Strong and weak interactions in the Standard Model (1)

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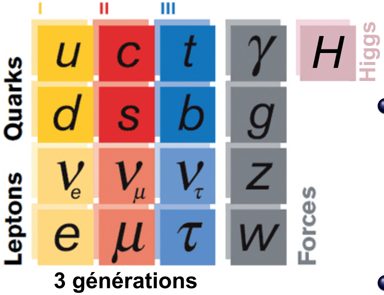
What the Standard Model is

Our current understanding of the basic constituents of matter



- 3 generations of
  - 2 quarks (u, d)
  - 1 charged lepton (e^-)
  - 1 neutrino (ν_e)
- 3 fundamental forces
  - Electromagnetism
  - Weak interaction (β decays)
  - Strong interaction (nucleus stability)
- A spin 0 particle: the Higgs boson

- 1st lecture: a few elements on weak and strong interactions
- 2nd lecture: techniques to tackle problems with both interactions

Back to basics with Quantum ElectroDynamics

Quantum field theory

- Combine special relativity and quantum mechanics
- Fields $\rightarrow$ operators acting on a state to modify particle content

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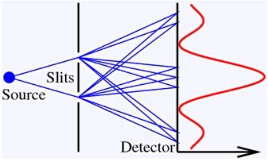

Young double-slit experiment

$$I = |A|^2 = |A_{\text{path1}} + A_{\text{path2}}|^2$$

Diff. in length path, hence interferences

$$A(a \to b) = \sum_{\text{all paths}} e^{i \cdot \text{phase}} = \int Dx(t)\, e^{i \cdot \text{phase}[x(t)]}$$

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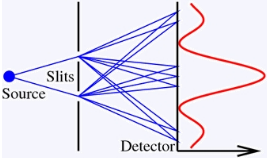

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- Classic: Only solutions are classical paths,
  minimising the action defined from a Lagrangian $\mathcal{L}$

$$\frac{\delta}{\delta x(t)} S[x(t)]\bigg|_{x_{classical}} = 0 \qquad S = \int dt\, L = \int d^4x\, \mathcal{L}$$

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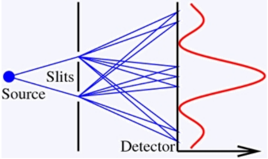

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- Quantum: Take phase $= S/\hbar$ $\qquad A(a \to b) = \int Dx(t)\ e^{iS[x(t)]/\hbar}$
  classical sols recovered in the limit $S \gg \hbar$ (other paths cancel)

Free lagrangians

Lagragian to recover equation of motion in classical limit

$$S = \int d^4x \mathcal{L}(\phi, \partial_\mu \phi) \qquad \delta S = 0 \Longrightarrow \frac{\partial \mathcal{L}}{\partial \phi} - \partial^\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)} \right) = 0$$

which yields the corresponding free Lagrangians

- Klein-Gordon spin 0: $(\partial_\mu \partial^\mu + m^2)\phi = 0 \qquad \mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi$
- Dirac spin 1/2: $\qquad (i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$
  with $\bar{\psi} = \psi^\dagger \gamma^0$

- description of free scalars and fermions
- theory can be quantized to analyse free propagation
- and to compute (free) correlation functions $\langle 0|\phi(x_1)\phi(x_2)\ldots|0\rangle$
  which can be related to Scattering matrix elements
- but not interaction at that stage, so very little dynamics !

Classical electrodynamics

- Maxwell equations $\begin{cases} \vec{\nabla} \cdot \vec{E} = \rho & \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} - \dfrac{\partial \vec{E}}{\partial t} = \vec{J} & \vec{\nabla} \times \vec{E} + \dfrac{\partial \vec{B}}{\partial t} = \vec{0} \end{cases}$

  with the potentials $\vec{E} = -\vec{\nabla} V - \dfrac{\partial \vec{A}}{\partial t} \quad \vec{B} = \vec{\nabla} \times \vec{A}$

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- Summarised in relativistic notation: $\partial_\mu F^{\mu\nu}=J^\nu$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu=\begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix} \qquad \begin{matrix} A^\mu=(V,\vec{A}) \\ J^\mu=(\rho,\vec{J}) \end{matrix}$$

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- For a free field (no current, $J=0$), comes from $\mathcal{L}=-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$
- Gauge invariance: same equation and same physics

  if arbitrary shift in potential $A^\mu\to A^\mu+\partial^\mu\Lambda$

Gauge principle: coupling the fermions

- Free (Dirac) theory: $\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$ with global phase inv.

$$\psi(x) \to e^{i\alpha Q}\psi(x)$$

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- Solving by introducing the covariant derivative

$$D_\mu \psi = (\partial_\mu - ieQA_\mu)\psi \to e^{i\alpha(x)Q} D_\mu \psi$$

with A_μ spin-1 field such as $A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$

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with A_μ spin-1 field such as $A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha$, like the potential !

Quantum Electrodynamics or QED

Invariance under local phase redefinition
yields QED, sum of the two Lagrangians

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\begin{aligned}
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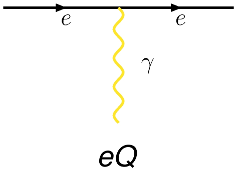

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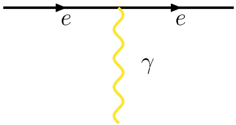

eQ

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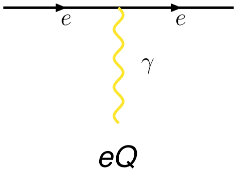

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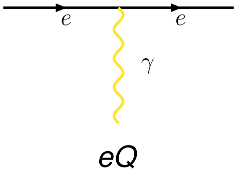

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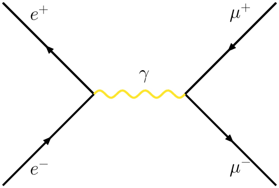
  Maxwell equation with current from electron
- Mass term $\mathcal{L}_M = \frac{1}{2} m_\gamma^2 A^\mu A_\mu$ forbidden by gauge invariance,
  so $m_\gamma = 0$ [exp $< 2 \cdot 10^{-16}$ eV]

Loop effects

- Each photon exchange comes with a power of $\alpha = e^2/(4\pi)$
- Perturbation theory (if α small enough to ensure convergence)

$$A = A^{(0)} + \alpha A^{(1)} + \alpha^2 A^{(2)} + \dots$$



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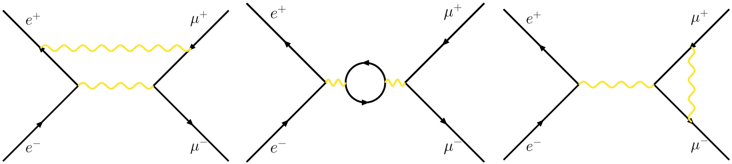

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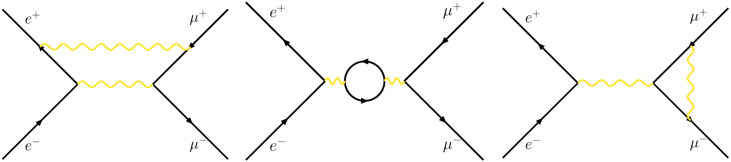

- Integration over momenta of internal particles
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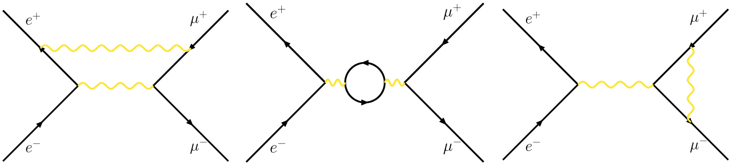

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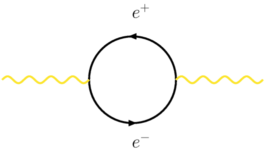
- Ultimately, UV divergences for large momenta to be renormalised

gauge symmetry constrains also structure of divergences

Experimental consequences: α



Em interaction between two electric probes
= photon exchange, sensitive to
pair creation of electron/positron from vacuum

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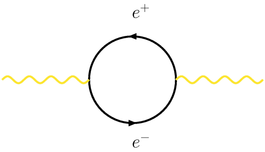

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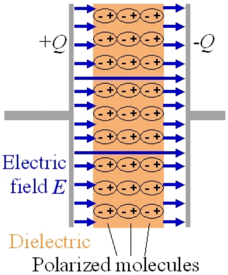

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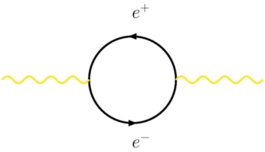

Em interaction between two electric probes
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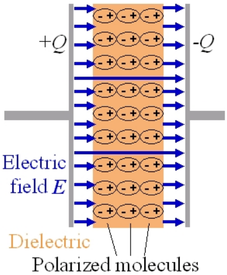
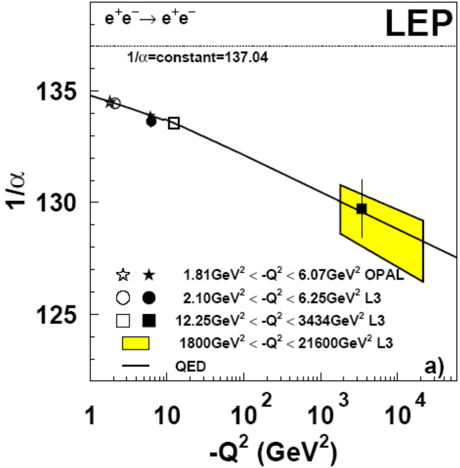
- Vacuum similar to a dielectric medium containing orientable dipoles which screen the electromagnetic field
- Virtual electron/positron loops make $\alpha = e^2/(4\pi)$ increases at short distances (or at large energies)

$$\frac{de(q)}{d\log(q)} = \beta(e) = \frac{e^3}{12\pi^2} + O(e^5) > 0$$



The fine-structure “constant” α is not constant



Tested at LEP
in 1990-2000

A coloured world with Quantum ChromoDynamics

Colours

- Quark model: proton *uud*, neutron *udd*...
- Among states discovered in 50's $\Delta^{++}(J = 3/2, J_3 = 3/2) = u^\uparrow u^\uparrow u^\uparrow$
- But Δ is a fermion, with antisymmetric wave function (Pauli)

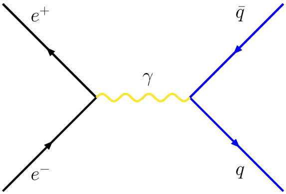
$\Longrightarrow$additional d.o.f.: colour (green, blue, red)

$$\Delta^{++}(J = 3/2, J_3 = 3/2) = \epsilon^{\alpha\beta\gamma} u^\uparrow_\alpha u^\uparrow_\beta u^\uparrow_\gamma$$

More generally, if i, j, k flavour and α, β, γ colour, hadrons combine quarks in colourless combination

- Baryons consist of $\epsilon^{abc} q^i_\alpha q^j_\beta q^k_\gamma$
- Mesons consist of $\delta^{\alpha\beta} q^i_\alpha \bar{q}^j_\beta$
- Exotics (tetraquarks, pentaquarks recently observed at Babar, Belle, LHCb...) combining the previous structures

How many colours ?

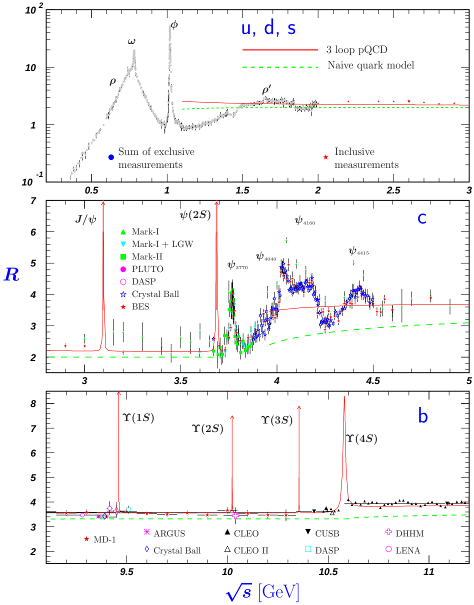


$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq N_c \sum_q Q_q^2$$

$$= \begin{cases} 2/3 \cdot N_c & (u,d,s) \\ 10/9 \cdot N_c & (u,d,s,c) \\ 11/9 \cdot N_c & (u,d,s,c,b) \end{cases}$$

vary when a $q\bar{q}$ threshold production is crossed

3 colours



Resonances after each $q\bar{q}$ threshold, then asymptotic value with $N_c = 3$

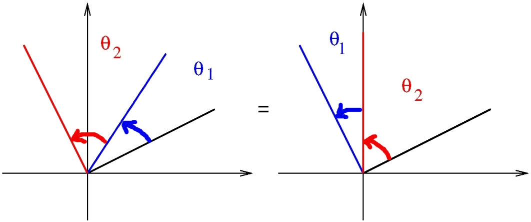
Colours have to do with the dynamics of quarks since coloured quarks bound in white objects

A few words on symmetries

- In QED, symmetry under phase redefinition

$$\psi \to e^{i\alpha Q}\psi$$

- $U(1)$ equivalent to $O(2)$ symmetry, rotations in 2 dimensions



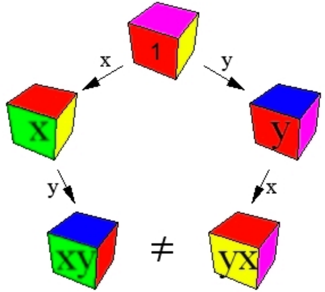
abelian (i.e.m commuting) group:

$$R(\theta_1)R(\theta_2) = R(\theta_2)R(\theta_1) = R(\theta_1 + \theta_2)$$

Not always the case !

Nonabelian symmetries

Rotations in larger spaces are nonabelian,
for instance $O(3)$: rotations and reflexions in 3 dimensions



- A group: $R_1 R_2$ still a rotation, belongs to $O(3)$
- But not abelian: $R_1 R_2 \neq R_2 R_1$
- Structure of the group specified by $[R_1, R_2] = R_1 R_2 - R_2 R_1$

Group transformation

- Representation of the group: "how the object transforms"

  For instance, under a $SO(3)$ (three-dimensional) rotation
  - scalar S: $S \to S$

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  - vector A: $A^i \to R^{ij} A^j \equiv [\exp[-i\theta_a J^a]]^{ij} A^j$

$$J^a = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

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  - spinor ψ: $\psi^\alpha \to [S_{1/2}(R)]^{\alpha\beta} \psi^\beta \equiv [\exp[-i\theta_a \sigma^a/2]]^{\alpha\beta} \psi^b$

$$\sigma^a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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  - spinor ψ: $\psi^\alpha \to [S_{1/2}(R)]^{\alpha\beta} \psi^\beta \equiv [\exp[-i\theta_a \sigma^a/2]]^{\alpha\beta} \psi^b$

$$\sigma^a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Lie Algebra: "how the group is characterised" (indep of repres.)

  $U = \exp(-i\theta_a T^a)$ with T^a traceless hermitian generators
  where $[T^a, T^b] = i f^{abc} T^c$ $\qquad f^{abc}$ group structure csts

Group transformation

- Representation of the group: "how the object transforms"

  For instance, under a $SO(3)$ (three-dimensional) rotation
  - scalar S: $S \to S$
  - vector A: $A^i \to R^{ij} A^j \equiv [\exp[-i\theta_a J^a]]^{ij} A^j$

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  Rotations: $\quad [J^a, J^b] = i\epsilon^{abc} J^c \qquad [\sigma^a/2, \sigma^b/2] = i\epsilon^{abc}\sigma^c/2$

  $\Longrightarrow$Infinitesimal version of the "table of multiplication" of the group

$SU(2)$ and $SU(3)$ groups

$U = \exp(-i\theta_a T^a) \in SU(N)$ parametrised by θ_a

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  Fundamental represent. $T^a = \frac{1}{2}\sigma^a$ from Pauli matr ($f^{abc} = \epsilon^{abc}$)

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- $SU(3)$: $a = 1 \ldots 8$ matrices 3×3
  Fundamental represent. $T^a = \frac{1}{2}\lambda^a$ from Gell-Mann matrices

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \ldots$$

Colour symmetry

- Free coloured quarks $q = \begin{pmatrix} q \\ q \\ q \end{pmatrix}$ $\qquad \mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m)q$

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  $G^\mu_{\alpha\beta} = G^\mu_a (\lambda^a)_{\alpha\beta}/2$ collects eight gluons !

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QED	QCD
One phase	Three colours
$U(1)$	$SU(3)$
Abelian symmetry	Nonabelian symmetry
1 parameter	8 parameters

QCD Lagrangian

Invariance under local colour rotations yields QCD Lagrangian

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- A term for the quarks: free + interaction

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- but also a kinetic term for the gluons

$$\mathcal{L}_F = -\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} = -\frac{1}{2}Tr[G^{\mu\nu}G_{\mu\nu}]$$

where $G^{\mu\nu}$ is the analogue of electromagnetic $F^{\mu\nu}$

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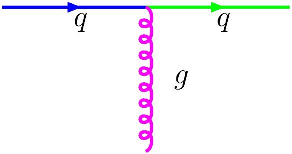
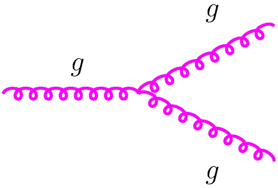
- No mass term (not gauge invariant), hence gluons are massless
- Interactions: q-q-g from $\mathcal{L}_D$, 3 gluons and 4 gluons from $\mathcal{L}_F$ [new !]

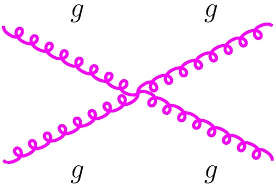
QCD interactions



$g_s \gamma^\mu \lambda_{\alpha\beta}/2$



$g_s f^{abc}$



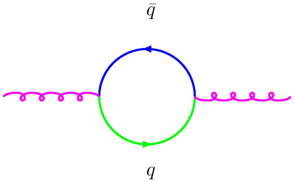
$g_s^2 f_{abc} f_{ade}$

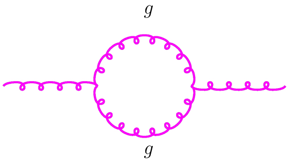
Differences from electromagnetism

- Gluons themselves sensitive to strong interaction
- Universal coupling g_s (no "colour-electric charge")
  related to the existence of 3- and 4-gluon interactions

Asymptotic freedom

And vacuum polarisation, e.g. gluon exchange between 2 quarks ?





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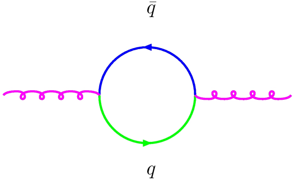

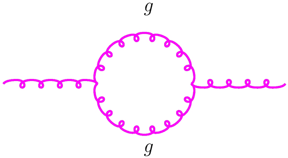

Pairs of virtual quarks AND gluons from the vacuum

- modification of $\alpha_s = g_s^2/(4\pi)$ with the distance/energy

$$\frac{dg_s(q)}{d\log(q)} = \beta(g) = -\frac{g^3}{4\pi^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right] + \ldots$$

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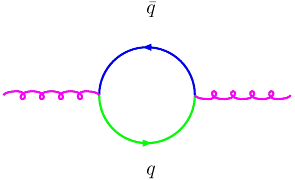

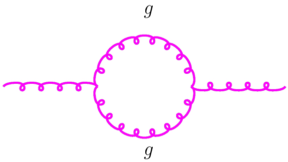

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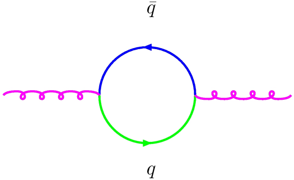

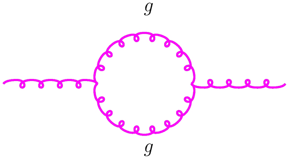

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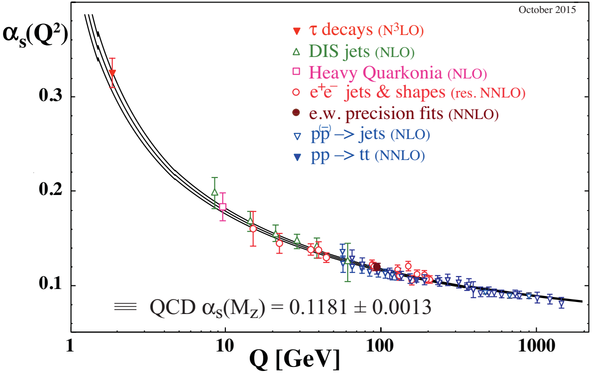
- N_c from gluons: α_s decreases at small distances
  (antiscreening from gluons, like magnets along colour field lines)
- in our world ($N_c = 3$, $N_f \leq 6$), the gluons win and $\beta < 0$!

α_s decrease at small distances

α_s at various scales



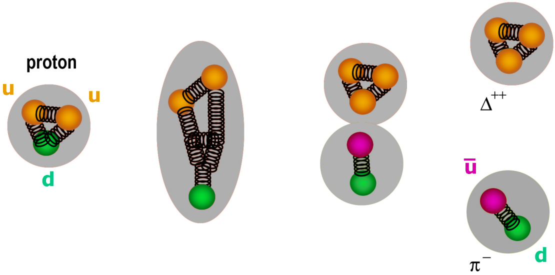
$\Longrightarrow$asymptotic freedom:
at large energies, interactions (prop to g_s) small perturbations

Consistency over a very large range of energies
(from m_τ up to LHC pp collisions)

Confinement

At distances of order 1 fm, α_s becomes of $O(1)$

$\Longrightarrow$breakdown of perturabtive picture



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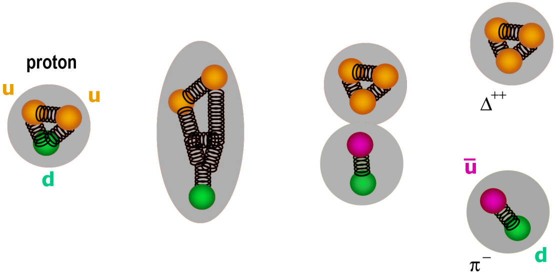

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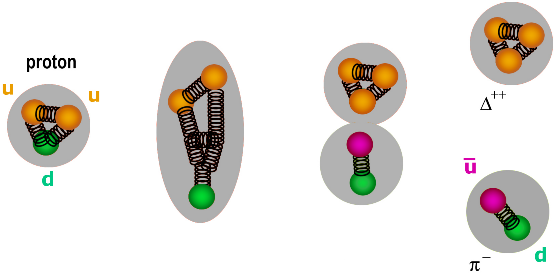

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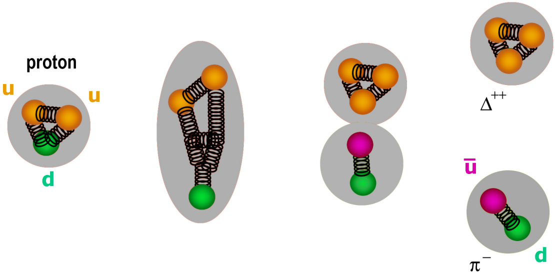

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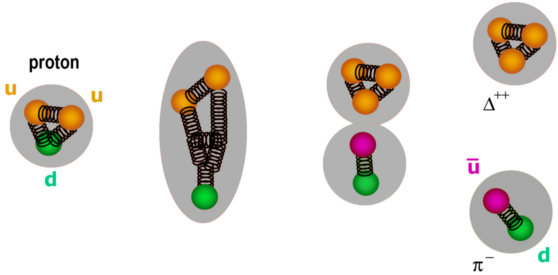

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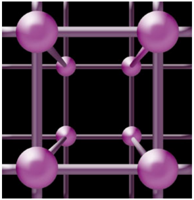
  $\Longrightarrow$quark decays weakly into another quark inside a hadron
- Hard to connect theory (quarks) and experiment (hadrons)
  - solve numerically the equations (lattice gauge theory)
  - build a theory of more limited scope (effective field theory)

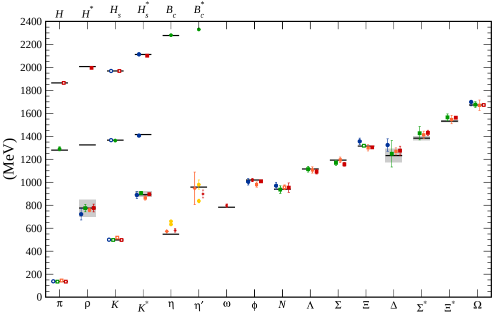
Lattice gauge theories



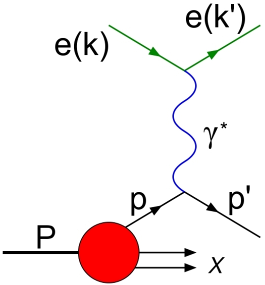
Compute propagation and decay of a particle

- Discretise space and time (lattice spacing)
- Finite 4D box (finite-volume effects) with Euclidean metric
- Sum over all possible configurations (Monte Carlo methods)

Recent progress in understanding effect of (virtual) sea quarks, finite volume, lattice spacing and renormalisation...



Deep inelastic scattering: parton model



$$e^-(k)p(P) \to e^-(k') + X$$

- $q = k - k'$ momentum transfer
- $s = (P + k)^2$ cms energy
- $x = -\frac{q^2}{2P\cdot q}$ scaling var
- $y = \frac{P\cdot q}{P\cdot k}$ relative energy loss

In parton model, energetic proton made of nearly collinear partons

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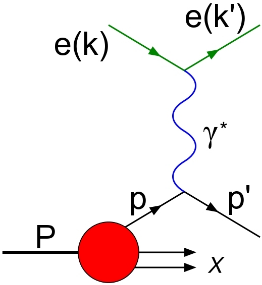

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$$\frac{d^2\sigma}{dxdy} = \sum_f [x f_f(x) Q_f^2] \times \frac{2\pi\alpha^2 s}{q^4}[1+(1-y)^2]$$

$f_f(x)$: parton distribution function, probability of finding a constituent f with a longitudinal fraction x of momentum

$\Longrightarrow$Parton model: pdf scale with x, parton cross-section depend on y

Deep inelastic scattering: QCD

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QCD provides corrections to the parton scaling

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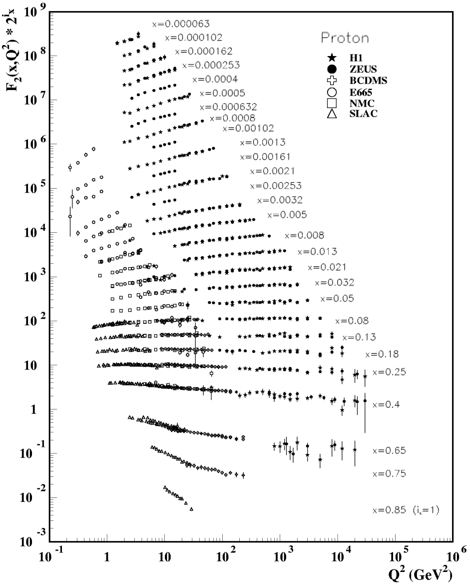
Two types of QCD correction

- $O(\alpha_s)$ and higher-order corrections to vertex
- variation of $f_f(x,q)$ with q

F_2 measurements



Measurements of

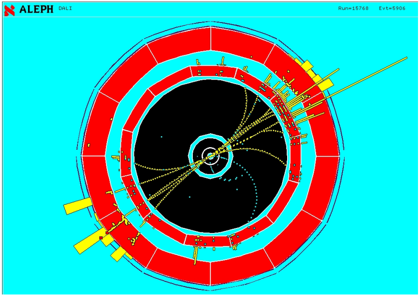
$$F_2 = \sum_f x Q_f^2 f_f(x, q)$$

Variations with q in agreement with QCD

Jets

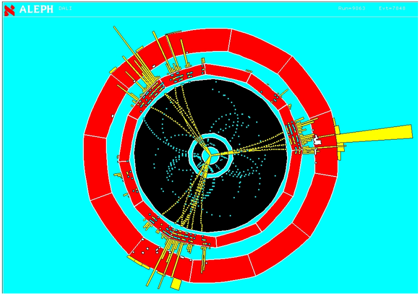
In collisions, quarks/gluons emit further gluons/quarks and lose energy, until they become soft (around 1 GeV) and bind into hadrons

$e^+e^- \to q\bar{q}$



Two jets

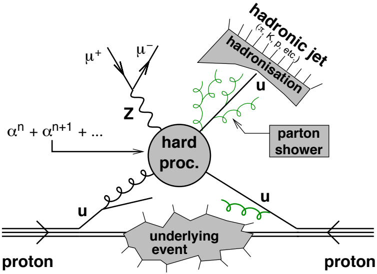
$e^+e^- \to q\bar{q}g$

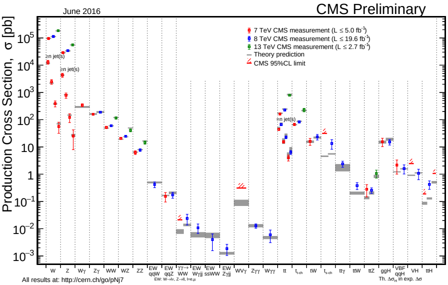


Three jets

$\Longrightarrow$Global observables, dependent on high energies (infrared safe), well described by perturbative QCD: total σ, thrust, sphericity

QCD@LHC





- Separation of scales between hard (perturbative) and soft (hadronic) dynamics
- Probe QCD and approximate models for Monte Carlo simulations
- Constraining α_s and/or parton distribution functions
- Good agreement with NLO QCD over 11 orders of magnitude
- Next steps: NNLO (already for $t\bar{t}$ production), processes with H

From left to right and back with the weak interactions

A detour through chirality and helicity

- Helicity: Projection of spin on direction of motion (frame-depend)

$$h = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|}$$

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A detour through chirality and helicity

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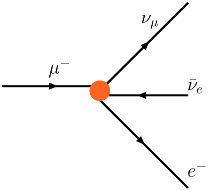
$$\begin{aligned} \mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \qquad\qquad \bar{\psi} = \psi^\dagger\gamma^0 \\ &= \bar{\psi}_L i\gamma^\mu\partial_\mu\psi_L + \bar{\psi}_R i\gamma^\mu\partial_\mu\psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \end{aligned}$$

A few observations

Charged weak currents
(charged boson exchange ?)



- Only left-handed fermions produced (right-handed antifermions)
  parity symmetry violated by the weak interactions
- Doublet partners (ℓ, ν_ℓ): $\nu_\mu X \to \mu^- X'$ but not $\nu_\mu X \to e^- X'$
- Universal strength: $\Gamma(\ell \to \nu_\ell \ell' \bar{\nu}_{\ell'}) \sim G_F^2 m_\ell^5$
- Charged bosons require embedding both weak and em interactions

Neutral weak currents (neutral boson exchange ?)

- $\nu_\mu(p) + N(q) \to \nu_\mu(p') + N(q')$
- Tiny flavour-changing neutral currents

Can we build a theory of weak interactions
embedding such elements ?

Fermion content

Free massless fermions

$$\mathcal{L} = \sum_{j=L,R,S} i\bar{\psi}_j \gamma^\mu \partial_\mu \psi_j$$

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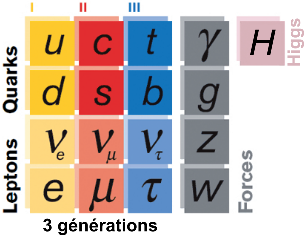

- Left-hd doublets: $\psi_L = \begin{pmatrix} f_u \\ f_d \end{pmatrix}_L$
- Right-hd singlets of two types

$$\psi_R = (f_u)_R \qquad \psi_S = (f_d)_R$$

with difference between "high" and "low" electric charge fermions

$$f_u = u, c, t, \nu_e, \nu_\mu, \nu_\tau \qquad f_d = d, s, b, e^-, \mu^-, \tau^-$$

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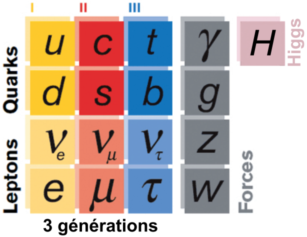

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in order to distinguish between

- left-handed doublets involved in charged currents
- right-handed fermions of different charges

Symmetries

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We introduce the $SU(2)_L \otimes U(1)_Y$ symmetry

- $U(1)_Y$: phase β, somehow related to QED
- $SU(2)_L$: rotation $U_L = \exp[i\frac{\vec{\alpha}\vec{\sigma}}{2}]$ affecting only left-hd doublets
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which yields the transformation of the doublets and singlets

$$\psi_L \to e^{iy_L\beta} U_L \psi_L \qquad \psi_R \to e^{iy_R\beta}\psi_R \qquad \psi_S \to e^{iy_S\beta}\psi_S$$

Historically, many attempts with different symmetry groups

Gauge bosons

Promoting $SU(2)_L \otimes U(1)_Y$ to local symmetry, thus covariant deriv

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- $U(1)_Y$: $\beta = \beta(x) \Longrightarrow$ 1 boson B_μ

$$\begin{aligned} D_\mu\psi_R &= [\partial_\mu - ig' y_R B_\mu]\psi_R \to e^{iy_R\beta(x)} D_\mu\psi_R \qquad [id \text{ for } \psi_S] \\ & B_\mu(x) \to B_\mu(x) + \frac{1}{g'}\partial_\mu\beta(x) \end{aligned}$$

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- $SU(2)_L$: $\vec{\alpha} = \vec{\alpha}(x) \Longrightarrow$ 3 bosons W_μ^i in $W_\mu(x) = \frac{\vec{\sigma}}{2}\vec{W}_\mu(x)$

$$D_\mu\psi_L = [\partial_\mu - igW_\mu - ig' y_L B_\mu]\psi_L \to e^{iy_L\beta(x)} U_L(x) D_\mu\psi_L$$
$$W_\mu(x) \to U_L(x) W_\mu(x) U_L^\dagger(x) - \frac{i}{g}\partial_\mu U_L(x) U_L^\dagger(x)$$

Charged currents

Write down the Lagrangian for fermions with covariant derivatives

$$\mathcal{L} = \sum_{j=L,R,S} i\bar{\psi}_j\gamma^\mu D_\mu\psi_j \Longrightarrow \text{free} + g\bar{\psi}_L\gamma^\mu W_\mu\psi_L + g' B_\mu \sum_{j=L,R,S} y_j\bar{\psi}_j\gamma^\mu\psi_j$$

The interaction term involves the

- 3 bosons related to $SU(2)_L$

$$W_\mu = \frac{\vec{\sigma}}{2}\vec{W}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^\dagger \\ \sqrt{2}W_\mu & -W_\mu^3 \end{pmatrix} \qquad W_\mu = (W_\mu^1 + iW_\mu^2)/\sqrt{2}$$

- 1 boson related ot $U(1)_Y$: B_μ

Charged currents

In the interaction term $g\bar{\psi}_L\gamma^\mu W_\mu\psi_L + g' B_\mu \sum_{j=L,R,S} y_j\bar{\psi}_j\gamma^\mu\psi_j$

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select charged current processes

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^\dagger [\bar{q}_u \gamma^\mu (1-\gamma_5) q_d + \bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\ell] + h.c.$$

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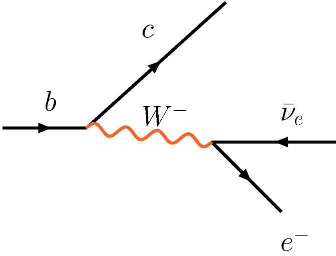
- Mediated by two charged bosons W^+ and W^-
- Quark and lepton universality (one coupling)
- At low energies, reduces to $g^2/M_W^2 \to G_F$ (Fermi constant)
- Left-handed interaction



Neutral currents: the photon

$$\mathcal{L}_{NC} = g\bar{\psi}_L\gamma^\mu W^3_\mu \frac{\sigma_3}{2}\psi_L + g' B_\mu \sum_{j=L,R,S} y_j \bar{\psi}_j\gamma^\mu\psi_j$$

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2 neutral bosons mixing to yield physical gauge bosons

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

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provided that the following relations hold

- Weinberg angle: $e = g\sin\theta_W = g'\cos\theta_W = \frac{gg'}{\sqrt{g^2+g'^2}}$
- Hypercharge: $y_L = Q_{f_u} - \frac{1}{2} = Q_{f_d} + \frac{1}{2}$, $y_R = Q_{f_u}$, $y_S = Q_{f_d}$

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Neutral currents: the Z boson

In addition to the photon part, $\mathcal{L}_{NC}$ contains interactions for Z^μ

$$
\begin{aligned}
\mathcal{L}_{NC}^Z &= \frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \left[\bar\psi_L \gamma^\mu \frac{\sigma_3}{2} \psi_L - \sin^2\theta_W \sum_{j=L,R,S} \bar\psi_j \gamma^\mu Q_j \psi_j \right] \\
&= \frac{e}{\sin 2\theta_W} Z_\mu \sum_f \bar f \gamma^\mu [v_f - a_f \gamma_5] f
\end{aligned}
$$

where fermions f are quarks and leptons containing both f_L and f_R

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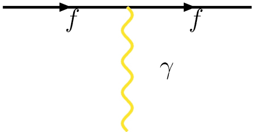
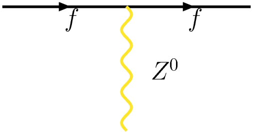
where fermions f are quarks and leptons containing both f_L and f_R

	u, c, t	d, s, b	ν_e, ν_μ, ν_τ	e^-, μ^-, τ^-
$2v_f$	$1-\frac{8}{3}\sin^2\theta_W$	$-1+\frac{4}{3}\sin^2\theta_W$	1	$-1+4\sin^2\theta_W$
$2a_f$	1	-1	1	-1

Neutral currents: gauge bosons



eQ_f



$\frac{e}{2\sin\theta_W}(v_f - a_f\gamma_5)$

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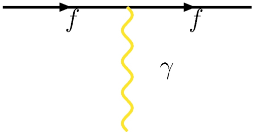

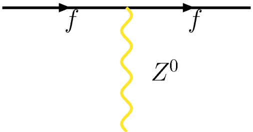

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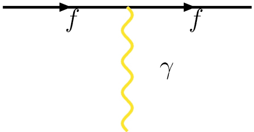
Neutral currents: gauge bosons



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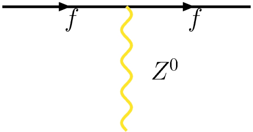

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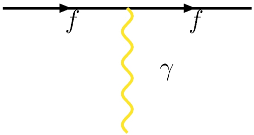
  $\Longrightarrow$Sterile (component of) neutrinos

Neutral currents: gauge bosons



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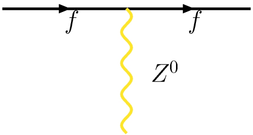

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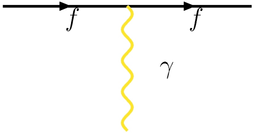
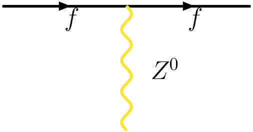
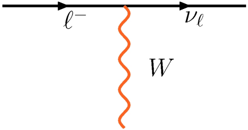
- $v_{\nu_\ell} = a_{\nu_\ell}$: no interaction for right-handed neutrinos
  $\Longrightarrow$Sterile (component of) neutrinos
- No flavour-changing neutral currents from Z and γ-exchange
  $\Longrightarrow$Occur only through loop effects in the SM (small)



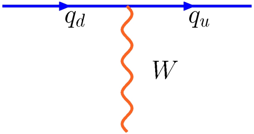
$$eQ_f$$



$$\frac{e}{\sin 2\theta_W}(v_f - a_f\gamma_5)$$



$$\frac{g}{2\sqrt{2}}(1-\gamma_5)$$



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The mass issue

Too symmetric a theory: problems with the mass

- $\mathcal{L}_{m_b} = \frac{1}{2} m_b^2 b^\mu b_\mu$ not invariant under gauge transformations:

$$W_\mu(x) \to U_L(x) W_\mu(x) U_L^\dagger(x) - \frac{i}{g'} \partial_\mu U_L(x) U_L^\dagger(x)$$

All the masses of gauge bosons should vanish but

$$m_\gamma = 0 \qquad m_W = 80\ \mathrm{GeV} \qquad m_Z = 91\ \mathrm{GeV}$$

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- $\mathcal{L}_{m_f} = -m_f \bar{f} f = -m_f (\bar{f}_L f_R + \bar{f}_R f_L)$ not invariant under gauge tf:

$$\psi_L \to e^{iy_L \beta} U_L \psi_L, \ \psi_R \to e^{iy_R \beta} \psi_R$$

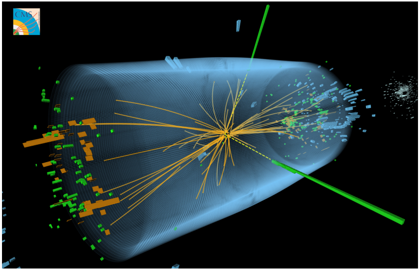
All the masses of the fermions should vanish but

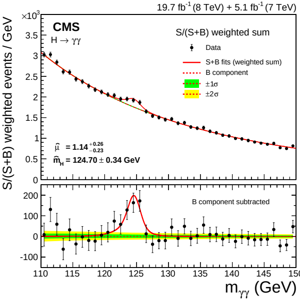
$$m_e = 0.5 \text{ MeV} \ldots m_t \simeq 170 \text{ GeV}$$

Not covered here: the Higgs mechanism

Mass issues by the introduction of a doublet ϕ of scalar complex fields

- Spontaneous breakdown: $SU_L(2) \otimes U_Y(1) \to U_{QED}(1)$
- Yukawa interaction of ϕ with fermions provide their mass terms
- 3 d.o.f. of ϕ provide longitudinal polarisations of the (massive) W, Z
- One d.o.f. remaining as particle in the spectrum, the H boson





This does not explain the mass hierarchy among the various fermions

Fermion mass matrices

- Yukawa interactions, but 3 generations

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- Interactions defined in terms of q'
  leading to U, V in interactions when expressed in terms of q

Charged & neutral currents

- Flavour-conserving neutral: $\bar{f}_L \Gamma f_L = \bar{f}'_L \Gamma f'_L$, $\bar{f}_R \Gamma f_R = \bar{f}'_R \Gamma f'_R$

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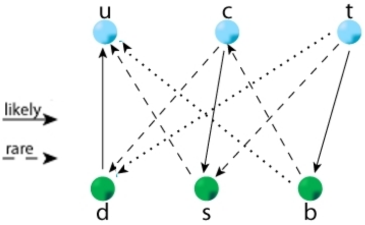

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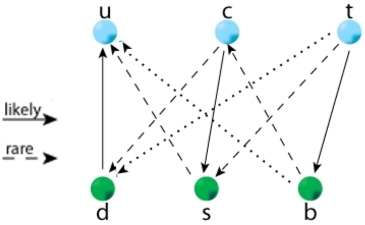

- For 3 generations, Cabibbo-Kobayahi-Maskawa matrix V contains one imaginary term, (only) source of CP violation in SM
- If no ν_R, $m_\nu = 0$, ℓ rotation absorbed in ν, no lepton mixing matrix, otherwise Pontecorvo-Maki-Nakagawa-Sakata matrix

Pure gauge: self-couplings

Let us introduce the field tensors

$$W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu - ig[W^\mu, W^\nu] \to U_L W^{\mu\nu} U_L^\dagger$$

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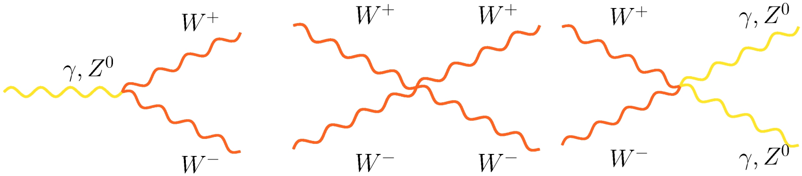
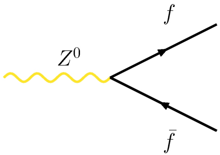
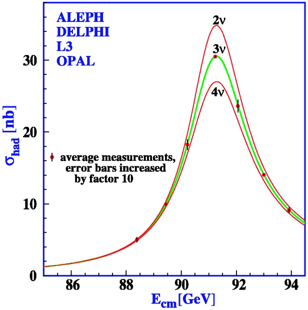
The kinetic part of the Lagrangian $\mathcal{L}_K = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} \vec{W}^{\mu\nu} \vec{W}_{\mu\nu}$



contains three and four-boson couplings
(W^3, B combinations of A, Z)

Z coupling to neutrinos





$$\Gamma(Z \to f\bar{f}) \propto |v_f|^2 + |a_f|^2$$

$$\frac{\Gamma(Z \to \text{invisible})}{\Gamma(Z \to \ell^+\ell^-)} = N_\nu \frac{\Gamma(Z \to \nu_\ell \bar{\nu}_\ell)}{\Gamma(Z \to \ell^+\ell^-)} = N_\nu \frac{2}{1 + (1 - 4\sin^2\theta_W)^2} = 1.96 N_\nu$$

LEP measurements: Only 3 light neutrinos !

Z related observables

Consider the cross section for $e^+e^- \to \gamma, Z \to f\bar{f}$
with an angle θ between in and out states in center of mass

$$\frac{d\sigma}{d\Omega} = \frac{\alpha_{em}^2}{8s} N_f[A(1+\cos^2\theta) + B\cos\theta - h_f[C(1+\cos^2\theta) + D\cos\theta]]$$

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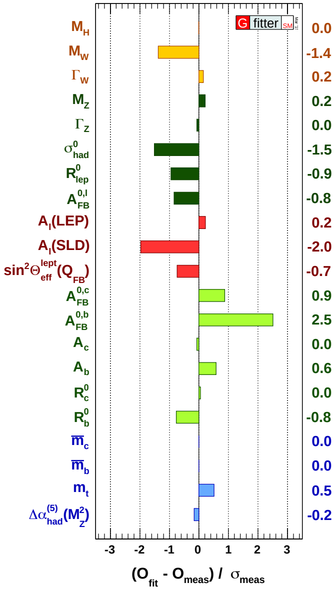
$$\sigma = \frac{4\pi\alpha_{em}^2}{3s} N_f A \qquad A_{FB}^f = \frac{N_F - N_B}{N_F + N_B} = \frac{3}{8}\frac{B}{A}$$

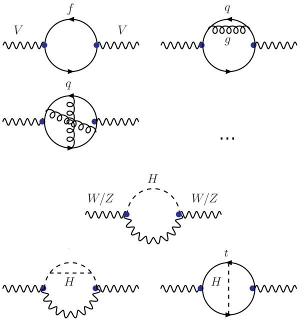
$$A_{LR}^f = \frac{\sigma^{h_f=1} - \sigma^{h_f=-1}}{\sigma^{h_f=1} + \sigma^{h_f=-1}} = -\frac{C}{A}$$

- At the Z peak, $A_{FB}^f = \frac{3}{4} A_{LR}^e A_{LR}^f$ (measures polarisation of quarks)

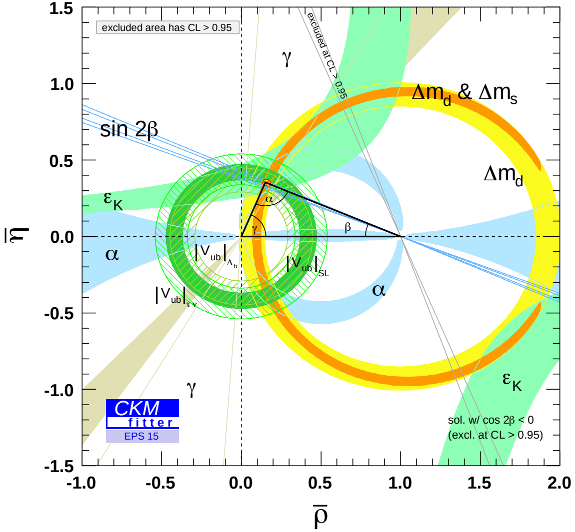
Electroweak precision measurements



- Fitting the previous observables and others, depending on M_H and m_t
- Good overall agreement



The CKM matrix



- V^{CKM} depends on 4 parameters $A, \lambda, \bar{\rho}, \bar{\eta}$
- Each band is a constraint from one (or several) weak process involving quarks
- Agree, lead to accurate $\bar{\rho}, \bar{\eta}$
- $\bar{\eta} \neq 0$ indicates CP-violation

Important constraint for any theory beyond the Standard Model

As conclusions

- QFT framework powerful for particle physics
- Requires gauge symmetry to describe 3 interactions in the Standard Model
- QED template of all three interactions for the Standard Model
- QCD non-abelian structure yields confinement, with running of α_s well tested
- Weak interactions described together with electromagnetism, distinguishing left and right chiralities
- Accurately tested through several ways (electroweak precision tests, CKM matrix structure)

Any questions ?