin}} &= - \frac{2}{(4\pi)^2 v_{\text{EW}}^4} \text{Re} \left[3M_t^2 (M_H^2 - 4M_t^2) \right. \\ &\quad \left. \times B_0(M_H; M_t, M_t) + 3M_H^2 A_0(M_t) \right] \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{4} (M_H^4 - 4M_H^2 M_W^2 + 12M_Z^4) \\ &\quad \times B_0(M_H; M_Z, M_Z) \\ &\quad + \frac{M_H^2 (7M_W^2 - 4M_Z^2)}{2(M_Z^2 - M_W^2)} A_0(M_Z) \\ &+ \frac{1}{2} (M_H^4 - 4M_H^2 M_W^2 + 12M_W^4) \\ &\quad \times B_0(M_H; M_W, M_W) \\ &\quad - \frac{3M_H^2 M_W^2}{2(M_H^2 - M_W^2)} A_0(M_H) \\ &\quad + \frac{M_H^2}{2} \left(-11 + \frac{3M_H^2}{M_H^2 - M_W^2} \right. \\ &\quad \left. - \frac{3M_W^2}{M_Z^2 - M_W^2} \right) A_0(M_W) \\ &\quad + \frac{9}{4} M_H^4 B_0(M_H; M_H, M_H) \\ &\quad + \frac{1}{4} (M_H^4 + M_H^2 (M_Z^2 + 2M_W^2 - 6M_t^2) \\ &\quad \left. - 8(M_Z^4 + 2M_W^4)) \right]. \end{aligned} \tag{A9}$$

Using (A2), it is easy to show that (A9) leads to:

$$\begin{aligned} \left[\frac{\beta_\lambda}{8} - \frac{\gamma_\phi \lambda}{2} \right]_{\overline{MS}} &= \frac{\mu_{\overline{MS}}}{2(4\pi)^2 v_{\text{EW}}^4} \frac{\partial}{\partial \mu_{\overline{MS}}} \left[-12M_t^4 B_0(M_H; M_t, M_t) \right. \\ &\quad + \frac{1}{4} (M_H^4 + 12M_Z^4) B_0(M_H; M_Z, M_Z) \\ &\quad + \frac{1}{2} (M_H^4 + 12M_W^4) B_0(M_H; M_W, M_W) \\ &\quad \left. + \frac{9}{4} M_H^4 B_0(M_H; M_H, M_H) \right]. \end{aligned} \tag{A10}$$

To obtain the corresponding object in the MOM scheme, we only have to reinstate $B_0(M_H; M_1, M_2) \rightarrow B_0(p; M_1, M_2)$, make a finite subtraction at $p^2 = -\mu^2$ and calculate the derivative $\mu\partial/\partial\mu$.⁹ After that, we arrive at (52)

$$\left[\frac{\beta_\lambda}{8} - \frac{\gamma_\phi \lambda}{2} \right]_{\text{MOM}} = \sum_i \frac{n_i \kappa_i^2}{32\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}. \tag{A11}$$

Similarly, the Higgs mass term is corrected as [26]:

$$\begin{aligned} (\delta m_{os}^2 - \delta m_{\overline{MS}}^2)|_{\text{fin}} \\ = - \frac{1}{(4\pi)^2 v_{\text{EW}}^2} \text{Re} \left[6M_t^2 (M_H^2 - 4M_t^2) B_0(M_H; M_t, M_t) \right. \\ + 24M_t^2 A_0(M_t) \\ \left. + (M_H^4 - 4M_H^2 M_W^2 + 12M_W^4) B_0(M_H; M_W, M_W) \right] \end{aligned}$$

⁹ When converting (A9) to the MOM scheme, there will be an additional external momentum dependence due to kinematics. However, as discussed, only momentum dependence of the function $B_0(p; M, M)$ contributes to the beta function in the MOM scheme.

$$\begin{aligned}
 & -2(M_H^2 + 6M_W^2)A_0(M_W) \\
 & + \frac{1}{2}(M_H^4 - 4M_H^2M_Z^2 + 12M_Z^4)B_0(M_H; M_Z, M_Z) \\
 & - (M_H^2 + 6M_Z^2)A_0(M_Z) \\
 & + \frac{9}{2}M_h^4B_0(M_H; M_H, M_H) - 3M_H^2A_0(M_H) \Big], \quad (A12)
 \end{aligned}$$

and repeating the same steps as in the λ case above, we obtain (53):

$$\left[\frac{1}{2}\gamma_m - \gamma_\phi \right]_{\text{MOM}} = \sum_i \frac{n_i \kappa_i \kappa_i^m}{16\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}. \quad (A13)$$

The $\beta_\lambda, \beta_{m^2}$ and γ_ϕ are one-loop β - and γ -functions:

$$\begin{aligned}
 16\pi^2\beta_\lambda &= 12(\lambda^2 - y_t^4 + \lambda y_t^2) - (3g'^2 + 9g^2)\lambda \\
 &+ \frac{9}{4} \left[\frac{1}{3}g'^4 + \frac{2}{3}g'^2g^2 + g^4 \right], \\
 16\pi^2\beta_{m^2} &= 16\pi^2\gamma_m m^2 = m^2 \left[6\lambda + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2 \right], \\
 16\pi^2\gamma_\phi &= 3 \left(y_t^2 - \frac{1}{4}g'^2 - \frac{3}{4}g^2 \right). \quad (A14)
 \end{aligned}$$

Appendix B: Generalization to constant curvature space

We work in the constant curvature space $R_{\mu\nu} = (R/4)g_{\mu\nu}$ and in the linear curvature approximation we consider

$$V(\rho_\Lambda^{\text{vac}}, \phi, m^2, \lambda_i, \kappa, \xi, \mu) \equiv V_0 + V_1 + \dots, \quad (B1)$$

where $\lambda_i \equiv (g, g', \lambda, h_t)$ runs over SM dimensionless couplings and V_0, V_1 are respectively the tree level potential and the one-loop correction, namely [27]

$$V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4 + \frac{1}{2}\xi R\phi^2, \quad (B2)$$

$$\begin{aligned}
 V_1 &= \sum_i \frac{n_i}{64\pi^2} \left(\bar{M}_i^4(\phi) - \frac{\bar{M}_i^2(\phi)R}{3} \right) \log \frac{\bar{M}_i^2(\phi)}{\mu_{MS}^2} \\
 &+ \rho_\Lambda^{\text{vac}} + \kappa R, \quad (B3)
 \end{aligned}$$

where we showed only logarithmic term relevant for us and defined

$$\bar{M}_i^2(\phi) = \kappa_i\phi^2 - \kappa_i^m m^2 + \kappa_i^R R, \quad (B4)$$

with the parameters $n_i, \kappa_i, \kappa_i^m$ and κ_i^R shown in Table 2.

$\bar{M}_i^2(\phi)$ are the tree-level expressions for the background-dependent and curvature-dependent masses of the particles that enter in the one-loop radiative corrections. Also $\kappa = (16\pi G)^{-1} = M_{pl}^2/2$ and ξ is the non-minimal coupling. It

Table 2 Contributions to the effective potential (B5) from the SM particles W^\pm, Z^0 , top quark t , Higgs ϕ and the Goldstone bosons $\chi_{1,2,3}$ [28]

Φ	i	n_i	κ_i	κ_i^m	κ_i^R
$W^\pm(\text{ghost})$	1	-2	$g^2/4$	0	1/2
W^\pm	2	8	$g^2/4$	0	1/2
$Z^0(\text{ghost})$	3	-1	$(g^2 + g'^2)/4$	0	1/2
Z^0	4	4	$(g^2 + g'^2)/4$	0	1/2
t	5	-12	$y_t^2/2$	0	1/4
ϕ	6	1	$3\lambda/2$	1	1/2
χ_i	7	3	$\lambda/2$	1	1/2

is convenient to redefine $\mathcal{M}_i^2 \equiv \bar{M}_i^2 - R/6$ so that (up to R^2 -terms)

$$V_1 = \sum_i \frac{n_i}{64\pi^2} \mathcal{M}_i^4(\phi) \log \frac{\mathcal{M}_i^2(\phi)}{\mu_{MS}^2} + \rho_\Lambda^{\text{vac}} + \kappa R. \quad (B5)$$

We again split the potential to vacuum and ϕ -dependent pieces

$$\begin{aligned}
 V(\rho_\Lambda^{\text{vac}}, \phi, m^2, \lambda_i, \kappa, \xi, \mu) &= V_{\text{vac}}(\rho_\Lambda^{\text{vac}}, m^2, \lambda_i, \kappa, \xi, \mu) \\
 &+ V_{\text{scal}}(\phi, m^2, \lambda_i, \xi, \mu). \quad (B6)
 \end{aligned}$$

and the RG equations (12–13) get now modified as follows:

$$\begin{aligned}
 \left(\mu \frac{\partial}{\partial \mu} + \beta_{\lambda_i} \frac{\partial}{\partial \lambda_i} + \gamma_m m^2 \frac{\partial}{\partial m^2} - \gamma_\phi \phi \frac{\partial}{\partial \phi} + \beta_{\rho_\Lambda^{\text{vac}}} \frac{\partial}{\partial \rho_\Lambda^{\text{vac}}} \right. \\
 \left. + \beta_\kappa \frac{\partial}{\partial \kappa} + \beta_\xi \frac{\partial}{\partial \xi} \right) V(\rho_\Lambda^{\text{vac}}, \phi, m^2, \lambda_i, \kappa, \xi, \mu) = 0 \quad (B7)
 \end{aligned}$$

$$\begin{aligned}
 \left(\mu \frac{\partial}{\partial \mu} + \beta_{\lambda_i} \frac{\partial}{\partial \lambda_i} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \beta_{\rho_\Lambda^{\text{vac}}} \frac{\partial}{\partial \rho_\Lambda^{\text{vac}}} \right. \\
 \left. + \beta_\kappa \frac{\partial}{\partial \kappa} + \beta_\xi \frac{\partial}{\partial \xi} \right) V_{\text{vac}}(\rho_\Lambda^{\text{vac}}, m^2, \lambda_i, \kappa, \xi, \mu) = 0, \quad (B8)
 \end{aligned}$$

$$\begin{aligned}
 \left(\mu \frac{\partial}{\partial \mu} + \beta_{\lambda_i} \frac{\partial}{\partial \lambda_i} + \gamma_m m^2 \frac{\partial}{\partial m^2} - \gamma_\phi \phi \frac{\partial}{\partial \phi} + \beta_\xi \frac{\partial}{\partial \xi} \right) \\
 V_{\text{scal}}(\phi, m^2, \lambda_i, \xi, \mu) = 0. \quad (B9)
 \end{aligned}$$

These equations are valid for any value of ϕ . However, for the extremum value $\phi = \langle \phi \rangle$ defined via $\left. \frac{\partial V_{\text{scal}}(\phi)}{\partial \phi} \right|_{\langle \phi \rangle} = 0$, the term containing anomalous dimension of the Higgs γ_ϕ will drop out and we have:

$$\begin{aligned}
 \left(\mu \frac{\partial}{\partial \mu} + \beta_{\lambda_i} \frac{\partial}{\partial \lambda_i} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \beta_\xi \frac{\partial}{\partial \xi} \right) \\
 V_{\text{scal}}(\langle \phi \rangle, m^2, \lambda_i, \xi, \mu) = 0. \quad (B10)
 \end{aligned}$$

Using the tree-level potential (B2), it is useful to define parameter $\rho_{ind}(\mu) = V_0(\langle\phi\rangle) = -\frac{(m^2(\mu)-\xi(\mu)R)^2}{2\lambda(\mu)}$ which will play the role similar to the $\rho_\Lambda^{vac}(\mu)$. The running of this parameter reads:

$$\mu \frac{\partial \rho_{ind}(\mu)}{\partial \mu} = -\frac{m^2 - \xi R}{\lambda} (\gamma_m m^2 - \beta_\xi R) + \frac{(m^2 - \xi R)^2}{2\lambda^2} \beta_\lambda. \tag{B11}$$

By equating the terms with R and different powers of ϕ :

$$\mu \frac{dV}{d\mu} \sim (\phi^4[\dots] + m^2 \phi^2[\dots] + m^4[\dots] + m^2 R[\dots] + \phi^2 R[\dots]) = 0, \tag{B12}$$

it is straightforward to check that the requirement (B7) applied to the full one-loop effective potential leads to

$$\frac{1}{8} \beta_\lambda - \frac{1}{2} \gamma_\phi \lambda = \sum_i \frac{n_i \kappa_i^2}{32\pi^2}, \tag{B13}$$

$$\frac{1}{2} \gamma_m - \gamma_\phi = \sum_i \frac{n_i \kappa_i \kappa_i^m}{16\pi^2}, \tag{B14}$$

$$\mu \frac{\partial \rho_\Lambda^{vac}}{\partial \mu} = m^4 \sum_i \frac{n_i (\kappa_i^m)^2}{32\pi^2}, \tag{B15}$$

$$\mu \frac{\partial \kappa}{\partial \mu} = m^2 \sum_i \frac{n_i \kappa_i^m}{16\pi^2} \left(\kappa_i^R - \frac{1}{6} \right), \tag{B16}$$

$$\frac{1}{2} \beta_\xi - \xi \gamma_\phi = \sum_i \frac{n_i \kappa_i}{16\pi^2} \left(\kappa_i^R - \frac{1}{6} \right), \tag{B17}$$

where SM β - and γ -functions are given above in (A14), while

$$16\pi^2 \beta_\kappa = \frac{4}{3} m^2, \tag{B18}$$

$$16\pi^2 (\beta_\xi - 2\xi \gamma_\phi) = 2\lambda - y_t^2 + \frac{g'^2}{2} + \frac{3g^2}{2}, \tag{B19}$$

with β_ξ as reported in [28]. Working to linear order in R and using (B13), (B14) and (B17) in order to reconstruct B11, we finally obtain

$$\mu \frac{\partial (\rho_\Lambda^{vac} + \rho_{ind} + \kappa R)}{\partial \mu} = \sum_i \frac{n_i}{32\pi^2} \mathcal{M}_i^4(\langle\phi\rangle) \tag{B20}$$

where we used

$$\begin{aligned} \mathcal{M}_i^2(\langle\phi\rangle) &= \kappa_i \langle\phi\rangle^2 - \kappa_i^m m^2 + \left(\kappa_i^R - \frac{1}{6} \right) R \equiv \tilde{m}_i^2(\langle\phi\rangle) \\ &+ \left(\kappa_i^R - \frac{1}{6} \right) R, \end{aligned} \tag{B21}$$

with $\langle\phi\rangle^2 = \frac{2(m^2 - \xi R)}{\lambda}$. Notice that for the Goldstones $\mathcal{M}_\chi^4(\langle\phi\rangle) \sim R$ and therefore $\mathcal{M}_\chi^4(\langle\phi\rangle) \sim R^2$ so that to linear R order they do not contribute to the sum. The masses \tilde{m}_i^2 formally look identical to the flat space analogues $M_i^2(\langle\phi\rangle)$ in (44) but, however, contain the curvature corrections via the $\langle\phi\rangle$:

$$\begin{aligned} \tilde{m}_i^2(\langle\phi\rangle) &= \kappa_i \langle\phi\rangle^2 - \kappa_i^m m^2 = \kappa_i \frac{2(m^2 - \xi R)}{\lambda} - \kappa_i^m m^2 \\ &= M_i^2(\langle\phi\rangle) - 2\kappa_i \frac{\xi R}{\lambda}. \end{aligned} \tag{B22}$$

Generalizing to the mass-dependent scheme we obtain:

$$\begin{aligned} \mu \frac{\partial (\rho_\Lambda^{vac} + \rho_{ind} + \kappa R)}{\partial \mu} &= \sum_i \frac{n_i}{32\pi^2} \mathcal{M}_i^4(\langle\phi\rangle) \\ &\int_0^1 \frac{x(1-x)\mu^2 dx}{\mathcal{M}_i^2(\langle\phi\rangle) + x(1-x)\mu^2}. \end{aligned} \tag{B23}$$

Since the mass of the ghost is equal to the corresponding vector boson mass, the ghost cancels the unphysical gauge mode (see Table 2) and we obtain:

$$\begin{aligned} \mu \frac{\partial (\rho_\Lambda^{vac} + \rho_{ind} + \kappa R)}{\partial \mu} &= \sum_j \frac{n_j (\mathcal{M}_{light}^4)_j}{32\pi^2} \\ &+ \frac{\mu^2}{12(4\pi)^2} \sum_i n_i \mathcal{M}_i^2(\phi) - \frac{\mu^4}{60(4\pi)^2} \sum_i n_i \\ &= \sum_j \frac{n_j (\mathcal{M}_{light}^4)_j}{32\pi^2} + \frac{\mu^2}{12(4\pi)^2} \\ &\times \left[-12\tilde{m}_t^2 + 6\tilde{m}_W^2 + 3\tilde{m}_Z^2 + \tilde{m}_H^2 + \frac{7}{3}R \right] + \frac{\mu^4}{30(4\pi)^2}. \end{aligned} \tag{B24}$$

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