Strong and weak interactions in the Standard Model (2)

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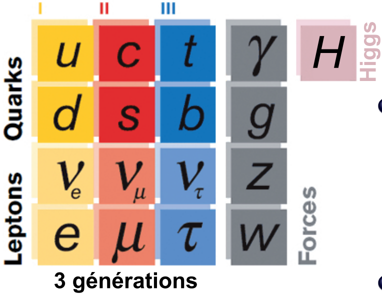
What the Standard Model is

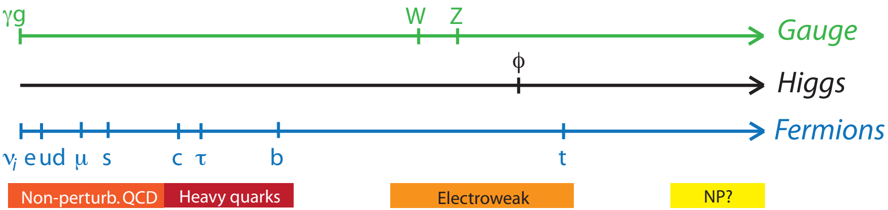
Our current understanding of the basic constituents of matter



- 3 generations of
  - 2 quarks (u, d)
  - 1 charged lepton (e^-)
  - 1 neutrino (ν_e)
- 3 fundamental forces
  - Electromagnetism
  - Weak interaction (β decays)
  - Strong interaction (nucleus stability)
- A spin 0 particle: the Higgs boson

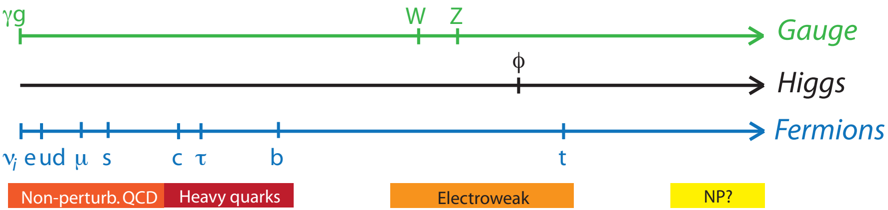
- 1st lecture: a few elements on weak and strong interactions
- 2nd lecture: techniques to tackle problems with both interactions

A multi-scale problem



- Transition from one quark to another through weak interaction: a tough multi-scale challenge with 3 interactions intertwined
- Several steps to separate/factorise scales
  BSM $\to$ SM+$1/\Lambda_{NP}$ $(\Lambda_{EW}/\Lambda_{NP})$ $\to \mathcal{H}_{eff}$ (m_b/Λ_{EW}) $\to$ *eff. theories* (Λ_{QCD}/m_b)

A multi-scale problem



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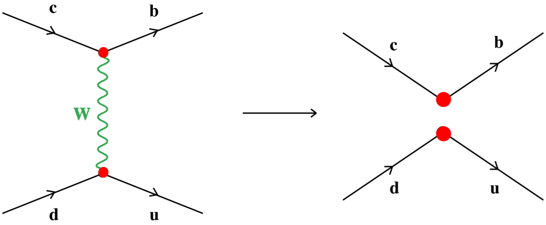
- Several steps to separate/factorise scales
  BSM $\to$ SM+$1/\Lambda_{NP}$ $(\Lambda_{EW}/\Lambda_{NP})$ $\to$ $\mathcal{H}_{eff}$ (m_b/Λ_{EW}) $\to$ *eff. theories* (Λ_{QCD}/m_b)
- Main theo problem from hadronisation of quarks into hadrons
  description/parametrisation in terms of QCD quantities
  decay constants, form factors, bag parameters...
- Long-distance non-perturbative QCD: source of uncertainties
  lattice QCD simulations, sum rules, effective theories...

Effective Hamiltonian

Fermi-like approach : μ separation between low and high energies

- Short distances : (perturbative) Wilson coefficients
- Long distances : local operator



$$V_{ud} V_{cb}^* \frac{G_F}{\sqrt{2}} \frac{m_W^2}{m_W^2 - p_W^2} \bar{u}\gamma_\mu(1-\gamma_5)d\bar{b}\gamma^\mu(1-\gamma_5)c$$

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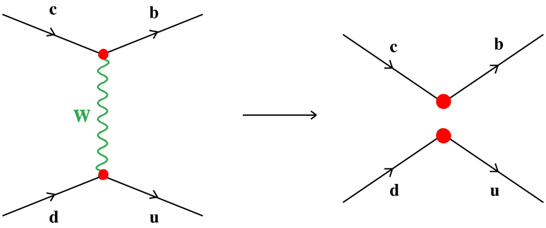

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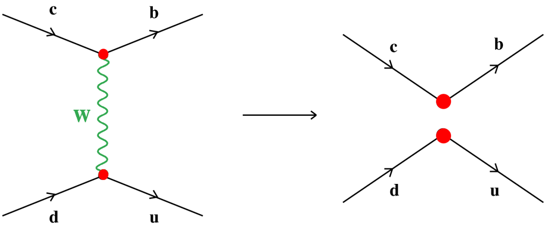

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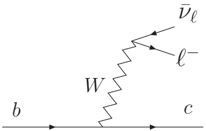
$$\mathcal{A}(B \to H) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i\, C_i(\mu)\, \langle H|\mathcal{O}_i|B\rangle(\mu)$$

- λ_i collect CKM-matrix elements,
- $C_i(\mu)$ Wilson coefficients (physics above m_b)
- matrix-elements of local operators $\mathcal{O}_i$

Computing processes

$$\mathcal{H}^{\text{eff}} = CKM \times \mathcal{C}_i \times \mathcal{O}_i$$
$$\langle M|\mathcal{H}^{\text{eff}}|B\rangle = CKM \times \mathcal{C}_i \times \langle M|\mathcal{O}_i|B\rangle$$



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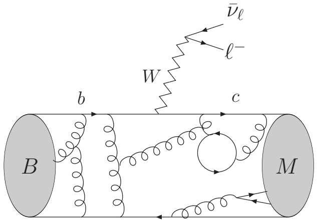

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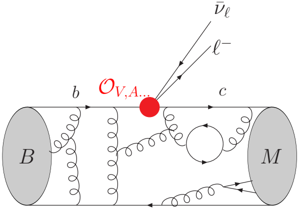

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  - Strong interactions below $\mu = O(m_b)$
  - No general method to compute these contributions
  - Lattice QCD, effective theories, dispersive approaches...

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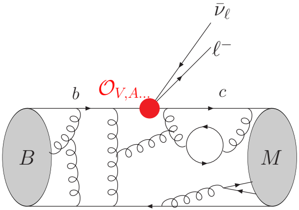

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  - Strong interactions below $\mu = O(m_b)$
  - No general method to compute these contributions
  - Lattice QCD, effective theories, dispersive approaches...
- Wilson coefficients
  - Weak and strong interactions above $\mu = O(m_b)$
  - Perturbatively computable
  - Can involve large logarithms $\alpha_s \log(M_W/\mu)$

Wilson coefficients

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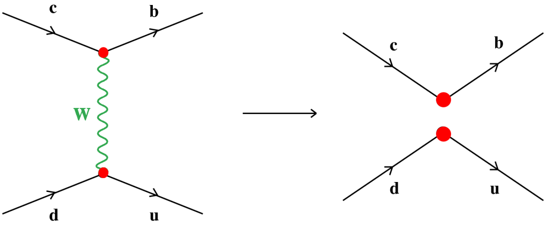

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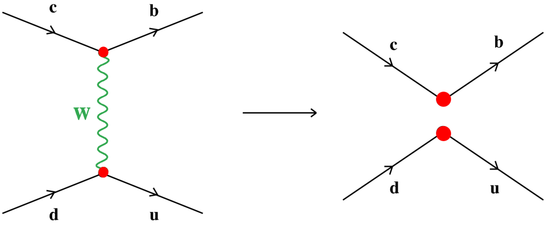

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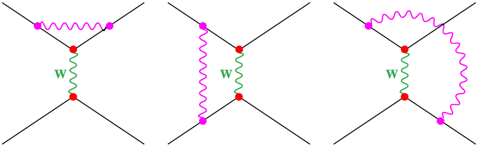
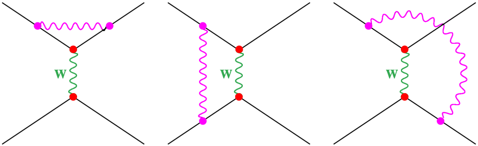


When we take into account one (or more) gluons

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} \qquad Q_2(\mu)$$

$$Q_2 \;=\; (\bar{b}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A} \qquad (\bar{b}c)_{V-A} = \bar{b}\gamma_\mu(1-\gamma_5)c$$

QCD effects



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$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu)]$$

$$Q_1 = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$
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- new colour structures (flipped indices α, β)

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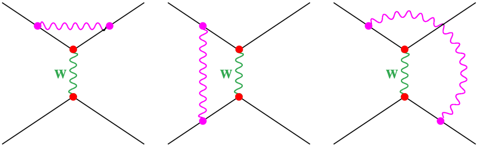

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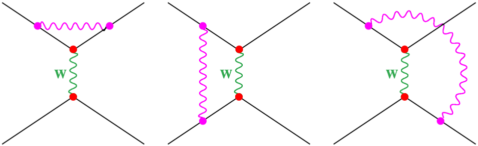

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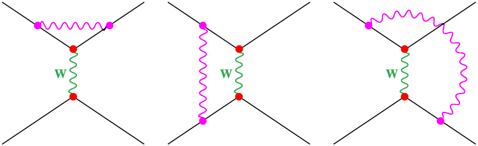
- new colour structures (flipped indices α, β)
- Without QCD $C_1 = 0$, $C_2 = 1$
- C_1 and C_2 calculable fonctions of μ as perturbative series in α_s

$\bar{b} \to \bar{c}\bar{d}u$ at one loop: fundamental theory

C high-energy part, independent of state :

take massless quarks, off-shell by $p^2 < 0$



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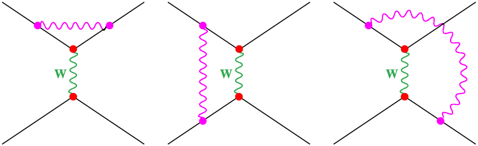

In "full" (SM) theory, taking into account quark renormalisation,

$$A_{\rm full} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} \left[M_2 + \frac{3}{N_c}\frac{\alpha_s}{4\pi} \log\frac{M_W^2}{-p^2} M_2 - 3\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{-p^2} M_1 \right]$$

at leading logarithms, with the matrix elements

$$\begin{aligned} M_1 &= \langle Q_1 \rangle^{LO} = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \\ M_2 &= \langle Q_2 \rangle^{LO} = (\bar{b}_\alpha c_\alpha)_{V-A} (\bar{u}_\alpha d_\alpha)_{V-A} \end{aligned}$$

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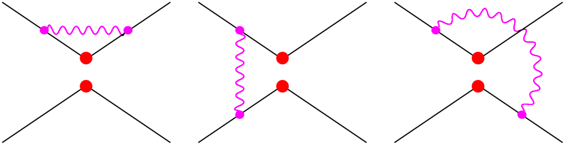
In the effective theory (effective Hamiltonian)



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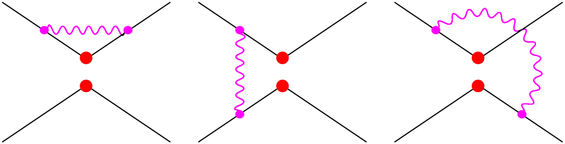

we obtain, taking also into account quark-field renormalisation

$$\langle Q_1\rangle^{(0)} = M_1 + \frac{3}{N_c}\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{-p^2}\right)M_1 - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{-p^2}\right)M_2$$

$$\langle Q_2\rangle^{(0)} = M_2 + \frac{3}{N_c}\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{-p^2}\right)M_2 - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{-p^2}\right)M_2$$

- Dimensional regularisation $d = 4 - 2\epsilon$ to treat UV divergences
- Introduction of a renormalisation scale μ: $g_s \to g_s\mu^\epsilon$
- Effective theory more singular than fundamental theory ($1/\epsilon$, absorbed by renormalising operators of eff. Hamiltonian)
- Involve only low scales (p^2 and μ, but not M_W)

Matching and Wilson coefficients

Matching: C_1 and C_2 so that full and effective theories yield same result

$$A_{\rm full} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_1(\mu)\langle Q_1(\mu)\rangle + C_2(\mu)\langle Q_2(\mu)\rangle]$$

At NLO in α_s, leading logarithms

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2), \qquad C_2(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2)$$

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Matching performed separation of scales $-p^2 < \mu^2 < M_W^2$

$$\left(1+\alpha_s X\log\frac{M_W^2}{-p^2}\right) = \left(1+\alpha_s X\log\frac{M_W^2}{\mu^2}\right)\times\left(1+\alpha_s X\log\frac{\mu^2}{-p^2}\right)$$

$$\int_{-p^2}^{M_W^2}\frac{dk^2}{k^2} = \int_{\mu^2}^{M_W^2}\frac{dk^2}{k^2} + \int_{-p^2}^{\mu^2}\frac{dk^2}{k^2}$$

Resumming large logarithms

At $\mu = m_b$ (separation between low and high energies)

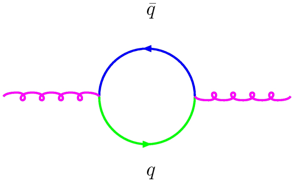
$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2) = -0.3 + \ldots$$

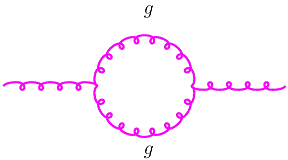
$$C_2(\mu) = 1 + \frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2) = 1 + 0.1 + \ldots$$

better to sum all leading-logs $\left(\alpha_s(\mu)\log\frac{M_W^2}{\mu^2}\right)^n$

$\Longrightarrow$ How can we perform this ?

Back to α_s



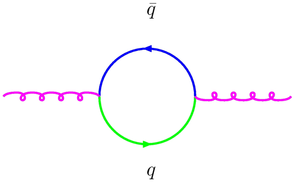


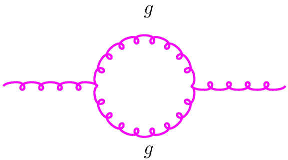
- Dependence on μ (*renormalisation group equation or RGE*)

$$\frac{d\alpha_s(\mu)}{d\log\mu} = -2\beta_0\frac{\alpha_s^2}{4\pi} - 2\beta_1\frac{\alpha_s^3}{(4\pi)^2} + \dots$$

  - $\beta_0 = (11N_c - 2N_f)/3$ from 1-loop computation
  - $\beta_1 = (34N_c^2 - 10N_cN_f - 3(N_c^2-1)N_f/N_c)/3$ from 2 loops

Back to α_s





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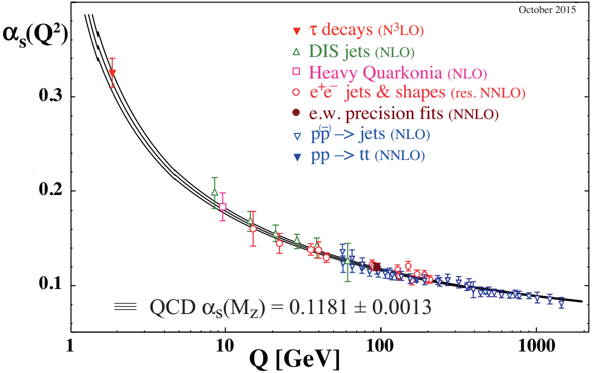
- Solution introduces a scale $\Lambda \simeq 200 - 250$ MeV

$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0\log(\mu^2/\Lambda^2)} - \frac{\beta_1}{\beta_0^3}\frac{\log\log(\mu^2/\Lambda^2)}{\log^2(\mu^2/\Lambda^2)} + \dots$$

with $\log\mu$ dependence very well satisfied experimentally

α_s at various scales



$\Longrightarrow$asymptotic freedom:
at large energies, interactions (prop to g_s) small perturbations

Consistency over a very large range of energies
(from m_τ up to LHC pp collisions)

Leading logarithms

- Keeping only first order in $d\alpha_s/d\log\mu$:

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 - \beta_0 \frac{\alpha_s(\mu_0)}{2\pi} \log(\mu_0/\mu)} = \alpha_s(\mu_0) \left[1 + \sum_{n=1}^{\infty} \left(\beta_0 \frac{\alpha_s(\mu_0)}{2\pi} \log \frac{\mu_0}{\mu} \right)^n \right]$$

- resummation of leading logs $\alpha_s^n(\mu_0) \log^n(\mu_0/\mu)$
  - needed for $\mu = O(m_b) \ll \mu_0 = O(M_W)$:
  - $\alpha_s(\mu_0) \ll 1$ but $\alpha_s(\mu_0) \log(\mu_0/\mu) = O(1)$

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LO	1			
NLO	$\alpha_s(\mu_0)\log(\mu_0/\mu)$	$\alpha_s(\mu_0)$		
$NNLO$	$\alpha_s^2(\mu_0)\log^2(\mu_0/\mu)$	$\alpha_s^2(\mu_0)\log(\mu_0/\mu)$	$\alpha_s^2(\mu_0)$	
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
	Leading Logs	Next – to – Leading Logs	NNLL	$\ldots$
	RGE LO	RGE NLO	RGE NNLO	$\ldots$

Solution of RGE for $d\alpha_s/d\log\mu$ at N^{k}LO in perturbation theory provides the resumation of N^k leading log contributions

Scale dependence of the Wilson coefficients

We can use the same trick for Wilson coefficients

- Absorbing $1/\epsilon$ poles from "bare quantities" $X^{(0)} = ZX$ into renormalisation factors Z, leading to renormalised X (without $1/\epsilon$)

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- Renormalising $\langle Q_i \rangle^{(0)} = Z_{ij} \langle Q_j \rangle, \; Z = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\begin{pmatrix} 3/N_c & -3 \\ -3 & 3/N_c \end{pmatrix}$

  which is diagonal in $Q_\pm = \frac{Q_2 \pm Q_1}{2}$, $C_\pm = C_2 \pm C_1$:

$$Q_\pm^{(0)} = Z_\pm Q_\pm, \qquad C_\pm^{(0)} = Z_\pm^{-1} C_\pm$$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_+^{(0)} Q_+^{(0)} + C_-^{(0)} Q_-^{(0)}]$$

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$$Q_\pm^{(0)} = Z_\pm Q_\pm, \qquad C_\pm^{(0)} = Z_\pm^{-1} C_\pm$$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_+(\mu) Q_+(\mu) + C_-(\mu) Q_-(\mu)]$$

Scale dependence of the Wilson coefficients

We can use the same trick for Wilson coefficients

- Absorbing $1/\epsilon$ poles from "bare quantities" $X^{(0)} = ZX$ into renormalisation factors Z, leading to renormalised X (without $1/\epsilon$)
- Renormalising $\langle Q_i\rangle^{(0)} = Z_{ij}\langle Q_j\rangle, \ Z = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\begin{pmatrix} 3/N_c & -3 \\ -3 & 3/N_c \end{pmatrix}$

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$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_+(\mu) Q_+(\mu) + C_-(\mu) Q_-(\mu)]$$

- Renormalising Wilson coefficient: $C_\pm(\mu) = C_\pm^{(0)} Z_\pm(\alpha_s)$

  $C_\pm^{(0)}$ independent of μ, $Z_\pm$ function of μ through α_s

$$\frac{dC_\pm(\mu)}{d\log\mu} = \gamma_\pm(\mu) C_\pm(\mu) \qquad \gamma_\pm = \frac{1}{Z_\pm}\frac{dZ_\pm}{d\log\mu} = \pm\frac{\alpha_s(\mu)}{4\pi}\frac{6(N_c \mp 1)}{N_c}$$

Resumming through RGE

- Solving the RGE knowing the dependence of α_s on μ

$$\frac{dg_s(\mu)}{\log\mu} = \beta(g_s(\mu)) = -\beta_0 \frac{g_s^3}{16\pi^2} + \dots \qquad \beta_0 = \frac{11N_c - 2N_f}{3}$$

$$\longrightarrow C_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{\gamma_\pm^{(0)}}{\beta^{(0)}}} C_\pm(M_W) \qquad \gamma_\pm^{(0)} = \frac{6(N_c \mp 1)}{N_c}$$

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$$C_+(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{23}} \qquad C_-(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{-\frac{12}{23}}$$

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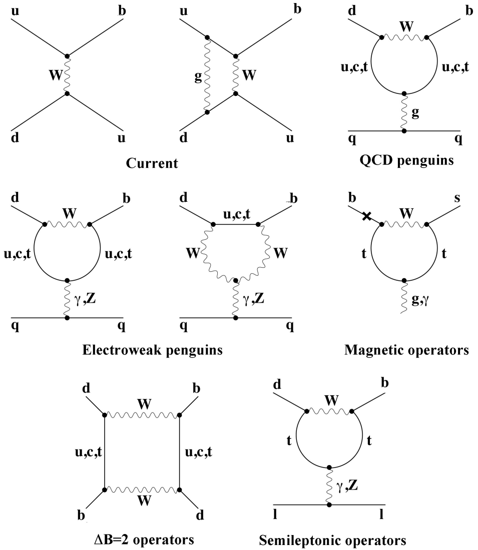
- Mixing between the operators Q_1 and Q_2 from M_W down to μ
- General framework to compute Wilson coefficients at the scale μ: matching at M_W, determining the RGE, evolving down to μ

Operators of interest

- Current-curent
  - $(\bar{b}u)_{V-A}(\bar{u}d)_{V-A}$,
  - $(\bar{b}_i u_j)_{V-A}(\bar{u}_j d_i)_{V-A}$



Buras et al.

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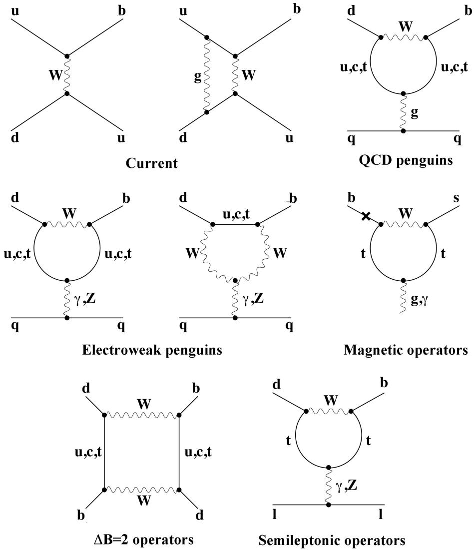

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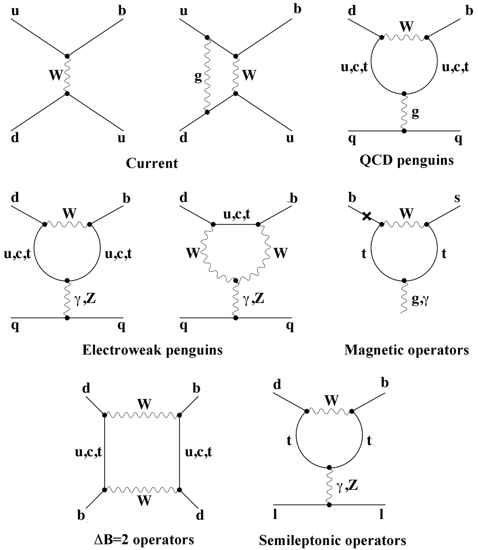

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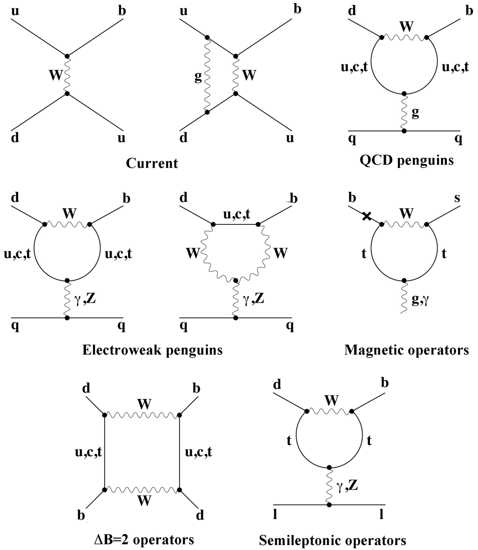

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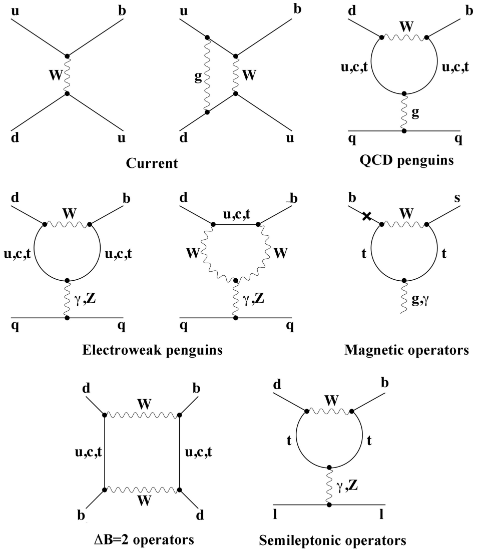
- $\Delta B = 2$ operators
  - $(\bar{b}d)_{V-A}(\bar{b}d)_{V-A}$



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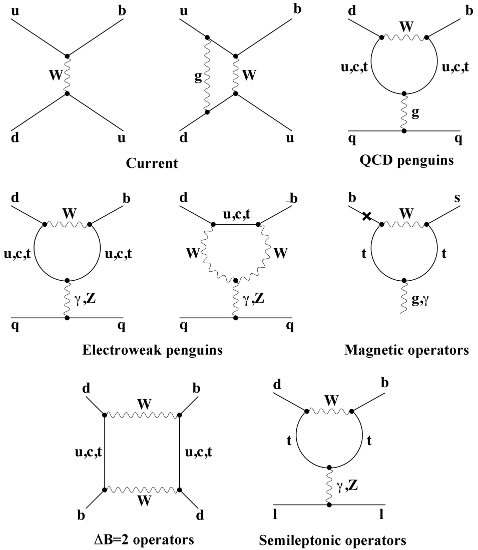
- $\Delta B = 2$ operators
  - $(\bar{b}d)_{V-A}(\bar{b}d)_{V-A}$
- Semileptonic operators
  - $(\bar{b}d)_{V-A}(\bar{\ell}\ell)_{V/A}$



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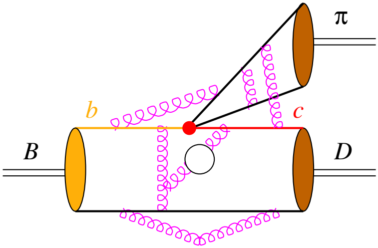
Hadronic quantities

Hadronic matrix elements

Effective Hamiltonian yields $A(B \to H) = \sum \lambda_i \, C_i(\mu) \langle H|\mathcal{O}_i|B\rangle(\mu)$

- above m_b, perturbative Wilson coefficients $C_i(\mu)$
- below m_b, operators yielding matrix elements $\langle H|\mathcal{O}_i|B\rangle(\mu)$



Strong interaction
in nonperturbative regime

How to compute $\langle H|\mathcal{O}_i|B\rangle$?

- Model building
- Lattice simulations
- Sum rules
- Light flavour symmetries (isospin, SU(3)...)
- Heavy flavour symmetries (HQET...)

Hadronic quantities

Describe hadronic matrix elements in terms of hadronic quantities

- simple (handled/computable theoretically if not perturbatively)
- universal (common to several processes)

$\Longrightarrow$Exploit Lorentz symmetry to simplify them whenever possible
$\Longrightarrow$The more mesons, the more complicated the quantity
(here, only decay constants and form factors)

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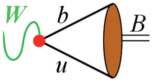
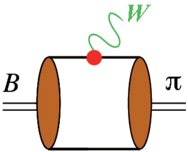
Decay constant
$\langle 0|\bar{u}\gamma_\mu\gamma_5 b|B^-(p)\rangle = ip_\mu F_B$ (real number)



- probability amplitude of hadronising quark pair into given hadron
- related (among others) to purely leptonic decay

$$\Gamma(B^- \to \ell\nu_\ell) \propto |V_{ub}|^2 F_B^2$$

Form factors



$$\langle \pi(p')|\bar{u}\gamma_\mu b|B(p)\rangle = (p+p')_\mu F_+(q^2) + (p-p')_\mu [F_0 - F_+](q^2)\frac{m_B^2-m_\pi^2}{q^2}$$

- transition from meson to another through flavour change
- projection over available Lorentz stuctures $(p \pm p')_\mu$
- form factors $F_{+,0}$ scalar functions of $q^2 = (p-p')^2$
- more complicated for vector mesons, since polarisation available

$$\frac{d\Gamma(B \to \pi\ell\nu)}{d(q^2)} \propto |V_{ub}|^2 \times |F_+(q^2)|^2 \qquad (m_\ell \to 0)$$

General statements about form factors

Not much known, apart from structure of *S*cattering matrix

$$S_{\beta\alpha} = \langle \beta_{out} | \alpha_{in} \rangle = \langle \beta | \alpha \rangle$$

and its related *T*ransition matrix $S = 1 + iT$

$$\langle \beta | iT | \alpha \rangle = (2\pi)^4 \delta(\sum p_\alpha - \sum p_\beta) \cdot iA(\alpha \to \beta)$$

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Almost only one thing known for sure

from conservation of probability, *S*-matrix is unitary

$$\begin{aligned} (S^\dagger S)_{\gamma\alpha} &= \sum_\beta \langle \beta_{out} | \gamma_{in} \rangle^* \langle \beta_{out} | \alpha_{in} \rangle \\ &= \sum_\beta \langle \gamma_{in} | \beta_{out} \rangle \langle \beta_{out} | \alpha_{in} \rangle = \langle \gamma_{in} | \alpha_{in} \rangle = \delta(\alpha - \gamma) \end{aligned}$$

since sum over complet state of states $|\beta\rangle$

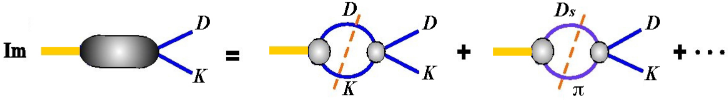
Cuts

Translation for *T*ransition matrix $S = 1 + iT$

$$S^\dagger S = 1 \Longrightarrow T - T^\dagger = iT^\dagger T$$

or in terms of amplitude

$$-i[A(\alpha \to \beta) - A^*(\alpha \to \beta)] = \sum_f A^*(\beta \to f) A(\alpha \to f)$$



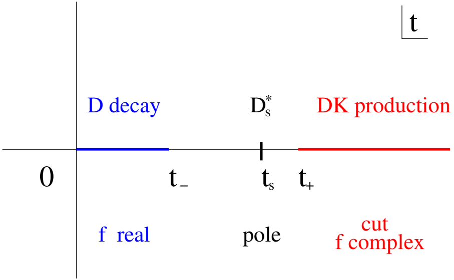
Form factors for $\alpha \to \beta$ acquire an imaginary part

- if there are (real) intermediate states f between α and β
- which depends on the value of the transfer momenta q^2

Analytic structure of a form factor

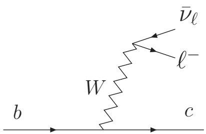
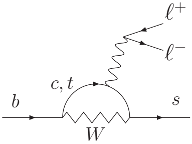
Taking for instance form factor describing $D \to K\ell\nu$

- Two physical regions, accessible to experiment
  - real for $t = q^2$ between m_ℓ^2 and $t_- = (m_D - m_K)^2$ $\quad D \to K$ decay
  - complex for $t \geq (m_D + m_K)^2$ $\quad W \to DK$ production
- Same form factor involved
  - Analytic function for almost every value of t in the complex plane
  - apart from poles for resonances (like D_s^*)
  - and cuts along the real axis due to imaginary part for open channels



- If info on the cut (from measurements), possible to reconstruct the form factor
- Otherwise, other approaches needed (lattice simulations, effective theories)

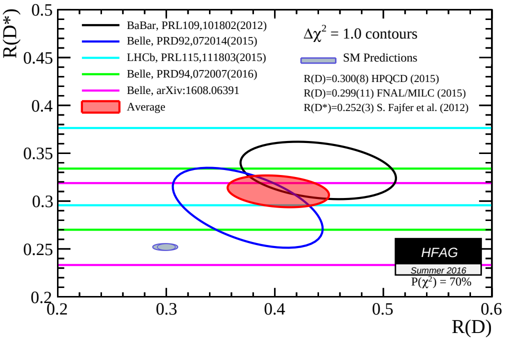
Two playgrounds

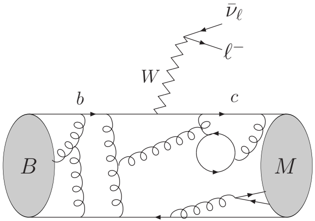
	$b \to c\ell\bar{\nu}_\ell$	$b \to s\ell^+\ell^-$
		
SM	tree (charged) ($V - A$)	loop (neutral)
Spin 0	$\overline{B} \to D\ell\bar{\nu}_\ell$	$B \to K\ell\ell$
Spin 1	$\overline{B} \to D^*\ell\bar{\nu}_\ell$	$B \to K^*\ell\ell$, $B_s \to \phi\ell\ell$
Observables	Total Br	$d\Gamma/dq^2$ + Angular obs
with	$\ell = \tau, \mu, e$	$\ell = \mu, e$
Tensions	$R_{D(*)} = \frac{Br(B \to D(*)\tau\nu)}{Br(B \to D(*)\ell\bar{\nu}_\ell)}$	$R_K = \frac{Br(B \to K\mu\mu)}{Br(B \to Kee)}$ $Br\ (K, K^*, \phi + \mu\mu)$ angular obs (e.g., P_5')
Tools	Lattice, HQET	Lattice, HQET, SCET

Patterns of deviations from SM analysed using effective Hamiltonian once form factors constrained thanks to effective theories

$b \to c$: Heavy-Quark Effective Theory

$b \to c\ell\bar{\nu}_\ell$: R_D and R_{D^*}





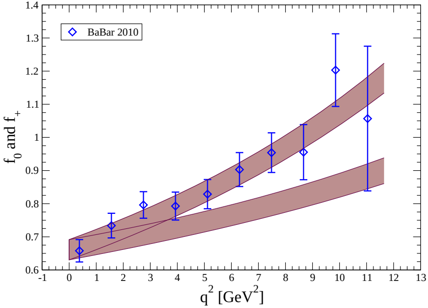
$$R_{D(*)} = \frac{Br(B \to D(*)\tau\nu)}{Br(B \to D(*)\ell\bar{\nu}_\ell)}$$

- different identification techniques of the τ for LHCb and B-factories
- $R(D)$ and $R(D^*)$ exceed SM predictions by 1.9 σ and 3.3 σ
- p-value=5.2×10^{-5}, difference with SM preds at 4.0σ level
- $|V_{cb}|$ drops from the ratios
- consistent with 15% enhancement for $b \to c\tau\bar{\nu}_\tau$

What is the basis for these predictions ?

$B \to D\ell\bar{\nu}_\ell$ branching ratio

$$\frac{d\Gamma(B \to D\ell\bar{\nu}_\ell)}{dq^2} \propto |V_{cb}|^2 \left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\vec{p}|^2$$
$$\left[\left(1 - \frac{m_\ell^2}{2q^2}\right)^2 M_B^2 |\vec{p}|^2 f_+^2(q^2) + \frac{3m_\ell^2}{8q^2}(M_B^2 + M_D^2)^2 f_0^2(q^2)\right]$$



- $\vec{p}$ D-momentum in B-frame, $q^2 = (p_B - p_D)^2$ lepton invariant mass
- Two form factors $f_+(q^2)$ (vector) and $f_0(q^2)$ (scalar)
  NP extension requires one more form factor f_T (tensor)
- From lattice QCD, extrapolated over whole kinematic range

[HPQCD, Fermilab collaborations]

[Nierste, Trine, Westhoff, Kamenik, Mescia]

$B \to D^* \ell \bar{\nu}_\ell$ branching ratio

$$\frac{d\Gamma(B \to D^* \ell \bar{\nu}_\ell)}{dq^2} \propto |V_{cb}|^2 \left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\vec{q}| q^2$$
$$\left[\left(1 + \frac{m_\ell^2}{2q^2}\right)^2 (|H_+|^2 + |H_-|^2 + |H_0|^2) + \frac{3m_\ell^2}{2q^2} |H_t|^2\right]$$

- H_λ describing $B \to D^*(\to D\pi)\ell\bar{\nu}_\ell$ with D^* helicity
- Interferences in principle accessible via angular analyses (but ν !)
- Four form factors $V, A_{0.1,2}$ (vector and axial)

  NP extension requires 3 more form factors $T_{1,2,3}$ (tensor)

$B \to D^*\ell\bar{\nu}_\ell$ branching ratio

$$\frac{d\Gamma(B \to D^*\ell\bar{\nu}_\ell)}{dq^2} \propto |V_{cb}|^2 \left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\vec{q}|q^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)^2 (|H_+|^2 + |H_-|^2 + |H_0|^2) + \frac{3m_\ell^2}{2q^2}|H_t|^2\right]$$

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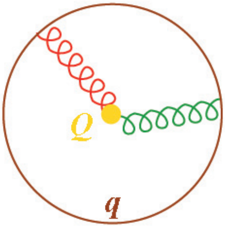
- Four form factors $V, A_{0.1,2}$ (vector and axial)
  NP extension requires 3 more form factors $T_{1,2,3}$ (tensor)
- No complete lattice determination, need other approaches !
  - HQET: Form factors related in the limit $m_b \to \infty$,
    providing ratios of form factors up to $O(\Lambda/m_B)$ corrections
  - Normalisation from Belle on $B \to D^*\ell\bar{\nu}_\ell$ ($\ell = e, \mu$)
    assuming no NP for light leptons

[Fajfer, Kamenik, Nisandzic]

Heavy-quark symmetry

Hierachy of scale in heavy-light systems

- heavy quark of mass M_Q,
- light quark dynamics interacting through soft gluons
- dynamics with energy of order $\Lambda \ll M_Q$



In reference frame of B hadron, heavy quark practically at rest
$\Longrightarrow$Heavy quark static source of gluons,
characterised by spin and colour numbers, but not mass

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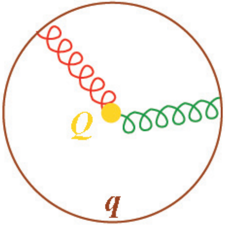

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On top of that, spin-flip transitions with gluons induced by
magnetic moment transitions, suppressed by $O(g_s/M_Q)$

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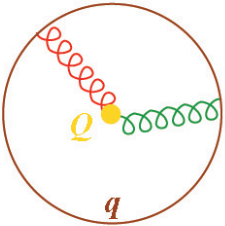
- light quark dynamics interacting through soft gluons
- dynamics with energy of order $\Lambda \ll M_Q$



In reference frame of B hadron, heavy quark practically at rest
$\Longrightarrow$Heavy quark static source of gluons,
characterised by spin and colour numbers, but not mass

On top of that, spin-flip transitions with gluons induced by
magnetic moment transitions, suppressed by $O(g_s/M_Q)$

When $M_Q \gg$ other scales in presence, properties of heavy hadrons independent of spin and mass of the heavy source of colour

Effective theory of an infinitely heavy quark

Heavy quark with momentum: $p^\mu = M_Q v^\mu + k^\mu$
where v^μ velocity of hadron ($p_B^\mu = m_B v^\mu, v^2 = 1$) and $k = O(\Lambda)$

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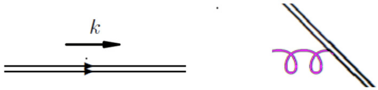

- Propagation of heavy quark

$$\frac{i}{\not p - M_Q} = \frac{i(\not p + M_Q)}{p^2 - M_Q^2} = \frac{i[M_Q(\not v + 1) + \not k]}{2(v\cdot k) + k^2} = \frac{i}{v\cdot k} P_+ + O(k/M_Q)$$

with projectors $P_\pm = \frac{1 \pm \not v}{2}$ $\qquad P_+^2 = P_+, P_-^2 = P_-, P_\pm P_\mp = 0$

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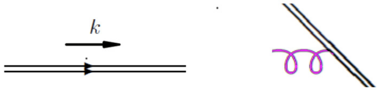

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- Interaction with gluons, taking into account that only P_+ at LO

$$i g_s P_+ \gamma_\mu T^a P_+ = i g_s P_+ (P_- \gamma_\mu + 2 v_\mu) T^a = i g_s v_\mu T^a$$

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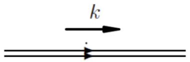

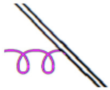

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Effective theory for the projection of heavy quark (only 1 spin d.o.f) ?

$$h_v(x) = \exp(im_Q v \cdot x) P_+ Q(x)$$

Heavy-Quark Effective Theory

Infinitely heavy quark described by Lagrangian

$$\mathcal{L} = \bar{h}_v(iv^\mu\partial_\mu + gT^a v^\mu G^a_\mu)h_v = \bar{h}_v(iv^\mu D_\mu)h_v$$

can be extended to two heavy flavours (b and c) at the same velocity v

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- Spin $SU(2)$: No Dirac structure in the Lagrangian
  $\Longrightarrow$spin of heavy-quark irrelevant for interaction with (soft) gluons
- No explicit Lorentz invariance (preferred direction: v_μ)
- $1/m_Q$ corrections (P_- modes integrated out $\to$ local operators)

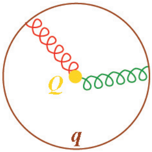
$$\mathcal{L} = h_v(iv\cdot D)h_v+\frac{1}{2M_Q}h_v\left[D^2-(v\cdot D)^2+\frac{g_s}{2}\sigma^{\mu\nu}G_{\mu\nu}\right]h_v+O\left(\frac{1}{M_Q^2}\right)$$

  - Corrections to kinetic term (motion of heavy quark in meson)
  - Chromomag. moment (mass splitting among heavy-light mesons)

Spectrum

In the rest frame of the heavy meson: $J = L + S$

- L angular momentum of light d.o.f.
- S angular momentum for heavy quark
  actually irrelevant in HQET Lagrangian



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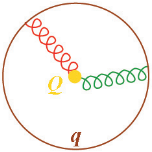

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Hadrons fully characterised by $(l, m_l; s, m_s)$
spectrum degenerate in m_s, organised in doublets

- $l = 0$ $j = 1/2$ $\Lambda_b(5620)$
- $l = 1/2$ $j = 0, 1$ degenerate pseudoscalar and vector

$$B(5279), B^*(5325) \qquad B_s(5366), B_s^*(5412)$$
$$D(1869), D^*(2010) \qquad D_s(1968), D_s^*(2112)$$

- $l = 1$ $j = 1/2, 3/2$ $\Sigma_b(5807), \Sigma_b^*(5829)$
- $l = 3/2$ $j = 1, 2$ $D_1(2420), D_2^*(2460)$ $D_{s1}(2536), D_{s2}(2573)$

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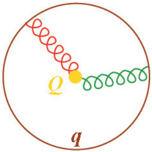

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Splitting is spin breaking $\propto \Lambda^2/m_Q$: $\frac{m_{B^*}-m_B}{m_{D^*}-m_D} = \frac{m_{B_s^*}-m_{B_s}}{m_{D_s^*}-m_{D_s}} = \frac{m_c}{m_b} = 1/3$

Dynamics for $B \to D^{(*)}\ell\nu$

$B \to D^{(*)}$ described by form factors, function of $q^2 = (p - p')^2$

$$\langle D(p')|\bar{c}\gamma_\mu b|\bar{B}(p)\rangle = (p + p')_\mu f_+ + \frac{M_B^2 - M_D^2}{q^2} q_\mu [f_0 - f_+]$$

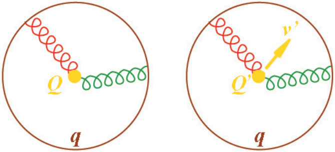
$$\langle D^*(p', \epsilon)|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}(p)\rangle = [M_B + M_{D^*}]\epsilon^*_\mu A_1 + \frac{\epsilon^* \cdot q}{M_B + M_{D^*}}(p + p')_\mu A_2$$

$$+\frac{\epsilon \cdot q}{q^2} q_\mu \left[(M_B + M_{D^*})A_1 - (M_B - M_{D^*})A_2 - 2M_{D^*}A_0\right]$$

$$\langle D^*(p', \epsilon)|\bar{c}\gamma_\mu b|\bar{B}(p)\rangle = \frac{-2i}{M_B + M_{D^*}} \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} p^\rho p'^\sigma V$$

- Meson velocities $v_\mu = p_{B\mu}/M_B$, $v'_\mu = p_{D\mu}/M_D$
- Recoil energy of D in B rest frame $E = m_D(v \cdot v' - 1)$
- $q^2 = m_B^2 + m_D^2 - 2m_B m_D(v \cdot v')$ up to $q^2_{\max} = (m_B - m_D)^2$
- $v \cdot v'$ varies between no-recoil limit $(v \cdot v' - 1)_{\min} = 0$
  and $(v \cdot v' - 1)_{\max} = \frac{(m_B - m_D)^2}{2m_B m_D} \simeq 0.6$

Physical picture



In the heavy quark limit, for $B \to D^{(*)}\ell\nu$

- Relations between D and D^* by heavy-quark symmetry on c spin
- In no-recoil limit $v = v'$, $b \to c$ unnoticed by light quark
- For $v \neq v'$, exchange of (soft) gluons
  to reorganise light cloud, still remaining quite soft
- ...decreasing the overlap between initial B and final D

Form factors and Isgur-Wise function

Embodiment of Wigner-Eckart theorem

$$\langle D(v')|\bar{c}\Gamma b|B(v)\rangle \quad \to \quad -\xi(v\cdot v')\mathrm{Tr}[\bar{\tilde{D}}(v')\Gamma\tilde{B}(v)]$$
$$\langle D^*(v',\epsilon)|\bar{c}\Gamma b|B(v)\rangle \quad \to \quad -\xi(v\cdot v')\mathrm{Tr}[\bar{\tilde{D}}^*(v',\epsilon)\Gamma\tilde{B}(v)]$$

- $Tr(\ldots) \equiv$ Clebsch-Gordan (configuration of spin projections)
- $\tilde{B}(v)$, $\bar{\tilde{D}}(v')$ and $\bar{\tilde{D}}^*(v',\epsilon)$ describe configurations of heavy and light quarks corresponding to each meson for $m_Q \to \infty$
- $\xi \equiv$ reduced matrix element
  Isgur-Wise function, depending only on $v \cdot v'$ (since $v^2 = v'^2$=1)

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In heavy-quark limit, form factors expressed in terms of ξ

$$\frac{M_B+M_D}{2\sqrt{M_B M_D}}\xi(v\cdot v') = f_+ = \left(1-\frac{q^2}{M_B+M_D)^2}\right)^{-1} f_0$$
$$\frac{M_{B^*}+M_D}{2\sqrt{M_{B^*} M_D}}\xi(v\cdot v') = V = A_0 = A_2 = \left(1-\frac{q^2}{M_{B^*}+M_D)^2}\right)^{-1} A_1$$

Isgur-Wise function ξ

- ξ also arises in

$$\langle B(v)|\bar{c}_v\gamma^0 b_v|B(v)\rangle = -\xi(v^2=1)\mathrm{Tr}[\bar{\tilde{B}}(v)\gamma^0\tilde{B}(v)] \Longrightarrow \xi(1)=1$$

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  - If $v\cdot v'=1$, $q^2_{\mathrm{max}}=(M_B-M_D)^2$ ($b\to c$ unnoticed by light quark)

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- Models to get away from $\omega = v\cdot v' = 1$

$$\xi(\omega) = 1-\rho^2(\omega-1)+O[(\omega-1)^2]$$

$$\ldots\left(\frac{2}{\omega+1}\right)^{2\rho^2}, e^{-\rho^2(\omega-1)}, \frac{2}{\omega+1}\exp\left[-(2\rho^2-1)\frac{\omega-1}{\omega+1}\right]\ldots$$

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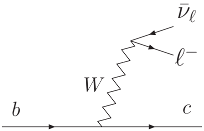
- Corrections to these relations among form factors
  - Hard-gluon exchanges $O(\alpha_s)$
  - Power corrections $O(\Lambda/m_B)$

$b \to c\ell\bar{\nu}_\ell$: effective Hamiltonian



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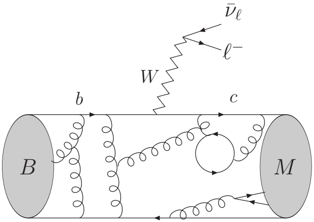

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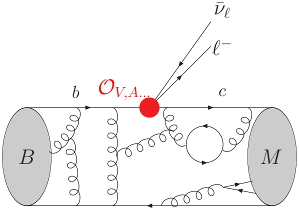

$\mathcal{H}^{\text{eff}}$ to determine short-distance couplings and look for NP model-independently

$$\mathcal{H}^{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_{\ell=e,\mu,\tau} (\bar{\ell}\gamma^\mu P_L \nu_\ell)$$
$$\times[\bar{c}\gamma^\mu P_L b + g_V \bar{c}\gamma^\mu b + g_{SL} i\partial^\mu(\bar{c}P_L b) + \ldots]$$

[with $P_{L,R} = (1 \mp \gamma_5)/2$]

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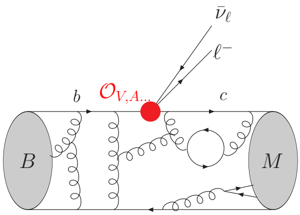

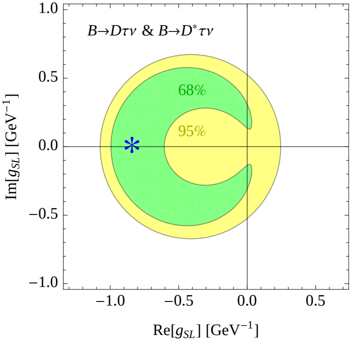

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- Scalar operators

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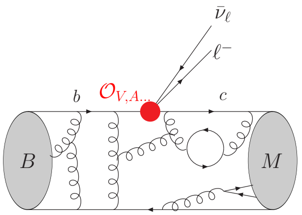

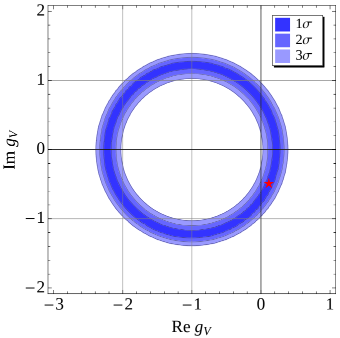

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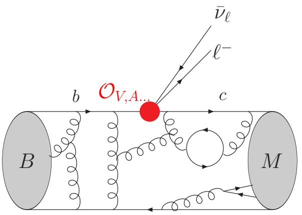

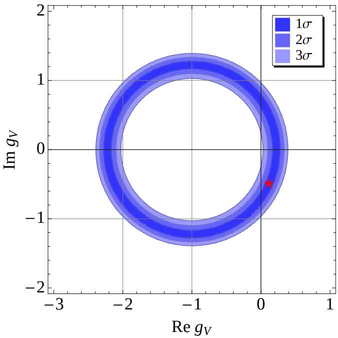

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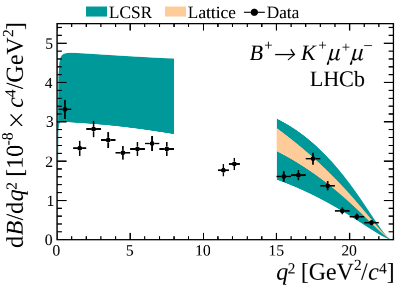
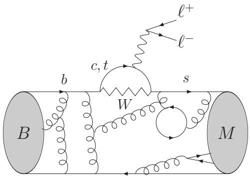
- Fit to R_D and R_{D^*} leading to viable explanation
- Scalar operators or vector operators
- However only few observables measured (neutrino in final state)
- Improving on $B \to D^*$ form factors ?

[Fajfer, Kamenik, Nisandzic, Becirevic, Tayduganov, Pokorski, Crivellin, Freytsis, Ligeti, Ruderman. . .]

$b \to s$: Soft-Collinear Effective Theory

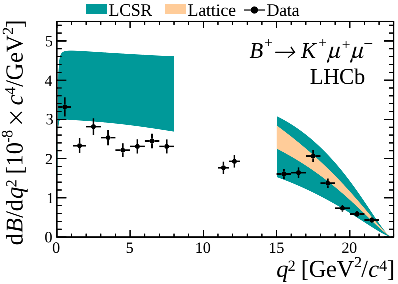
$b \to s\ell^+\ell^-$: $B \to K\ell\ell$

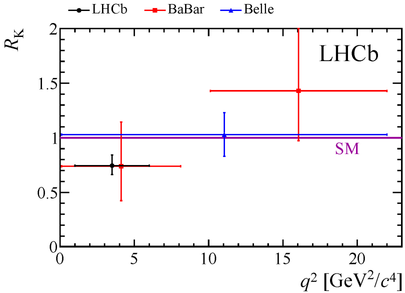


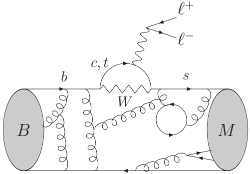


- $Br(B \to K\mu\mu)$ too low compared to SM

$b \to s\ell^+\ell^-$: $B \to K\ell\ell$

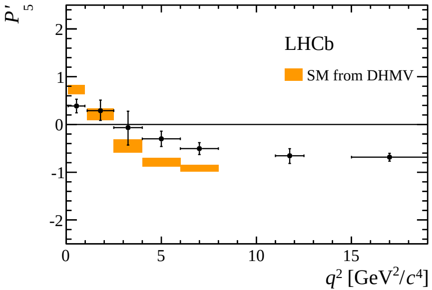






- $Br(B \to K\mu\mu)$ too low compared to SM
- $R_K = \left.\frac{Br(B\to K\mu\mu)}{Br(B\to Kee)}\right|_{[1,6]} = 0.745^{+0.090}_{-0.074} \pm 0.036$
- equals to 1 in SM (universality of lepton coupling), 2.6 σ dev
- would require NP coupling differently to μ and e

$b \to s\ell^+\ell^-$: $B \to K^*\mu\mu$ (1)



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- Measured at LHCb with 1 fb^{-1} (2013) and 3 fb^{-1} (2015)
- Discrepancies for some (but not all) observables,
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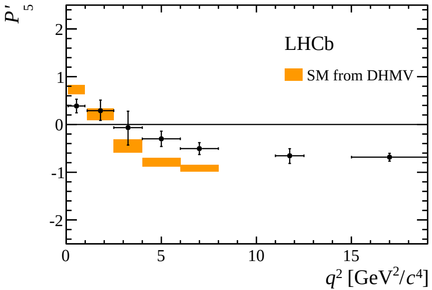

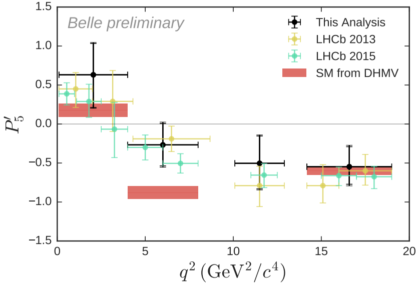

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- . . . confirmed by Belle in 2016

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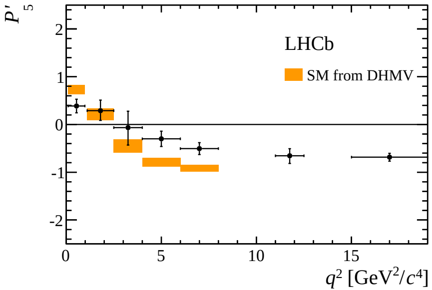

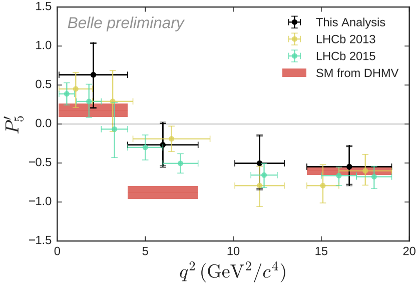

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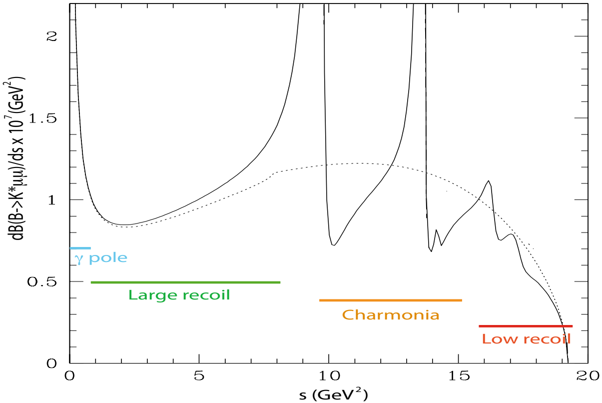
- Discrepancies for some (but not all) observables,
  in particular two bins for P'_5 deviating from SM by 2.8 σ and 3.0 σ
- ...confirmed by Belle in 2016
- Also deviations in $BR(B \to K^*\mu\mu)$ and $BR(B_s \to \phi\mu\mu)$ at low recoil

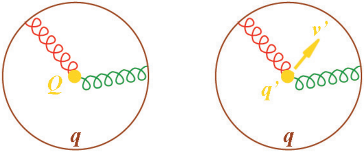
$b \to s\ell^+\ell^-$: $B \to K^*\mu\mu$ (2)



- Very large K^*-recoil ($4m_\ell^2 < q^2 < 1$ GeV^2) γ almost real
- Large K^*-recoil ($q^2 < 9$ GeV^2) energetic K^* ($E_{K^*} \gg \Lambda_{QCD}$)
  LCSR, SCET, QCD factorisation
- Charmonium region ($q^2 = m^2_{\psi,\psi'\ldots}$ between 9 and 14 GeV^2)
- Low K^*-recoil ($q^2 > 14$ GeV^2) soft K^* ($E_{K^*} \simeq \Lambda_{QCD}$)
  Lattice QCD, HQET, Operator Product Expansion

Two different regions

$B \to K^*\ell\ell$, i.e., $b \to s\ell\ell$ at the quark level



Two different regions for $B \to K^*\ell\ell$

- low K^* recoil: most of the energy is emitted by the lepton pair, the soft cloud is rearranged after the decay, but it remains soft
  $\Longrightarrow$HQET can be used
- large K^* recoil: little energy is emitted by the lepton pair the soft cloud undergoes a drastic change, the two light quarks must become collinear (along the K^* recoil direction)
  $\Longrightarrow$A different effective theory is needed

SCET

Soft-Collinear Effective Theory = Effective theory of QCD
with energetic/collinear light mesons

[Stewart et al., Beneke et al.]

- Relevant degrees of freedom
  - soft gluons/quarks : $p_s = O(\Lambda, \Lambda, \Lambda, \Lambda)$
    [light quarks, but also heavy quarks $p = Mv + p_s$]
  - collinear gluons/quarks : $p_c = (M, 0, 0, M) + O(\Lambda, \Lambda, \Lambda, \Lambda)$
    [energetic, but along one direction, with $p_c^2 = \Lambda^2$)]
- explains how soft and collinear quarks/gluons communicate
- hard d.o.f. are integrated out (corrections as local operators)
- interactions organised in an expansion in Λ/M

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$\Longrightarrow$SCET much more complicated Lagrangian than HQET

- large number of d.o.f. involved
- various interactions through soft/collinear gluons
- for inclusive, radative, nonleptonic decays, also collider physics...

$B \to K^{(*)}$ form factors

- Vector or pseudoscalar meson V or P
- Vector currents $V, A = q\gamma_\mu b, q\gamma_\mu\gamma_5 b$
- Tensor currents $T, T_5 = q[\gamma_\mu, \gamma_\nu]b, q[\gamma_\mu, \gamma_\nu]\gamma_5 b$

$$
\begin{aligned}
\langle P|V^\mu|B\rangle &= f_+ \left[p^\mu + p'^{\,\mu} - \frac{M^2 - m_P^2}{q^2} q^\mu\right] + f_0 \frac{M^2 - m_P^2}{q^2} q^\mu \,, \\
\langle P|T^{\mu\nu} q_\nu|B\rangle &= i \frac{f_T}{M + m_P} \left[q^2(p^\mu + p'^{\,\mu}) - (M^2 - m_P^2) q^\mu\right] \,, \\
\langle V|V^\mu|B\rangle &= i \frac{2V}{M + m_V} \epsilon^{\mu\nu\rho\sigma} p^\nu p'^{\,\rho} \epsilon^{*\sigma} \,, \\
\langle V|A^\mu|B\rangle &= 2m_V A_0 \frac{\epsilon^* \cdot q}{q^2} q^\mu + (M + m_V) A_1 \left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu\right] \\
&\quad - A_2 \frac{\epsilon^* \cdot q}{M + m_V} \left[p^\mu + p'^{\,\mu} - \frac{M^2 - m_V^2}{q^2} q^\mu\right] \,, \\
\langle V|T^{\mu\nu} q_\nu|B\rangle &= -2T_1 \epsilon^{\mu\nu\rho\sigma} p^\nu p'^{\,\rho} \epsilon^{*\sigma} \,, \\
\langle V|T_5^{\mu\nu} q_\nu|B\rangle &= -iT_2 \left[(M^2 - m_V^2)\epsilon^{*\mu} - (\epsilon^* \cdot q)(p^\mu + p'^{\,\mu})\right] \\
&\quad - iT_3 (\epsilon^* \cdot q) \left[q^\mu - \frac{q^2}{M^2 - m_V^2}(p^\mu + p'^{\,\mu})\right] \,.
\end{aligned}
$$

Relations between form factors at leading order

For energetic $E = O(M_B)$ light mesons, all form factors expressed in terms of three form factors $\zeta, \zeta_{||}, \zeta_\perp$ at leading order in α_s and E/M

[Charles et al.]

$$f_+(q^2) = \zeta(E_P)\,, \qquad f_0(q^2) = \left(1 - \frac{q^2}{M^2 - m_P^2}\right)\zeta(E_P)\,,$$

$$f_T(q^2) = \left(1 + \frac{m_P}{M}\right)\zeta(E_P)\,, \qquad A_0(q^2) = \left(1 - \frac{m_V^2}{ME_V}\right)\zeta_{||}(E_V) + \frac{m_V}{M}\zeta_\perp(E_V)\,,$$

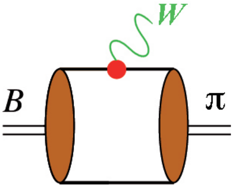
$$A_1(q^2) = \frac{2E_V}{M + m_V}\zeta_\perp(E_V)\,, \qquad A_2(q^2) = \left(1 + \frac{m_V}{M}\right)\left[\zeta_\perp(E_V) - \frac{m_V}{E_V}\zeta_{||}(E_V)\right]\,,$$

$$V(q^2) = \left(1 + \frac{m_V}{M}\right)\zeta_\perp(E_V)\,, \qquad T_2(q^2) = \left(1 - \frac{q^2}{M^2 - m_V^2}\right)\zeta_\perp(E_V)\,,$$

$$T_1(q^2) = \zeta_\perp(E_V)\,, \qquad T_3(q^2) = \zeta_\perp(E_V) - \frac{m_V}{E}\left(1 - \frac{m_V^2}{M^2}\right)\zeta_{||}(E_V)\,.$$

Leading-order results for Soft-Collinear Effective Theory

Higher-order corrections



Corrections in α_s can be computed

[Beneke, Feldmann, Seidel]

$$f_i(q^2) = C_i(q^2)\xi_i(q^2) + \phi_B \otimes T_i \otimes \phi_\pi$$

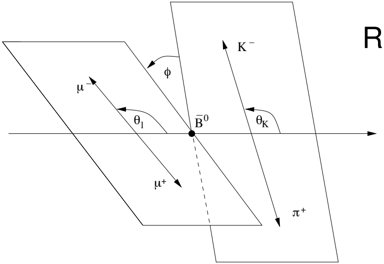
- $\xi_i = \xi_{||}, \xi_\perp$ are universal (soft) form factors
- C_i and T_i dominated by hard gluons and can be computed perturbatively: $C_i = 1 + O(\alpha_s)$, $T_i = O(\alpha_s)$
- ϕ_B and ϕ_π are light-cone distribution amplitudes

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(0)|\pi^+(p)\rangle = ip_\mu F_\pi \int_0^1 dx\ e^{ix(p\cdot z)}\phi(x) \qquad z^2 = 0$$

Hadronic quantity, corresponding to probability amplitude of finding in $\pi(p)$ a quark with longitudinal momentum xp

$\Longrightarrow$relations among form factors: $O(\alpha_s)$ and $O(\Lambda/M)$ corrections

$B \to K^*(\to K\pi)\mu\mu$ optimised observables



Rich kinematics

- differential decay rate in terms of 12 angular coeffs $J_i(q^2)$ with $q^2 = (p_{\ell^+} + p_{\ell^-})^2$
- interferences between 8 transversity amplitudes for $B \to K^*(\to K\pi)V^*(\to \ell\ell)$

[Ali, Hiller, Matias, Krüger, Mescia, SDG, Virto, Hofer, Bobeth, van Dyck, Buras, Altmanshoffer, Straub, Bharucha, Zwicky, Gratrex, Hopfer, Becirevic, Sumensari, Zukanovic-Funchal . . .]

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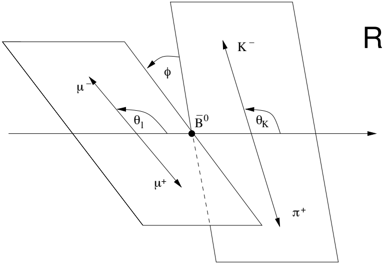

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- Transversity ampls.: Wilson coeffs $\times$ 7 form factors $A_{0,1,2}$, V, $T_{1,2,3}$
- Relations between form factors in limit $m_B \to \infty$,
  either when K^* very soft or very energetic (low/large-recoil)

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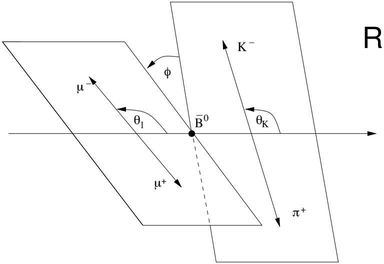

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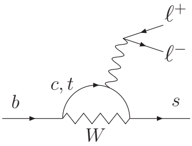
- Build ratios of J_i where form factors cancel in these limits
  (corrections by hard gluons $O(\alpha_s)$, power corrs $O(\Lambda/m_B)$)
- Optimised observables P_i with reduced hadronic uncertainties

[Matias, Krüger, Becirevic, Schneider, Mescia, Virto, SDG, Ramon, Hurth; Hiller, Bobeth, Van Dyk]

$b \to s\mu\mu$ effective hamiltonian



$$b \to s\gamma(^*) : \mathcal{H}^{SM}_{\Delta F=1} \propto \sum V^*_{ts} V_{tb} \mathcal{C}_i \mathcal{O}_i + \ldots$$

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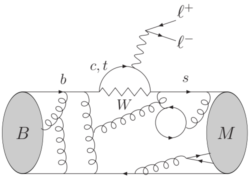

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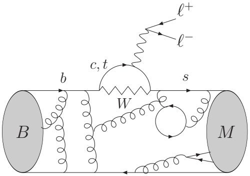

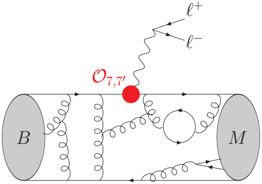

$$b \to s\gamma(^*) : \mathcal{H}^{SM}_{\Delta F=1} \propto \sum V^*_{ts} V_{tb} \mathcal{C}_i \mathcal{O}_i + \ldots$$

- $\mathcal{O}_7 = \frac{e}{g^2} m_b \, \bar{s}\sigma^{\mu\nu}(1+\gamma_5) F_{\mu\nu} \, b$ [real or soft photon]

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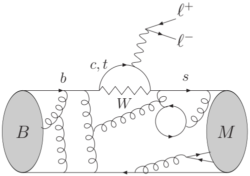

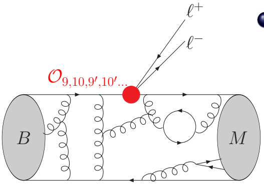

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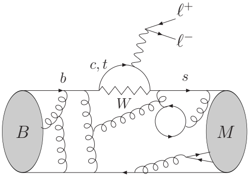
- $\mathcal{O}_9 = \frac{e^2}{g^2} \bar{s}\gamma_\mu(1-\gamma_5)b\ \bar{\ell}\gamma^\mu\ell$ [$b \to s\mu\mu$ via Z/hard $\gamma\dots$]
- $\mathcal{O}_{10} = \frac{e^2}{g^2} \bar{s}\gamma_\mu(1-\gamma_5)b\ \bar{\ell}\gamma^\mu\gamma_5\ell$ [$b \to s\mu\mu$ via Z]

$b \to s\mu\mu$ effective hamiltonian



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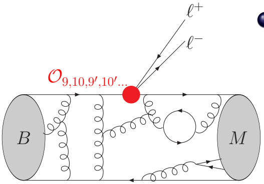
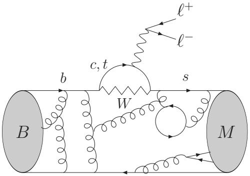
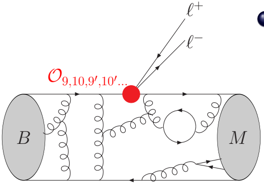


$$\mathcal{C}_7^{\rm SM} = -0.29,\ \mathcal{C}_9^{\rm SM} = 4.1,\ \mathcal{C}_{10}^{\rm SM} = -4.3 \text{ @ } \mu_b = m_b$$

$b \to s\mu\mu$ effective hamiltonian





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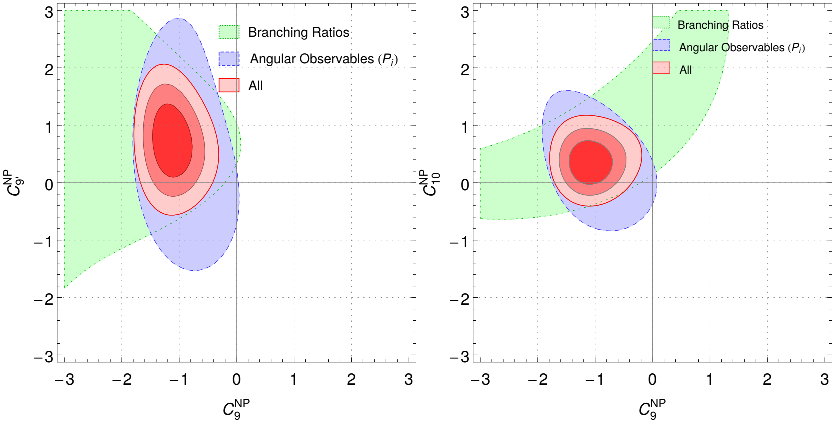
$$\mathcal{C}_7^{\rm SM} = -0.29,\ \mathcal{C}_9^{\rm SM} = 4.1,\ \mathcal{C}_{10}^{\rm SM} = -4.3 \ @\ \mu_b = m_b$$

NP changes short-distance $\mathcal{C}_i$ for SM or new long-distance ops $\mathcal{O}_i$

- Chirally flipped ($W \to W_R$) $\quad \mathcal{O}_7 \to \mathcal{O}_{7'} \propto \bar{s}\sigma^{\mu\nu}(1-\gamma_5)F_{\mu\nu}\, b$
- (Pseudo)scalar ($W \to H^+$) $\quad \mathcal{O}_9, \mathcal{O}_{10} \to \mathcal{O}_S \propto \bar{s}(1+\gamma_5)b\bar{\ell}\ell, \mathcal{O}_P$
- Tensor operators ($\gamma \to T$) $\quad \mathcal{O}_9 \to \mathcal{O}_T \propto \bar{s}\sigma_{\mu\nu}(1-\gamma_5)b\ \bar{\ell}\sigma_{\mu\nu}\ell$

Some favoured scenarios



$\mathcal{C}_9^{\rm NP}, \mathcal{C}_{9'}^{\rm NP}$ $\mathcal{C}_9^{\rm NP}, \mathcal{C}_{10}^{\rm NP}$

- BRs and angular obs both favour $\mathcal{C}_9^{NP} \simeq -1$ in "good" scenarios
- Convergence of effects when considering separtely several channels, low vs large recoil, BR versus angular
- results in agreement with [Altmanshoffer, Straub] and [Hurth, Mahmoudi, Neshatpour]

As conclusions

- Quark transitions involve both strong and weak interactions, with very different energy scales
- Separation possible through the effective Hamiltonian approach
- Short distances are embedded in Wilson coefficients, which can be computed perturbatively
- But this requires to resum potentially large logarithms through RGE
- Remaining hadronic quantities are decay constants, form factors...
- Not so many general properties known about form factors
- So often useful to simplify their structure thanks to effective theories, as illustrated with two sectors with deviations from the SM
    - $b \to c\ell\nu$, where Heavy Quark Effective Theory can be used
    - $b \to s\ell\ell$, where Soft Colllinear Effective Theory can be exploited

  leading to analyses in terms of contributions to Wilson coefficients

Any questions ?