

Do we have a chance for renormalizable and unitary quantum gravity?

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Semiclassical approach and higher derivatives in QG

General Relativity (GR) is a complete theory of classical gravitational phenomena. It proved valid in the wide range of energies and distances.

At the same time, singular behavior in general solutions of GR indicates the need for extending the theory.

The most natural is to assume that

- **GR is not valid at all scales.**

At very short distances and/or when the curvature becomes very large, the gravitational phenomena must be described by a more extensive theory of gravity.

Indeed, we expect that this unknown theory coincides with GR at the large distance & weak field limit.

The dimensional arguments hint that the origin of deviations from GR are most likely related to quantum effects.

Three choices for Quantum Gravity (QG)

The fundamental units probably indicate some new and fundamental physics at the Planck scale.

General classification of possible approaches into three distinct groups:

- **Quantize both gravity and matter fields. This is, definitely, the most fundamental possible approach.**
- **Quantize only matter fields on classical curved background (semiclassical approach).**
- **Quantize something else. E.g., in case of (super)string theory both matter and gravity are induced.**

Which approach is “better”?

Indeed, they have something in common.

Semiclassical approach: background gravity

The vacuum effective action includes contributions of fields Φ ,

$$e^{i\Gamma(g_{\mu\nu})} = e^{iS_{vac}(g_{\mu\nu})} \int d\Phi e^{iS_m(\Phi, g_{\mu\nu})}.$$

The vacuum action of renormalizable QFT in curved space is

$$S_{vac} = S_{EH} + S_{HD}, \quad \text{where} \quad S_{EH} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \{R + 2\Lambda\},$$

$$S_{HD} = \int d^4x \sqrt{-g} \{a_1 C^2 + a_2 E + a_3 \square R + a_4 R^2\},$$

$$\text{where} \quad C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3 R^2$$

Without HD terms in the vacuum sector there is no consistency (renormalizability, running etc) in the semiclassical gravity.

QG: covariant renormalization and power counting

As the first example consider quantum GR.

$$S_{EH} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + 2\Lambda).$$

Power counting: $D + d = 2 + 2p$.

At the 1-loop level we can expect the divergences like

$$\mathcal{O}(R^2) = R_{\mu\nu\alpha\beta}^2, R_{\mu\nu}^2, R^2.$$

t'Hooft and Veltman; Deser and van Nieuwenhuisen, (1974); ...

At the 2-loop level we have

$$\mathcal{O}(R^3) = R_{\mu\nu} \square R^{\mu\nu}, \dots R^3, R_{\mu\nu} R_{\alpha}^{\mu} R^{\alpha\nu}, R_{\mu\nu\alpha\beta} R^{\mu\nu}{}_{\rho\sigma} R^{\mu\nu\rho\sigma}.$$

M.H. Goroff and A. Sagnotti, NPB 266 (1986).

The last structure does not vanish on-shell and this proves that the theory is not renormalizable, at least within the standard perturbative approach.

Within the standard perturbative approach non-renormalizability means the theory has no predictive power.

Every time we introduce a new type of a counterterm, it is necessary to fix renormalization condition and this means a measurement. So, before making a single predictions, it is necessary to have an infinite amount of experimental data.

What are the possible solutions?

- Change standard perturbative approach to something else.** There are many options, but their consistency or their relation to the QG program are not clear, in all cases.
- Change the theory, i.e., take another theory to construct QG.**

The first option is widely explores in the asymptotic safety scenarios, in the effective approaches to QG, induced gravity approach (including string theory) and so on.

Let us concentrate on the second idea.

The most natural choice is four derivative model, because we need four derivatives anyway for quantum matter field.

Already known action: $S_{gravity} = S_{EH} + S_{HD}$

where S_{HD} includes square of the Weyl tensor and R

$$S_{HD} = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2\lambda} C^2 + \frac{\omega}{3\lambda} R^2 + \text{surface terms} \right\},$$

$$C^2(4) = R_{\mu\nu\alpha\beta}^2 - 2R_{\alpha\beta}^2 + R^2/3,$$

Propagators of metric and ghosts behave like $\mathcal{O}(k^{-4})$ and we have K_4, K_2, K_0 vertices.

The superficial degree of divergence

$$D + d = 4 - 2K_2 - 4K_0.$$

This theory is definitely renormalizable. Dimensions of counterterms are 4, 2, 0.

K. Stelle, Phys. Rev. D (1977).

Well, there is a price to pay: massive ghosts

$$G_{\text{spin-2}}(k) \sim \frac{1}{m^2} \left(\frac{1}{k^2} - \frac{1}{k^2 + m^2} \right), \quad m \propto M_P.$$

The tree-level spectrum includes massless graviton and massive spin-2 “ghost” with negative kinetic energy and a huge mass.

Particle with negative energy means instability of vacuum state.

For instance, the Minkowski space is not protected from the spontaneous creation of massive ghost and many gravitons from vacuum.

All in all, HDQG seems to have a very serious problem with massive ghosts.

One can include more than four derivatives,

$$S = S_{EH} + \sum_{n=0}^N \int d^4x \sqrt{-g} \left\{ \omega_n^C C_{\mu\nu\alpha\beta} \square^n C_{\mu\nu\alpha\beta} + \omega_n^R R \square^n R \right\} + \mathcal{O}(R^3, \dots).$$

Simple analysis shows that in this theory massive ghost-like states are still present. For the real poles:

$$G_2(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2 + m_1^2} + \frac{A_2}{k^2 + m_2^2} + \dots + \frac{A_{N+1}}{k^2 + m_{N+1}^2}.$$

For any sequence $0 < m_1^2 < m_2^2 < m_3^2 < \dots < m_{N+1}^2$, the signs of the corresponding terms alternate, $A_j \cdot A_{j+1} < 0$.

M. Asorey, J.-L. Lopez & I. Sh., IJMPPhA (1997), hep-th/9610006.

$$S = S_{EH} + \int d^4x \sqrt{-g} \left\{ \omega_N^C C_{\mu\nu\alpha\beta} \square^N C_{\mu\nu\alpha\beta} + \omega_N^R R \square^N R + \dots \right\}.$$

Again, let us consider only vertices with a maximal $K_\nu = 2k + 4$.

Then we have $r_l = K_\nu = 2k + 4$ and, combining

$$D + d = \sum_{l_{int}} (4 - r_l) - 4n + 4 + \sum_{\nu} K_\nu$$

$$\text{with } l_{int} = p + n - 1,$$

we can easily arrive at the estimate of d for $D = 0$

$$d = 4 + k(1 - p).$$

For $k = 0$ we meet the standard HDQG result, $d \equiv 4$. Starting from $k = 1$ we have superrenormalizable theory, where the divergences exist only for $p = 1, 2, 3$.

For $k \geq 3$ we have superrenormalizable theory, where divergences exist only for $p = 1$, that is at the one-loop level.

Exact β -functions in QG

In the superrenormalizable QG one can derive exact RG equations by working at the one-loop level!

M. Asorey, J.-L. Lopez & I. Sh., IJMPPhA (1997), hep-th/9610006.

$$\beta_{\Lambda} = \mu \frac{d}{d\mu} \left(-\frac{\Lambda}{8\pi G} \right) = \frac{1}{(4\pi)^2} \left(\frac{5\omega_{N-2,C}}{\omega_{N,C}} + \frac{\omega_{N-2,R}}{\omega_{N,R}} - \frac{5\omega_{N-1,C}^2}{2\omega_{N,C}^2} - \frac{\omega_{N-1,R}^2}{2\omega_{N,R}^2} \right).$$

L. Modesto, L. Rachwal & I.Sh., arXiv:1704.03988.

$$\beta_G = \mu \frac{d}{d\mu} \left(-\frac{1}{16\pi G} \right) = -\frac{1}{6(4\pi)^2} \left(\frac{5\omega_{N-1,C}}{\omega_{N,C}} + \frac{\omega_{N-1,R}}{\omega_{N,R}} \right).$$

Different from four-derivative version these β -functions are gauge-fixing independent expressions.

Once again: what is bad in the higher-derivative gravity?

For the linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (1)$$

we meet

$$G_{\text{spin-2}}(k) \sim \frac{1}{m^2} \left(\frac{1}{k^2} - \frac{1}{k^2 + m^2} \right), \quad m \propto M_P.$$

Tree-level spectrum includes massless graviton and massive spin-2 “ghost” with negative kinetic energy and huge mass.

- **Interaction between ghost and gravitons may violate energy conservation in the massless sector** (*M.J.G. Veltman, 1963*).
- **In classical systems higher derivatives generate exploding instabilities at the non-linear level** (*M.V. Ostrogradsky, 1850*).
- **Without ghost one violates unitarity of the S-matrix.**

Ghost-free HD models of gravity

There are two examples of ghost-free HD models of gravity.

- **In the (super)string theory, the object of quantization is a kind of non-linear sigma-model in two space-time dimensions.**

Both metric and matter fields are induced, implying unification of all fundamental forces.

The σ -model approach is close to QFT in curved space,

$$\mathcal{S}_{str} = \int d^2\sigma \sqrt{g} \left\{ \frac{1}{2\alpha'} g^{\mu\nu} G_{ij}(X) \partial_\mu X^i \partial_\nu X^j + \frac{1}{\alpha'} \frac{\varepsilon^{\mu\nu}}{\sqrt{g}} A_{ij}(X) \partial_\mu X^i \partial_\nu X^j + B(X)R + T(X) \right\}, \quad i, j = 1, 2, \dots, D.$$

The Polyakov approach: conditions of anomaly cancellation order by order in α' . Critical dimensions:

D=26 for bosonic string, D=10 for superstrings.

At the first order in α' the effective equations give GR !

E.S. Fradkin & A. Tseytlin (1985);

C. Callan, D. Friedan, E. Martinec, M. Perry, (1985).

● **Metric reparametrization remove ghosts at all orders in α' .**

In the torsionless case the effective action can be written as

$$S_M = \frac{2}{\kappa^2} \int d^D x \sqrt{G} e^{-2\phi} \left\{ -R + 4(\partial\phi)^2 \right. \\ \left. + \alpha' (a_1 R_{\lambda\mu\nu\rho} R^{\lambda\mu\nu\rho} + a_2 R_{\mu\nu} R^{\mu\nu} + a_3 R^2) \right\} + \dots$$

In order to remove ghosts one performs reparametrization of the background metric $G_{\mu\nu}$

$$G_{\mu\nu} \longrightarrow G'_{\mu\nu} = G_{\mu\nu} + \alpha' (x_1 R_{\mu\nu} + x_2 R G_{\mu\nu}) + \dots$$

where $x_{1,2,\dots}$ are specially tuned parameters.

B. Zweibach, S. Deser & A.N. Redlich, ... A. Tseytlin (1985-1987).

Ghost-killing reparametrization doesn't affect string S-matrix,

$$G_{\mu\nu} \longrightarrow G'_{\mu\nu} = G_{\mu\nu} + \alpha' (x_1 R_{\mu\nu} + x_2 R G_{\mu\nu}) + \dots$$

At the same time, Zweibach reparametrization is ambiguous and this actually produce ambiguous physical solutions.

A. Maroto & I.Sh., PLB, hep-th/9706179.

Another subtle point is that the really effectively working ghost-killing transformation must be absolutely precise!

Even an infinitesimal change produce a ghost with a huge mass. Moreover, smaller violation of fine-tuning leads to a greater mass of the ghost, hence (according to a “standard wisdom”) smaller violation of fine-tuning produce greater gravitational instability.

At low energies we know that the quantum effects are described by QFT, not string theory. Hence, string theory is ghost-free and unitary only if completely controls QFT, even in the very deep IR.

An alternative to Zweibach transformation

In the non-local theory

$$S = -\frac{1}{2\kappa} \int d^4x \sqrt{-g} \left\{ R + G_{\mu\nu} \frac{a(\square) - 1}{\square} R^{\mu\nu} \right\}, \quad a(\square) = e^{-\square/m^2}.$$

A. Tseytlin, *PLB*, *hep-th/9509050*.

In this and similar theories propagator of metric perturbations has a single massless pole, corresponding to gravitons.

With this choice there are no ghosts!

The idea is to use Zweibach-like transformation, but arrive at the non-local theory which is non-polynomial in derivatives, instead of “killing” all higher derivatives that one can kill.

One more ambiguity in the (super)string theory.

There was a proposal to use the same kind of non-local models to construct superrenormalizable and unitary models of QG.

E.T. Tomboulis, hep-th/9702146, unpublished. ...

L. Modesto, L. Rachwal, NPB B889 (2014), arXiv:1407.8036.

In order to explore the flat-space propagator, the relevant part of the classical action is at most bilinear in the curvature tensor,

$$S = \int_x \left\{ -\frac{1}{\kappa^2} R + \frac{1}{2} C_{\mu\nu\alpha\beta} \Phi(\square) C^{\mu\nu\alpha\beta} + \frac{1}{2} R \Psi(\square) R \right\}.$$

The equation for defining the poles of the propagator is

$$p^2 \left[1 + \kappa^2 p^2 \Phi(-p^2) \right] = 0.$$

There is always a massless pole corresponding to gravitons. It is easy to provide the absence of other poles.

Unfortunately, it is impossible to preserve the ghost-free structure at the quantum level.

Typically there are infinitely many poles on the complex plane.

No way to live without ghosts!

One can conclude that in all three approaches to QG, namely semiclassical, legitimate QG, induced gravity/strings, there is no reasonable way to get rid of massive ghost-like states.

What we can really do is to make all the ghosts complex, in the sense of complex “massive” poles in the propagator.

The complex poles always come in complex conjugate pair, which opens interesting possibilities, related to Lee-Wick quantization.

This is coherent with the previous attempts to solve the problem of higher derivative massive ghosts.

E. Tomboulis (1977, 1980, 1984), A. Salam and J. Strathdee (1978), I. Antonidis & E. Tomboulis (1986), D. A. Johnston (1988), S. Hawking et al (1990, ...),

Very short historical review

According to the works done in 70-ies and 80-ies, the main hope to have unitary & renormalizable fourth-derivative QG is related to the splitting of real massive pole of a fourth-derivative theory into a couple of complex conjugate poles, at the quantum level.

E. Tomboulis (1977,1980,1984), A. Salam and J. Strathdee (1978),

In this case one has to consider always a scattering of a pair of the conjugate particles, it opens the way to have unitary theory.

S. Hawking et al (1990, ...),

The main problem is that the definite resolution of the problem of unitarity in the fourth-derivative model requires complete information about the dressed propagator.

D. A. Johnston, NPB (1988).

Complex ghosts and Lee-Wick unitarity in QG

Starting from Tomboulis (1977) and Salam and Strathdee (1978) the main hope in the “minimal” fourth-derivative QG was that the real ghost pole splits into a couple of complex conjugate poles under the effect of quantum corrections.

One-loop effects, large- N approximation and lattice-based considerations indicated an optimistic picture, but unfortunately all of them are not conclusive, as shown by Johnston (1988).

However, for six- or more- derivative theory of QG, one can just start from the theory which has only complex massive poles.

L. Modesto, and I.Sh. PLB (2016), arXiv:1512.07600.

It turns out that such a theory is unitary and, moreover, this property may probably hold even at the quantum level.

Let us write the six-derivatives action in a slightly different form:

$$S = -\frac{2}{\kappa^2} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left\{ \frac{\alpha}{2} C_{\mu\nu\alpha\beta} \Pi_2 C^{\mu\nu\alpha\beta} + \alpha\omega R \Pi_0 R \right\},$$

where $\Pi_{0,2} = \Pi_{0,2}(\square) = 1 + \dots$ are some polynomials of order k .

The part of the action which is quadratic in the perturbations

$\kappa h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ has the form

$$S_{red}^{(2)} = - \int d^4x \left\{ \frac{1}{2} h^{\mu\nu} \left[\frac{\alpha\kappa^2}{2} \Pi_2(\partial^2) \partial^2 - 1 \right] \partial^2 P_{\mu\nu, \rho\sigma}^{(2)} h^{\rho\sigma} \right. \\ \left. + h^{\mu\nu} \left[\alpha\omega\kappa^2 \Pi_0(\partial^2) \partial^2 - 1 \right] \partial^2 P_{\mu\nu, \rho\sigma}^{(0-s)} h^{\rho\sigma} \right\}.$$

Here

$$P_{\mu\nu, \rho\sigma}^{(0-s)} = \frac{1}{3} \theta_{\mu\nu} \theta_{\rho\sigma}, \quad P_{\mu\nu, \rho\sigma}^{(2)} = \frac{1}{2} (\theta_{\mu\rho} \theta_{\nu\sigma} + \theta_{\nu\rho} \theta_{\mu\sigma}) - P_{\mu\nu, \rho\sigma}^{(0-s)},$$

are projectors to spin-0, 2 states, and $\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}$.

After the Wick rotation the equations for the poles are

$$\alpha \Pi_2(p^2) p^2 = 2M_P^2, \quad \alpha \omega \Pi_0(p^2) p^2 = M_P^2.$$

Let us consider the six-order theory,

$$\Pi_2(p^2) = 1 + \frac{p^2}{2A_2}, \quad \Pi_0(p^2) = 1 + \frac{p^2}{2A_0},$$

where A_0 and A_2 are constants of the $mass^2$ - dimension.

Let us present the solution only for the tensor part.

$$p^2 = m_2^2 = -A_2 \pm \sqrt{A_2^2 + \frac{4A_2 M_P^2}{\alpha}}.$$

Possible cases:

- Two real positive solutions $0 < m_{2+}^2 < m_{2-}^2$.
- Two pairs of complex conjugate solutions for the mass.

In QFT theory of the field $h_{\alpha\beta}$ the condition of unitarity of the S - matrix can be formulated in a usual way,

$$S^\dagger S = 1, \quad \text{or} \quad S = 1 + iT \quad \text{and} \quad -i(T - T^\dagger) = T^\dagger T.$$

By defining the scattering amplitude as

$$\langle f|T|i\rangle = (2\pi)^D \delta^D(p_i - p_f) T_{fi}.$$

we arrive at

$$-i(T_{fi} - T_{if}^*) = \sum_k T_{kf}^* T_{ki}.$$

Assuming that for the forward scattering amplitude $i = f$, previous equation simplifies to

$$2 \operatorname{Im} T_{ii} = \sum_k T_{ik}^* T_{ik} > 0.$$

The detailed analysis of tree-loop, one-loop and multi-loop diagrams shows that this relation is satisfied because massive poles always show up in a complex conjugate pairs.

The analysis performed in *L. Modesto and I.Sh. PLB (2016)* is mainly at the tree-level. Indeed, the proof of unitarity is complete only when it takes loops into account.

However, the analysis of loop contributions is essentially done for another Lee-Wick models, in Ref.

T. D. Lee and G. C. Wick, PRD 2 (1970) 1033; NPB 9 (1969) 209,

and especially in

R. E. Cutkosky, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne, NPB 12 (1969) 281

and

B. Grinstein, D.O'Connell and M.B. Wise, PRD, arXiv:0805.2156.

Exactly as in the $O(N)$ scalar model, treated in the last reference, in higher derivative gravity with complex massive poles, the theory is unitary, but there may be a violation of causality at the microscopic time scales, defined by the magnitude of masses.

The main issue is stability

Certainly, the unitarity of the S -matrix is not the unique condition of consistency of the quantum gravity theory.

The most important feature is the stability of physically relevant solutions of classical general relativity in the presence of higher derivatives and massive ghosts.

The problem is well explored for the cosmological backgrounds. Gravitational waves on de Sitter space (energy $\ll M_p$):

A. A. Starobinsky, Let. Astr. Journ. (in Russian) (1983).

S. Hawking, T. Hertog, and H.S. Reall, PRD (2001).

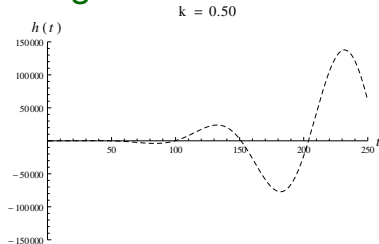
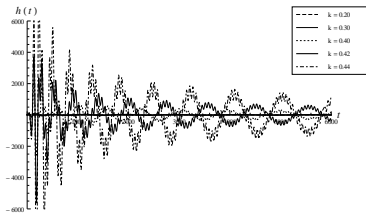
J. Fabris, A. Pelinson and I.Sh., NPB (2001).

J. Fabris, A. Pelinson, F. Salles and I.Sh., JCAP, arXiv:1112.5202.

More general FRW-backgrounds:

F. Salles and I.Sh., PRD, arXiv:1401.4583.

More general cosmological backgrounds



Example: radiation-dominated Universe. There are no growing modes until the frequency k achieves the value ≈ 0.5 in Planck units. Starting from this value, we observe instability as an effect of massive ghost.

The anomaly-induced quantum correction is $\mathcal{O}(R^3)$. Until the energy is not of the Planck order of magnitude, these corrections can not compete with classical $\mathcal{O}(R^2)$ - terms.

Massive ghosts are present only in the vacuum state. We just do not observe them “alive” until the energy scale M_P

Low-energy effects of higher derivatives

The study of IR effects of quantum and classical HD gravity is important for several reasons.

1) There is a chance that there is a Planck protection from the ghost-generated instabilities of an unknown origin. But for a more than four derivative gravity one can imagine the kind of see-saw mechanism, which makes a relatively light ghost-like state mass out of several Planck-scale dimensional parameters in the action.

2) If the mass of the gravitational ghost-like state is reduced, especially in the case of numerous complex poles, then what can be a manifestation of such a ghost-like states?

3) By the end of the day, can we observe some effect of higher derivatives, in one or another way?

Gravitational see-saw?

A.Accioly, B.L. Giacchini, I.Sh., arXiv:1604.07348.

Consider the simplest superrenormalizable action with the relevant part of the form

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \frac{2}{\kappa^2} R + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu}^2 + \frac{A}{2} R \square R + \frac{B}{2} R_{\mu\nu} \square R^{\mu\nu} \right\},$$

Here $\kappa^2 = 32\pi G = 2M_p^{-2}$, and α, β, A, B are free parameters, where the first two are dimensionless, $A, B \sim (\text{mass})^{-2}$ and we assume this mass has Planck order of magnitude.

In the weak-field limit, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ and $|\kappa h_{\mu\nu}| \ll 1$, the linearized field equations can be cast into the form

$$\left(\frac{2}{\kappa^2} - \frac{\beta}{2} \square - \frac{B}{2} \square^2 \right) \left(R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R \right) - \left(\alpha + \frac{\beta}{2} + A \square + \frac{B}{2} \square \right) (\eta_{\mu\nu} \square R - \partial_\mu \partial_\nu R) = -\frac{T_{\mu\nu}}{2}.$$

By introducing a suitable gauge condition, the weak gravitational field generated by a static point-like mass

$T_{\mu\nu}(\mathbf{r}) = M \eta_{\mu 0} \eta_{\nu 0} \delta^{(3)}(\mathbf{r})$ has non-zero components

$$h_{00} = \frac{M\kappa}{16\pi} \left(-\frac{1}{r} + \frac{4}{3}F_2 - \frac{1}{3}F_0 \right), \quad h_{11,22,33} = \frac{M\kappa}{16\pi} \left(-\frac{1}{r} + \frac{2}{3}F_2 + \frac{1}{3}F_0 \right),$$

where

$$F_k = \frac{m_{k+}^2}{m_{k+}^2 - m_{k-}^2} \frac{e^{-m_{k-}r}}{r} + \frac{m_{k-}^2}{m_{k-}^2 - m_{k+}^2} \frac{e^{-m_{k+}r}}{r}.$$

Here $k = 0, 2$ labels the spin of the particles, whose masses are defined by the positions of the poles of the propagator,

$$m_{2\pm}^2 = \frac{\beta \pm \sqrt{\beta^2 + \frac{16}{\kappa^2}B}}{2B}, \quad m_{0\pm}^2 = \frac{\sigma_1 \pm \sqrt{\sigma_1^2 - \frac{8\sigma_2}{\kappa^2}}}{2\sigma_2},$$

with $\sigma_1 \equiv 3\alpha + \beta$ and $\sigma_2 \equiv 3A + B$.

The see-saw requires a relation

$$m_{2+}^2 \ll m_{2-}^2 \implies 16|B| \ll \kappa^2 \beta^2,$$

In the theory where this condition is satisfied the masses can be approximated by

$$m_{2+}^2 \approx \frac{4}{\kappa^2 |\beta|} \ll m_{2-}^2 \approx \frac{\beta}{B}.$$

As in the original neutrino's seesaw mechanism one of the masses depends mainly on only one parameter, while the other depends on both. And this is a very general situation, indeed.

$$\frac{1}{m_0^4} k^6 - \frac{3}{m_1^2} k^4 + 3\beta k^2 - m_2^2 = 0.$$

The lightest mass depends only on β , while the largest one depends on both parameters. **No see-saw in HDQG !**

Is it a good news? **There is no threat to the Planck protection against ghosts, if such protection exist. But it will be certainly difficult to observe the effect of higher derivatives.**

Good news for traditional effective approach

The non-existence of the gravitational see-saw given supports the traditional effective IR approach to higher derivative gravity,

J.Z. Simon, Phys. Rev. D41 (1990);

L. Parker and J.Z. Simon, Phys. Rev. D47 (1993), gr-qc/9211002.

In this approach all higher derivative terms, including the renormalized terms in the classical action, quantum corrections, running parameter etc, are regarded as small perturbations over the much greater Einstein-Hilbert term.

Certainly, this approach is a kind of ad hoc one and it can work only for energies much below the Planck scale, that is not what we expect from the “theory of everything”, such as QG.

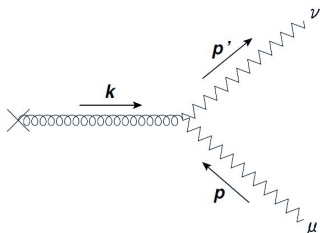
The idea of “IR protection from creating ghosts from vacuum” is coherent with this approach in the IR, partially explaining it by a special choice of initial conditions.

If the see-saw works, the threshold could shift to the IR.

How can it be observed? Light bending in HDQG

The light bending in HDQG has been explored in the paper by Accioly et al (see also refs. therein) in the semiclassical approximation.

A. Accioly, J. Helayël-Neto, B. Giacchini and W. Herdy, *PRD*, arXiv:1506.00270.



Method of “saturated propagators”: M. Veltman, in *Methods in Field Theory*, eds. R. Balian and J. Zinn-Justin, 1981.

It can be shown that only tensor mode of the gravitational perturbation affects the trajectory of the photon.

The results can be extended to the six-derivative model with real, complex, simple or multiple poles.

A. Accioly, B.L. Giacchini, I.Sh., arXiv:1604.07348, 1610.05260.

Important note. If we write the action in the form

$$S = \int_x \left\{ -\frac{1}{\kappa^2} R + \frac{1}{2} C_{\mu\nu\alpha\beta} \Phi(\square) C^{\mu\nu\alpha\beta} + \frac{1}{2} R \Psi(\square) R \right\},$$

and the light bending does not depend on the term $R \Psi R$.

$R \Psi R$ - term contributes to the propagation of only the trace $h = h_{\mu}^{\mu}$, which couples to the trace of $T_{\mu\nu}$. For photons it is zero.

No correspondence with massless BD theory, due to the presence of potential in the conformal mapping of the $R \Psi R$.

For the massive BD the finite rescaling of Newton constant is restricted to the region $r \ll m_0^{-1}$ which is far beyond the phenomenologically interesting regions for light bending.

C.M. Will, *Theory and experiment in gravitational physics*. (1993);
J. Alsing, E. Berti, C. Will, H. Zaglauer, PRD-2012.

General result valid for real simple, real multiple and also for complex poles:

$$\frac{1}{\theta_{GR}^2} = \frac{1}{\theta^2} + \frac{E^2}{(m_{2-}^2 - m_{2+}^2)^2} \left(\frac{m_{2-}^4}{E^2\theta^2 + m_{2+}^2} + \frac{m_{2+}^4}{E^2\theta^2 + m_{2-}^2} \right) + \frac{2E^2}{m_{2-}^2 - m_{2+}^2} \left[\frac{m_{2-}^2}{m_{2+}^2} \ln \left(\frac{E^2\theta^2}{E^2\theta^2 + m_{2+}^2} \right) - \frac{m_{2+}^2}{m_{2-}^2} \ln \left(\frac{E^2\theta^2}{E^2\theta^2 + m_{2-}^2} \right) - \frac{m_{2-}^2 m_{2+}^2}{(m_{2-}^2 - m_{2+}^2)^2} \ln \left(\frac{E^2\theta^2 + m_{2-}^2}{E^2\theta^2 + m_{2+}^2} \right) \right],$$

The correction to the GR bending angle is very small for $\mathcal{E} \ll m_{2\pm}$. Remember that typically $m_{2\pm} \sim M_P$.

Solving with respect to θ we gain an infinite series in $\mathcal{E}/m_{2\pm}$ which depends on θ_{GR} and hence on the impact parameter.

Similar result for qualitatively different model:
R. Caldwell and D. Green, PRL (2006-08).

The net result is that the effect of massive ghost makes the light bending dependent on the ratio(s) \mathcal{E}/m_2 .

For the “minimal” four-derivative gravity the ghost mass is huge $m_2 \approx M_P$ and the effect of frequency dependence in the light bending is strongly suppressed for the photons which have energies much below the Planck energy scale.

Things do not change dramatically in the theory with six or more derivatives, even if there may be much more sophisticated mass spectrum.

In the higher derivative theories it is not possible to have a kind of gravitational see-saw mechanism, with $m_{2+} \ll m_{2-} \approx M_P$.

As a result, there may be only an extremely small angular dependence on higher derivatives, and experimentally detecting the presence of higher derivatives in the gravitational action requires qualitatively new ideas, at least.

Conclusions

- The construction of QG theory which is **not** restricted to the IR region, is not possible without higher derivative terms.
- Most important: the higher derivative terms are requested for a consistent formulation of semiclassical theory, than means for the quantization of matter fields.
- Including more than four derivatives provides theoretical advantages: superrenormalizable QG and well-defined renormalization group flow, free from gauge-fixing ambiguities.
- Lee-Wick type unitarity of the **S**- matrix for the gravitational field takes place in case of complex massive poles.
- In the theory with $6+$ derivatives there is no chance for the gravitational see-saw mechanism and potentially observable energy-dependence, e.g., of the light bending is supposed to be very weak.