Short-Distance QCD corrections to $K\bar{K}$ mixing in Left-Right Models

[Bernard, Descotes-Genon, LVS - arXiv:1512.00543 [hep-ph]]

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Outline

1. Introduction: LRM
2. Short-Distance (SD) QCD corrections $K\bar{K}$: SM
   - $K\bar{K}$ mixing in SM
   - "Method of Regions" (MR)
   - Effective Field Theory
3. SD QCD corrections $K\bar{K}$: LRM
   - $K\bar{K}$ mixing in LRM
   - MR
   - EFT
   - Checks EFT, results
4. Conclusions

Introduction: beyond the SM

- Extend the *SM* beyond v_{EW}

$$\begin{array}{c}\text{New Physics (NP) model}\\ \downarrow \Lambda_{NP}\\ SM = SU(3)_c \times SU(2)_L \times U(1)_Y\\ \downarrow v_{EW}\\ SU(3)_c \times U(1)_{EM}\end{array}$$

- Two procedures:

  $\rightarrow$ model-independent way: operators suppressed by $\frac{1}{\Lambda_{NP}^n}$

  $\rightarrow$ pick a well-motivated model, where the *SM* is embedded in

- In our case, we start from the following observation:

  $\mathcal{P}$ **and** $\mathcal{C}$ **are violated in the** $\boldsymbol{SM}$,

  i.e. left- and right-handed fields are treated differently

Introduction: LRM

- Left-Right Models (*LRM*): left-handed nature of *SM* as the low-energy limit of a parity-sym. th. [Pati, Salam, Mohapatra, Senjanovic]

$$LRM = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\downarrow \Lambda_{LR}$$
$$SM = SU(3)_c \times SU(2)_L \times U(1)_Y$$
$$\downarrow v_{EW}$$
$$SU(3)_c \times U(1)_{EM}$$

- **RH fermions**: $SU(2)_R$ doublets, $SU(2)_L$ singlets

  and similarly for **LH fermions**: $\begin{pmatrix} U \\ D \end{pmatrix}_{R,L}$ and $\begin{pmatrix} \nu \\ \ell \end{pmatrix}_{R,L}$
- UV completion of LRM: not interested on what happens beyond Λ_{LR} (Pati-Salam, $SO(10)$, E_6, etc.)

Introduction: LRM phenomenology

- BEH mechanism:

  @ Λ_{LR} (heavy) new scalars: $H^0, H^\pm, \ldots$

  @ Λ_{LR} (heavy) new gauge bosons: W', Z'

  @ v_{EW} SM-like h^0, W, Z

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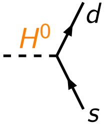
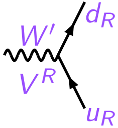
  @ $v_{\rm EW}$ SM-like h^0, W, Z
- LRM manifestations: scalar **FCN Couplings**, **charged weak right-handed currents**, etc.





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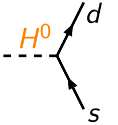

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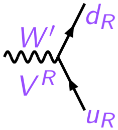
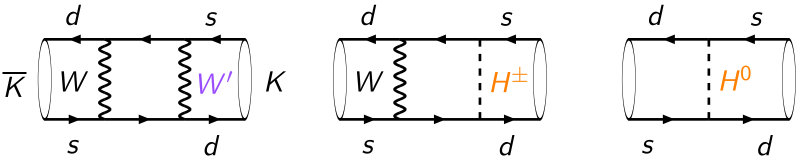


- Test viability and structure of the model:

  direct searches, EW precision tests, flavour physics,...

Introduction: constraints from $K\bar{K}$ mixing

- Sensitive to NP $\sim \mathcal{O}(100)$ TeV and beyond [Isidori etal '10, Charles etal '14]
- LRM: potentially large contributions [Beall etal '82, Frère etal '92, Barenboim etal '96 '97, Ball etal '00, Kiers etal '02, Zhang etal '07, Maiezza etal '10, Blanke etal '11, Bertolini etal '14]



- Powerful constraints, typically: $M_{W'} \gtrsim 3$ TeV, $M_{H^0} \gtrsim 15$ TeV
- Clear interest: direct searches W', H [Nemevsek etal '11, Mohapatra etal '14, Cheung etal '15, Dobrescu etal '15, Patra etal '16] , etc.

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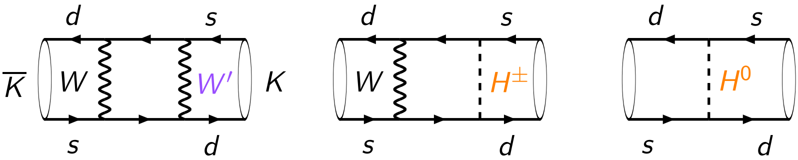

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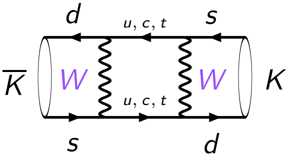
- **Revisit the Short-Distance QCD calculations for more reliable bounds coming from $|\Delta F| = 2$**

Outline

$K\bar{K}$ meson-mixing in SM



Ex.: Indirect CPV $|\varepsilon_K| \pm \delta|\varepsilon_K|$

Accurate measurements $\frac{\delta|\varepsilon_K|_{\mathrm{exp}}}{|\varepsilon_K|_{\mathrm{exp}}} \sim 0.5\%$ [PDG14]

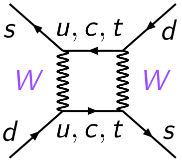
Theoretical control in SM: $\frac{\delta|\varepsilon_K|_{\mathrm{SM}}}{|\varepsilon_K|_{\mathrm{SM}}} \sim 20\%$ [CKMfitter15]

(of which about half of the error comes from $V^L_{cb}, \bar{\eta}, \bar{\rho}$

and other important contributions come from SD QCD η_{cc}, η_{ct})

SM contribution to $K\bar{K}$ mixing

Very well studied [review: Buchalla, Buras, Lautenbacher '96]



$$= i\frac{G_F^2 M_W^2}{4\pi^2}\sum_{i,j=c,t} V_{is}^L\, V_{id}^{L*}\, V_{js}^L\, V_{jd}^{L*}\, S(x_i,x_j)\, Q_V,\ \ x_i = \frac{m_i^2}{M_W^2}$$

V^L = CKM, unitary matrix

$$S(x_i,x_j) \overset{\text{unitarity}}{=} I(x_i,x_j) - I(x_i,x_u) - I(x_u,x_j) + I(x_u,x_u),$$

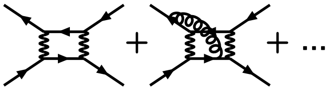
$I(x_i,x_j)$ are the Inami-Lim functions

Weak Left-Handed Currents: $Q_V = \bar{s}\gamma^\mu P_L d \cdot \bar{s}\gamma_\mu P_L d$

i,j	t, t	c, t	c, c
$S(x_i,x_j)$	$f_{tt}(x_t)$	$x_c[-\log x_c + f_{ct}(x_t)]$	x_c

Expansion for small $x_c = \frac{m_c^2}{M_W^2}$

SM $K\bar{K}$ mixing: Short-Distance QCD corrections



$$= i\frac{G_F^2 M_W^2}{4\pi^2}\sum_{i,j=c,t} V_{is}^L V_{id}^{L*} V_{js}^L V_{jd}^{L*} \, S(x_i,x_j)\,\bar{\eta}_{ij}\,Q_V$$

$\bar{\eta}_{ij}$ Short-Distance, perturbative QCD corrections

$\bar{\eta}_{ij} = 1 + \mathcal{O}(\alpha_s \cdot \log x_c) + \ldots$: mandatory for phenomenology

SM	t,t	c,t	c,c
$\eta_{\rm LL}$ [Herrlich, Nierste]	0.59	0.37	0.74
$\eta_{\rm NLL}$ [Herrlich, Nierste]	0.57 ± 0.01	$0.47^{+0.03}_{-0.04}$	$1.3^{+0.3}_{-0.2}$
$\eta_{\rm NNLL}$ [Buras etal '90, Brod, Gorbahn]	0.577 ± 0.007	0.50 ± 0.05	1.9 ± 0.8

Overview: Short-Distance QCD corrections

Many scales in $\mathcal{H}^{\rm SM}$: m_c, m_t, M_W

Factorization short/long distances @ μ

$$\mathcal{H}^{\rm SM} \overset{\rm OPE}{=} \sum_i C_i(\mu) \cdot Q_i(\mu), \quad \mathcal{H}^{\rm SM} \text{ independent of } \mu$$

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$C_i(\mu)$, $Q_i(\mu)$ not calculated at the same scale:
$C_i(\mu_{\rm high})$ at high energies, $\langle K|Q_i|\bar{K}\rangle \equiv \langle Q_i(\mu_{\rm had})\rangle$ on the Lattice

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$$\frac{d}{d\log\mu}\begin{pmatrix} C_1 \\ C_2 \\ \vdots \end{pmatrix} \overset{\rm RGE}{=} \gamma^{\rm T} \cdot \begin{pmatrix} C_1 \\ C_2 \\ \vdots \end{pmatrix}$$

γ: anom. dim. matrix

$$C(\mu_{\rm high})$$

$$\downarrow \text{running}$$

$$C(\mu_{\rm low}) \overset{\rm LL}{=} \left(\frac{\alpha_s(\mu_{\rm low})}{\alpha_s(\mu_{\rm high})}\right)^{\gamma} C(\mu_{\rm high})$$

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- $\left(\frac{\alpha_s(\mu_{\rm low})}{\alpha_s(\mu_{\rm high})}\right)^{\gamma} \overset{\rm LL}{=} \left[\sum_{n=0}^{\infty} \left(\beta_0 \frac{\alpha_s(\mu_{\rm low})}{2\pi} \log\left(\frac{\mu_{\rm low}}{\mu_{\rm high}}\right)\right)^n\right]^{\gamma}$

large $\alpha_s \cdot \log(\mu_{\rm low}/\mu_{\rm high})$ resummed to all orders in α_s by RGE

Methods to compute $\bar{\eta}$

In $\mathcal{H}_{\rm eff}$ language: $\bar{\eta} = 1 + \sum_{n=1}^{\infty} (\alpha_s \cdot \log \frac{\mu_{\rm low}}{\mu_{\rm high}})^n a_n + \ldots$
collects Short-Distance QCD

Two methods to compute $\bar{\eta}$:

- "Method of Regions" (MR): main QCD effects
  Initially designed for the SM @ LL [Vainshtein etal '77, Vysotskii '80]
  Considered for the LRM @ LL [Bigi, Frère '83, Ecker, Grimus '85, Bertolini etal '14]
  SM and LRM: NLL corrections **[Bernard, Descotes-Genon, LVS]**
- EFT: build eff. th. valid at low energies [Gilman, Wise '83]
  LL, NLL, NNLL [Buras, Jamin, Weisz '90, Herrlich, Nierste '94 '95 '96, Brod, Gorbahn '10 '12]
  LRM @ NLL for charm-charm WW' box **[Bernard, Descotes-Genon, LVS]**

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"Method of Regions" (MR)

Resum potentially large $\alpha_s \cdot \log$ in an **approximate** way

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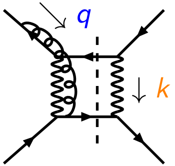

$q^2 \in [k^2, M_W^2]$

$\alpha_s \cdot \log(k^2/M_W^2)$

1. Fix k^μ of box: two $|\Delta S| = 1$ sides
2. Identify range of q^μ for potentially large $\alpha_s \cdot \log$

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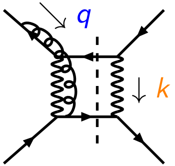

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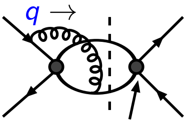
3. W is integrated out; quarks left dynamic
4. Over q^2 range, two $|\Delta S| = 1$ ops. of anom. dim. γ: $M_W^2 \xrightarrow{\text{run}} k^2$

$$C_{|\Delta S|=1}(M_W^2) \overset{\text{RGE}}{=} \left(\frac{\alpha_s(M_W^2)}{\alpha_s(k^2)}\right)^\gamma C_{|\Delta S|=1}(k^2)$$



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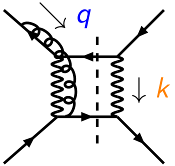

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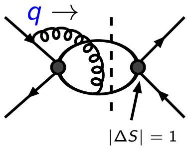
   $$C_{|\Delta S|=1}(M_W^2) \overset{\text{RGE}}{=} \left(\frac{\alpha_s(M_W^2)}{\alpha_s(k^2)}\right)^\gamma C_{|\Delta S|=1}(k^2)$$
5. Finally, integrate over k^μ

   Dominant k^2-range from the loop-functions

   e.g. $S(x_c, x_c) = x_c = \frac{m_c^2}{M_W^2} \to \alpha_s^\gamma(m_c^2)$

   and $S(x_c, x_t) \overset{\text{LL}}{=} -x_c \log x_c \to \int_{m_c^2}^{M_W^2} \frac{dk^2}{k^2} \alpha_s^\gamma(k^2)$



“Method of Regions” (MR)

- Possible if the anom. dim. of the local $|\Delta S| = 1$ operators already known

  $|\Delta S| = 2$ anom. dim. also required (example not shown)

- [Bernard, Descotes-Genon, LVS]

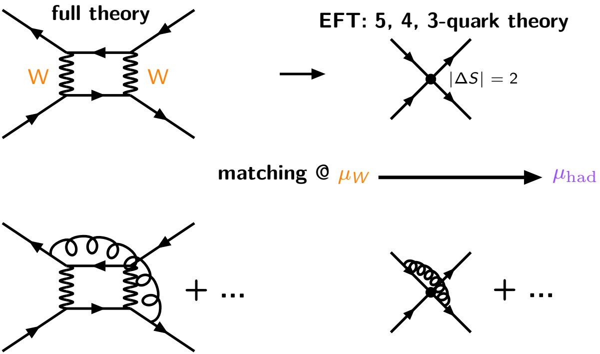
  $\rightarrow$ use known anom. dim. @ NLL

  $\rightarrow$ include matching corrections if known

  $\rightarrow$ appropriate counting for large log from the loop-functions

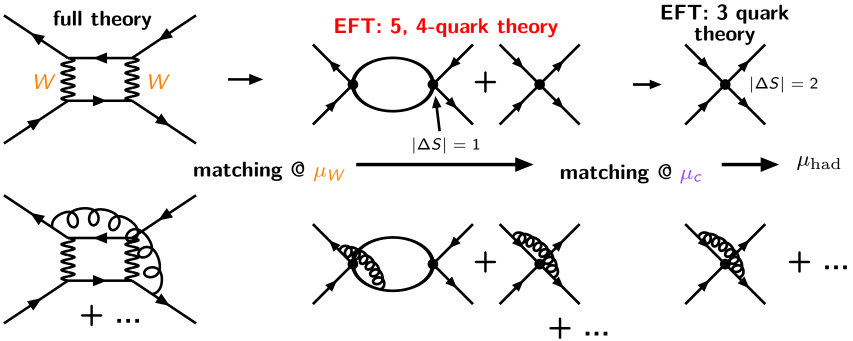
Outline

EFT method: $\bar{\eta}_{tt}$



- t, W integrated out @ $\mu_W = \mathcal{O}(M_W, m_t)$: single op. in EFT
- Matching corrected by the set of gluon exchanges in the full th.
- Anom. dim. in the EFT from loop diagrams
- Matrix element $\langle Q_V \rangle$ calculated @ $\mu_{\rm had}$

EFT method: $\bar{\eta}_{ct}$, $\bar{\eta}_{cc}$



- Hierarchy: t, W are integrated out @ μ_W, c is integrated out @ μ_c
- $\bar{\eta}_{ct}$: local operators in 5, 4-quark th. **ARE** required mixing between local and bi-local Wilson coefficients
- $\bar{\eta}_{cc}$: due to GIM, **NO** local operators in 5, 4-quark th.

Comparison MR and EFT for SM

$$S(x_c, x_t) = x_c[-\log x_c + f_{ct}(x_t)]$$

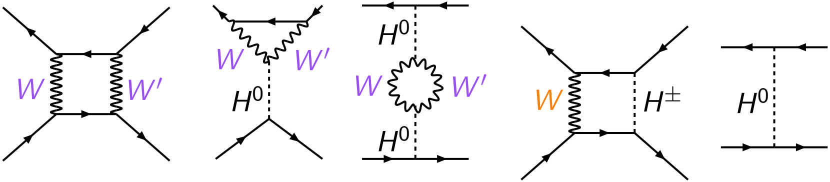
SM	η_{tt}	η_{ct}
Leading Log	$(\alpha_s \cdot \log(x_c))^n$	$\log x_c \cdot (\alpha_s \cdot \log(x_c))^n$
Next-to-LL	$\alpha_s \cdot (\alpha_s \cdot \log(x_c))^n$	$(\alpha_s \cdot \log(x_c))^n$
EFT (LL, NLL)	$0.612 - 0.038 = 0.574$	$0.368 + 0.099 = 0.467$
MR (LL, NLL)	$0.598 + 0.028 = 0.626$	$0.345 - 0.011 = 0.334$

- EFT from literature [Buras etal '90, Herrlich, Nierste]
- For η_{cc}: same expressions MR / EFT @ LL and NLL
- MR for η_{tt}: top is not integrated out in $[M_W, m_t]$ thus producing small differences w.r.t. EFT [LL: Datta etal '90, Herrlich, Nierste]
- LL: MR values are in good agreement w/ EFT ($\leqslant 6\%$)
- 30% difference MR / EFT @ NLL for a large $\log x_c$

Outline

LRM: $K\bar{K}$ mixing

SM +



Gauge inv. set

[Basecq etal '85, Hou etal '85, Kenmoku etal '88]

We focus in this set first:
Box, Vertex, Self-Energy
Two scalars: H^0, A^0

Gauge inv. set

[Gagyi-Palffy etal '98]

Other contributions suppressed by
$\beta \equiv M_W^2/M_{W'}^2$

LRM corrections to $K\bar{K}$ mixing: WW'

$$\mathcal{H}_{\text{eff}}^{WW'} = \beta \frac{2G_F^2}{\pi^2} \frac{g_R^2}{g_L^2} \sum_{i,j=u,c,t} V_{is}^L V_{id}^{R*} V_{js}^R V_{jd}^{L*}$$

$$\sqrt{m_i m_j}\, S^{WW'}(x_i, x_j, \beta, \omega)\, \bar{\eta}_{ij}^{LR}\, Q_2^{LR} + h.c., \quad \beta = \frac{M_W^2}{M_{W'}^2}$$

$\bar{\eta}_{ij}^{LR}$ collects QCD corrections

V^R is the RH analogous of V^L: no GIM

Weak RH currents: $Q_2^{LR} = \bar{s}P_L d \cdot \bar{s}P_R d$,

no need for $Q_1^{LR} = \bar{s}\gamma_\mu P_L d \cdot \bar{s}\gamma^\mu P_R d$

i,j	t, t	c, t	c, c
$S^{WW'}(x_i, x_j, \beta, \omega) - \frac{\log\beta + F(\omega)}{4}$	$f_{tt}(x_t)$	$f_{ct}(x_t)$	$\log(x_c) + 1$

$x_u = 0,\ x_c = \frac{m_c^2}{M_W^2},\ \beta = \frac{M_W^2}{M_{W'}^2},\ \omega = \frac{M_{W'}^2}{M_H^2} \lesssim \mathcal{O}(1)$. requires $\neq$ counting

Outline

MR: Short-Distance QCD corrections for WW'

Same overall strategy compared to SM:

- High energy scales: $M_{W'}, M_H$
- Matching: $|\Delta S| = 1$ ops. $\bar{s}\gamma_\mu P_X q_1 \cdot \bar{q}_2 \gamma^\mu P_X d, \ X = L, R$

  $C_{|\Delta S|=1}(M_W) = 1 + \frac{\alpha_s(M_W)}{4\pi} B$ from [Buras etal '92]
- Running:

  $|\Delta S| = 2$ ops. $Q_{1,2}^{LR}$ and $|\Delta S| = 1$ ops.

  w/ known anom. dim.:

  $\gamma = \frac{\alpha_s}{4\pi}\gamma^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \gamma^{(1)}$ [Buras etal '00]

  MR: no mixing between $|\Delta S| = 2$ and $|\Delta S| = 1$

Results for MR

$$S^{WW'}(x_c, x_c, \beta, \omega) = \log(x_c) + \mathcal{O}(1)$$

LRM	$\bar{\eta}_{tt}^{LR}$	$\bar{\eta}_{ct}^{LR}$	$\bar{\eta}_{cc}^{LR}$
LL	$(\alpha_s \cdot \log(x_c))^n$	$(\alpha_s \cdot \log(x_c))^n$	$\log x_c \cdot (\alpha_s \cdot \log(x_c))^n$
NLL	$\alpha_s \cdot (\alpha_s \cdot \log(x_c))^n$	$\alpha_s \cdot (\alpha_s \cdot \log(x_c))^n$	$(\alpha_s \cdot \log(x_c))^n$
MR (LL+NLL)	5.9 ± 2.0	2.74 ± 0.87	1.35 ± 0.49

$$M_{W'} = 1 \text{ TeV}, \; M_{W'}^2/M_H^2 = 0.1, \; \mu_{\text{had}} = 1 \text{ GeV}$$

- Worse comparison MR / EFT in the SM
  and different treatment of top w.r.t. EFT $\Rightarrow$ 30% error
- $\log x_c$: loop-function for cc includes a large logarithm
  SM example: bad comparison MR / EFT
  MR may not be accurate in LRM as well

Outline

Above the matching scale μ_W

Hierarchy $M_{W', H^{0,\pm}} \gg M_W, m_t$:

- EFT @ $\mu_{W'} = \mathcal{O}(M_{W', H^{0,\pm}})$ where $W', H^{0,\pm}$ are integrated out
- Running $\mu_{W'} \to \tilde{\mu}_W$ resums $\alpha_s(\tilde{\mu}_W) \log(\tilde{\mu}_W / \mu_{W'})$

  However, $\alpha_s \cdot \log$ not large:
- Pheno: $\alpha_s(M_W) \sim 0.1$ and $M_{W'} \in [1, 10]$ TeV
- take $\mu_W \in [\tilde{\mu}_W, \mu_{W'}]$ and add $\alpha_s(\tilde{\mu}_W) \log\left(\frac{\tilde{\mu}_W}{\mu_{W'}}\right) \sim 20\%$ errors

Above the matching scale μ_W

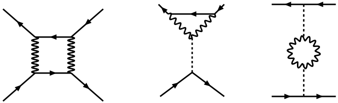
Hierarchy $M_{W',H^{0,\pm}} \gg M_W, m_t$:

- EFT @ $\mu_{W'} = \mathcal{O}(M_{W',H^{0,\pm}})$ where $W', H^{0,\pm}$ are integrated out
- Running $\mu_{W'} \to \tilde{\mu}_W$ resums $\alpha_s(\tilde{\mu}_W)\log(\tilde{\mu}_W/\mu_{W'})$

  However, $\alpha_s \cdot \log$ not large:
- Pheno: $\alpha_s(M_W) \sim 0.1$ and $M_{W'} \in [1, 10]$ TeV
- take $\mu_W \in [\tilde{\mu}_W, \mu_{W'}]$ and add $\alpha_s(\tilde{\mu}_W)\log\left(\frac{\tilde{\mu}_W}{\mu_{W'}}\right) \sim 20\%$ errors

---

@ $\mu_{\mathbf{W}} = \mathcal{O}(M_W, m_t, M_{W',H})$

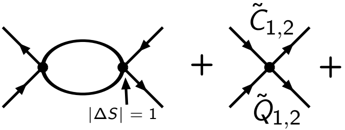


$$S^{WW'} = \log x_c + \left(1 + \frac{\log\beta + F(\omega)}{4}\right)$$

LL	NLL
$\log x_c$	$\frac{\left(1+\frac{\log\beta+F(\omega)}{4}\right)}{\log x_c}$
-8.2	$20 - 60\%$

for $M_{W'} \in [1, 10]$ TeV
and $\omega \in [0.1, 0.8]$

Matching @ μ_W



+ Evanescent ops.

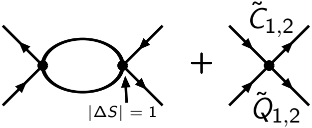
Divergences @ EFT w/o gluon exchanges: we define the operators $\tilde{Q}_{1,2}$ in the counter-terms as [Herrlich, Nierste]

$$\tilde{Q}_1 = \frac{m_c^2}{4\pi\alpha_s\,\mu^{2\epsilon}}(\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d), \quad \tilde{Q}_2 = \frac{m_c^2}{4\pi\alpha_s\,\mu^{2\epsilon}}(\bar{s}P_L d)(\bar{s}P_R d)$$

so that the counter-terms coefficients necessary for the ren. come as $\alpha_s \frac{1}{\epsilon}$

$$\rightarrow \gamma = \frac{\alpha_s}{4\pi}\gamma^{(0)} + \ldots$$

Matching @ μ_W



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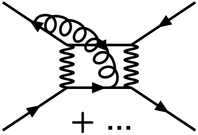
so that the counter-terms coefficients necessary for the ren. come as $\alpha_s \frac{1}{\epsilon}$ $\quad \rightarrow\ \gamma = \frac{\alpha_s}{4\pi}\gamma^{(0)} + \ldots$



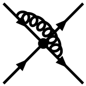
With this normalization
$\tilde{C}_{1,2}$ starts @ $\alpha_s(\mu_W)$
($\leftarrow$) these diagrams match @ $\alpha_s^2 \Rightarrow$ NNLL

Evanescent operators: dimensional reg.

- Regularization is made in $D = 4 - 2\epsilon$ dimensions
  New ops. are introduced: evanescent basis
  Example:

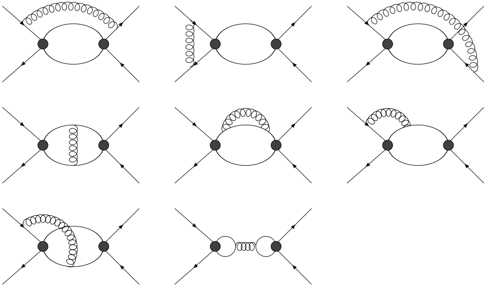
$$E \;=\; \gamma_\nu \gamma_\mu P_R \otimes \gamma^\nu \gamma^\mu P_L - (4 + a\epsilon) P_R \otimes P_L$$

- Evanescent ops. vanish identically when $D = 4 \Rightarrow E = 0$
- $\{a\}$: part of the renormalization scheme
- Necessary for determining the anomalous dimensions

Example: insert $\frac{E}{\epsilon}$ $\rightarrow$  $\rightarrow$ $\frac{\alpha_s}{4\pi}\frac{Q}{\epsilon}$

Loop diagrams

Insert physical $|\Delta S| = 1$
and $|\Delta S| = 2$,
and Evanescent ops.

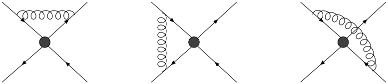


- FeynCalc [Mertig etal '91, Shtabovenko etal '16]

  Simplifies the Dirac algebra in $D \neq 4$ dims
- TARCER [Mertig, Scharf '98]

  Simplifies and calculates some integrals
- $\alpha_s \frac{1}{\epsilon}$ terms

  $\Rightarrow$ anom. dim. matrix



After $\mu_W \to \mu_c$: matching @ μ_c

4-flavour theory

$\tilde{Q}_1 = \frac{m_c^2}{g^2 \mu^{2\epsilon}} (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d)$, etc.

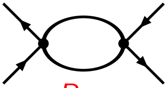
$|\Delta S| = 1$ ops. $\bar{s}\gamma_\mu P_X q_1 \cdot \bar{q}_2 \gamma^\mu P_X d$

3-flavour theory

$Q_1^{LR} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d)$, etc.

$$\Bigg(\underbrace{\frac{4\pi}{\alpha_s(\mu_c)} \tilde{C}_a(\mu_c)}_{\text{LL+NLL}} + \underbrace{\sum_{i,j=\pm} (R_{ij,a} C_i C_j)(\mu_c)}_{\text{NLL}} \Bigg) \overset{\text{matching}}{=} C_a(\mu_c)$$

- $\tilde{C}_{1,2}$, $C_\pm$, $C_{1,2}$: Wilson coefficients
- $R_{ij,a}$: loop-function
- $R_{ij,a}$ corrected by $\alpha_s(\mu_c) \times \epsilon^0$: 2-loop diagrams
  NNLL: test the convergence of the series
- **Last step**: running $C_{1,2}$ from μ_c to μ_{had}



$R_{ij,a}$

Review: EFT for $\bar{\eta}_{cc}^{LR}$ @ NLL

- Hierarchy $M_{W',H} \gg M_W, m_t \to \alpha_s(M_W) \log \frac{M_W^2}{M_{W'}^2}$ error $\surd$
- Matching @ $\mu_W = \mathcal{O}(M_W, m_t, M_{W',H})$
  $\mathcal{O}(\alpha_s(\mu_W))$ for $C_{|\Delta S|=1}(\mu_W)$ $\surd$ [Buras etal '92]
  $\mathcal{O}(\alpha_s(\mu_W))$ for $\tilde{C}_{|\Delta S|=2}(\mu_W)$ $\surd$
- Running $\mu_W \to \mu_c$
  $|\Delta S| = 1$ ops. $\bar{s}\gamma_\mu P_X q_1 \cdot \bar{q}_2 \gamma^\mu P_X d$, $X = L, R$ $\surd$ [Buras etal '00]
  Mixing of two $|\Delta S| = 1$ w/ single $|\Delta S| = 2$: $\surd$
  requires dedicated 2-loop computation
- Matching @ μ_c: NNLL estimate $\surd$
- Running $\mu_c \to \mu_{\rm had}$ of $Q_{1,2}^{LR}$ $\surd$ [Buras etal '00]
- Then $\bar{\boldsymbol{\eta}}_{\mathbf{cc}}^{\mathbf{LR}}$

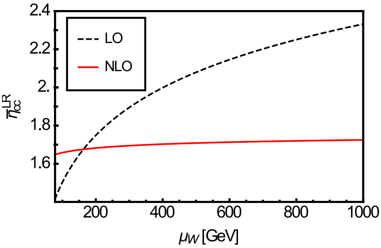
$\surd \to$ **[Bernard, Descotes-Genon, LVS]**

Outline

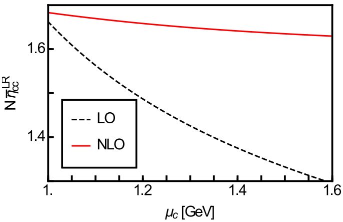
1. Introduction: LRM
2. Short-Distance (SD) QCD corrections $K\bar{K}$: SM
   - $K\bar{K}$ mixing in SM
   - "Method of Regions" (MR)
   - Effective Field Theory
3. SD QCD corrections $K\bar{K}$: LRM
   - $K\bar{K}$ mixing in LRM
   - MR
   - EFT
   - Checks EFT, results
4. Conclusions

Important checks

- Residual dependence on matching scales μ_W, μ_c in $\mathcal{H}_{\text{eff}}^{WW'}$
  - $\rightarrow$ Large uncertainty @ LL
  - $\rightarrow$ Much weaker uncertainty @ NLL



$$N = \frac{S^{WW'}(x_c(\mu_c), \beta, \omega)}{S^{WW'}(x_c(m_c), \beta, \omega)}$$



- Independence QCD gauge and IR reg. parameters ($m_{d,s}$)
- Ind. on particular choice of the evanescent basis [Herrlich, Nierste]

Results for $\bar{\eta}_{cc}^{LR}$

- Comparison LL / NLL:

  $M_{W'} = 1\ \mathrm{TeV}$, $\frac{M_{W'}^2}{M_H^2} = 0.1$, $\mu_{\mathrm{had}} = 1\ \mathrm{GeV}$, $\mu_W = M_W$, $\mu_c = m_c$,

  small shift $LL : 1.41 \to NLL : 1.65$,

  but LL value highly depends on choice $\mu_{W,c}$

$$\bar{\eta}_{cc}^{LR} = 1.65 \pm 0.50\ (\mathrm{NLL})$$

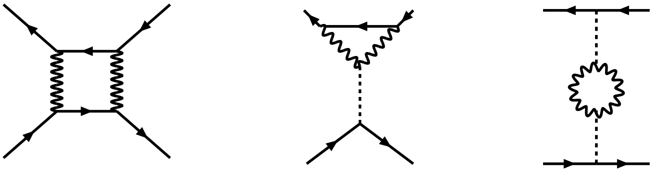
- Error bar from: size of NNLL estimate;

  μ_W, μ_c dependencies; $\alpha_s(\mu_W) \log \beta$ size

- Comparison EFT / MR:

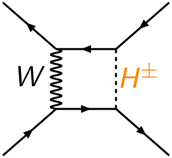
  MR gives a consistent central value $\bar{\eta}_{cc}^{LR}|_{MR} = 1.35 \pm 0.49$

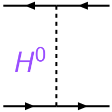
Final results for WW'



$$\bar{\eta}^{LR}_{tt} = 5.9 \pm 2.0 \quad (MR)$$
$$\bar{\eta}^{LR}_{ct} = 2.74 \pm 0.87 \quad (MR)$$
$$\bar{\eta}^{LR}_{cc} = 1.65 \pm 0.50 \quad (EFT)$$

Results for the other NP contributions in MR





Anom. dim. from [Buras etal '00]

$$\bar{\eta}_{tt}^{H^{\pm}Box} = 5.9 \pm 2.0\ (MR),$$
$$\bar{\eta}_{ct}^{H^{\pm}Box} = 2.76 \pm 0.90\ (MR),$$
$$\bar{\eta}_{cc}^{H^{\pm}Box} = 1.29 \pm 0.40\ (MR),$$

Anom. dim. from [Buras etal '00]

$$\bar{\eta}_{tt}^{H} = 5.66 \pm 0.30\ ,$$
$$\bar{\eta}_{ct}^{H} = 2.70 \pm 0.09\ ,$$
$$\bar{\eta}_{cc}^{H} = 1.28 \pm 0.04\ .$$

- $H^{\pm}$ Box: no large $\log x_c$

Outline

Conclusions

- LRM: extension to SM restoring $\mathcal{P}$ or $\mathcal{C}$
- Kaon meson-mixing provides essential information to probe the structure of LRM [..., Zhang etal '07, Maiezza etal '10, Blanke etal '11, Bertolini etal '14]
- Short-Distance QCD corrections for meaningful bounds
- Two methods in the literature

  MR: simplified method, easy when anom. dim./matchings known

  EFT: formal method, dedicated computations
- Here: NLL analyses of diagrams containing WW',

  the charged scalar box and tree level neutral scalar

Conclusions

- Next: perform pheno. analysis of LRM based on meson-mixing and other constraints

  Two scalar contents: doublets and triplets

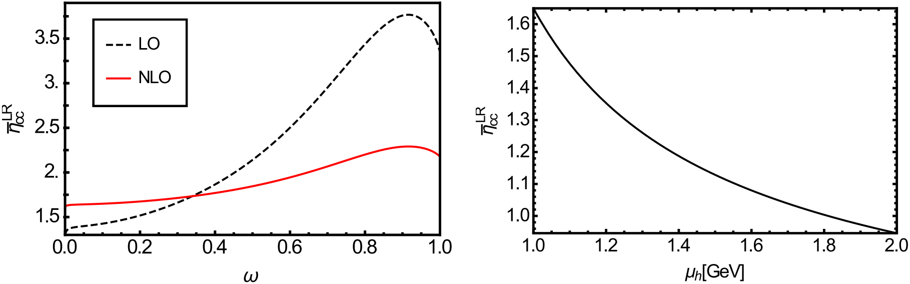
- Already considered EWPO for doublets [LVS '15]

  Global fit meson-mixing ($K\bar{K}$, $B\bar{B}$), $b \to c$, EWPO, direct searches

  w/ CKMfitter under way

Questions...

EFT: $\bar{\eta}_{cc}^{LR}$

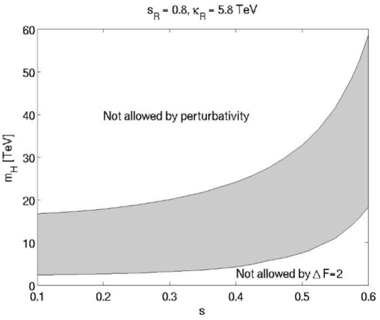


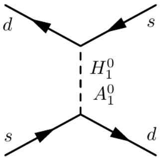
Dependence of $\bar{\eta}_{cc}^{LR}$ on $\omega = M_{W'}^2/M_H^2$ and on the hadronic scale in the EFT approach

Higgs content in the case of triplets

Triplets: $\langle \Delta_R \rangle = (0,\, 0,\, \kappa_R)$ and $\langle \Delta_L \rangle = (0,\, 0,\, \kappa_L)$

- 1 light Higgs + 3 H^0, 2 A^0, 2 $H^\pm$, 2 $H^{\pm\pm}$
- See-saw mechanism $m_{\nu_L} \propto \kappa^2/m_{\nu_R}$
- $\rho = M_W^2/(\cos^2(\theta_W)\cdot M_Z^2) \simeq 1 \Rightarrow \kappa_L/\sqrt{\kappa_1^2+\kappa_2^2} \ll 1$
- $K\overline{K}$ mixing: $M_{H,A} \gtrsim 10$ TeV, for gen. V_R, $\frac{g_L}{g_R}$, $s \equiv \frac{\kappa_2}{\sqrt{\kappa_1^2+\kappa_2^2}}$ [Blanke etal '11]





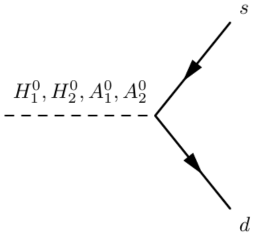
H_1^0, A_1^0 d.o.f. from the bi-doublet

Higgs content in the case of doublets

Different EW breaking pattern: Aim at probing the scalar and gauge energy scales, and the flavor mixing in the quark sector

Doublets: $\langle \chi_R \rangle = (0,\, \kappa_R)$ and $\langle \chi_L \rangle = (0,\, \kappa_L)$

- 1 light Higgs + 3 H^0, 2 A^0, 2 $H^\pm$
- $\rho = 1$ at tree-level: κ_L must be constrained by other means
- In this minimal picture, neutrinos are Dirac particles: no see-saw
- Other contributions to neutral meson mixing modulated by κ_L, modifying the constraint on $M_{H,A}$

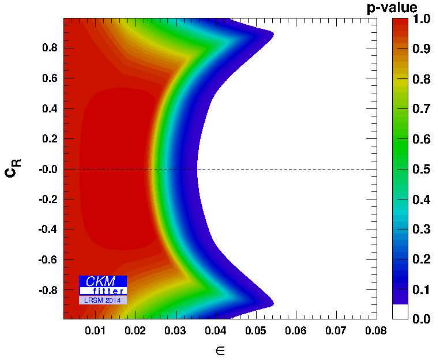


Higgses	d.o.f.	FCNC for sd
H_1^0, A_1^0	$h_1\, \phi + g_1\, \chi_L$	$h_C \sum_a m_u^a V_L^{as*} V_R^{ad}$
H_2^0, A_2^0	$h_2\, \chi_L + g_2\, \phi$	$g_C \sum_a m_u^a V_L^{as*} V_R^{ad}$

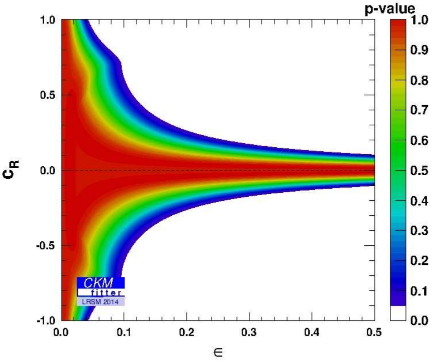
$h_{1,2,C}(\kappa_L) \to 1$, $g_{1,2,C}(\kappa_L) \to 0$ when $\kappa_L \to 0$

Direct $M_{W'} \simeq g_R \kappa_R / 2$

W/ $M_{W'}$ input

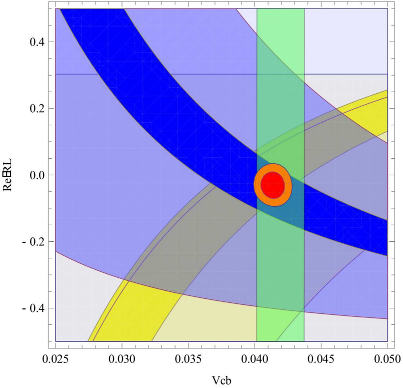


W/O $M_{W'}$ input



- Under the assumptions $g_L = g_R$ and manifest V_R, $M_{W'} \gtrsim 2$ TeV
- CMS and ATLAS as of 2014
- r, w and $c_R^2 \equiv 1 - (g_L/g_R)^2 (s_W/c_W)^2$ not much constrained

Preliminary $b \to c$



$B \to D\ell\bar{\nu}$, $B \to D\tau\bar{\nu}$
inclusive
(yellow) $B \to D^*\ell\bar{\nu}$
(light yellow) $B \to D^*\tau\bar{\nu}$
(lower side hor. line) $R(D^*)$
(over all) $R(D)$
combination 68 % CL
combination 95 % CL

- $V_{cb}^L, \epsilon_{RL} \equiv cs\epsilon^2 \frac{V_{cb}^R}{V_{cb}^L} e^{i\alpha}$
- **Preliminary**: no correlations, no scalar contributions, data from 2012
- Large tensions remain: $\chi^2(R(D^*)) = 4.8$, $\chi^2(R(D)) = 2.9$

Errors for $|\varepsilon_K|_{\rm SM}$

TABLE IX. Fractional error budget for $\varepsilon_K^{\rm SM}$ obtained using the AOF method, the exclusive V_{cb}, and the FLAG $\hat{B}_K$.

source	error (%)	memo
V_{cb}	40.7	FNAL/MILC
$\bar{\eta}$	21.0	AOF
η_{ct}	17.2	$c-t$ Box
η_{cc}	7.3	$c-c$ Box
$\bar{\rho}$	4.7	AOF
m_t	2.5	
ξ_0	2.2	RBC/UKQCD
$\hat{B}_K$	1.6	FLAG
m_c	1.0	
⋮	⋮	

- Left [Bailey etal '15]

  (individual sources of error over total error, multiply by ~ 0.2 for *relative* errors)
- Bottom [Ligeti etal '16]

  (*relative* errors)

	CKM inputs	η_{cc}	η_{ct}	$\kappa_\epsilon^{(\prime)}$	m_t	m_c	$\widehat{B}_K$	$\lvert V_{cb}\rvert$	$\bar{\eta}$	$\bar{\rho}$	$\lvert\Delta\epsilon_K/\epsilon_K\rvert_{\rm tot.}$
Usual evaluation	tree-level	7.3%	4.0%	1.1%	1.7%	0.8 %	1.3%	11.1%	10.4%	5.4%	18.4%
	SM CKM fit	7.4%	4.0%	1.7%	1.7%	0.8 %	1.3%	4.2%	2.0%	0.8%	10.1%
Our evaluation	tree-level	—	3.4%	5.2%	1.5%	1.2%	1.3%	9.5%	8.9%	4.5%	15.6%
	SM CKM fit	—	3.4%	5.9%	1.5%	1.3%	1.3%	3.6%	1.7%	0.7%	8.3%

TABLE IV. The present error budget of ϵ_K in the usual evaluation (upper part) and using our evaluation (lower part). The parameters with a corresponding uncertainty above 1% are shown.

Other meson-mixing observables

$\Delta m_d, \Delta m_s$ (Δm_K: no accurate SM prediction)

- $\frac{\delta(\Delta m_d)_{\rm exp}}{(\Delta m_d)_{\rm exp}} \sim 0.6\%$ [HFAG14]
- $\frac{\delta(\Delta m_d)_{\rm SM}}{(\Delta m_d)_{\rm SM}} \sim 8\%$ [CKMfitter15]
- $\frac{\delta(\Delta m_s)_{\rm exp}}{(\Delta m_s)_{\rm exp}} \sim 0.1\%$ [HFAG14]
- $\frac{\delta(\Delta m_s)_{\rm SM}}{(\Delta m_s)_{\rm SM}} \sim 5\%$ [CKMfitter15]

Indirect CP violation $\mathrm{Re}(\varepsilon'_K/\varepsilon_K)$ [Lehner, Lunghi, Soni '15]

- $\frac{\delta(\mathrm{Re}(\varepsilon'_K/\varepsilon_K))_{\rm exp}}{(\mathrm{Re}(\varepsilon'_K/\varepsilon_K))_{\rm exp}} \sim 10\%$
- $\frac{\delta(\mathrm{Re}(\varepsilon'_K/\varepsilon_K))_{\rm SM}}{(\mathrm{Re}(\varepsilon'_K/\varepsilon_K))_{\rm SM}} \sim 5-7$ times bigger

Bounds on LRM can still be derived from $\Delta m_K, \mathrm{Re}(\varepsilon'_K/\varepsilon_K)$ by saturating the experimental values

Matrix elements

$$\langle \overline{M} | Q_a^i | M \rangle(\mu) = \frac{2}{3} m_M F_M^2 P_a^i(\mu), \quad P_1^{VLL} = B_1^{VLL},$$

$$P_1^{LR} = -\frac{1}{2}\left(\frac{m_M}{m_{q_1} + m_{q_2}}\right)^2 B_1^{LR}, \quad P_2^{LR} = \frac{3}{4}\left(\frac{m_M}{m_{q_1} + m_{q_2}}\right)^2 B_2^{LR},$$

$$B_1^{VLL} = B_1, \quad B_1^{LR} = B_5, \quad B_2^{LR} = B_4$$

$\overline{\text{MS}}$ @ 2 GeV

- $B_1 = 0.53(2)$
- $B_5 = 0.57(4)$
- $B_4 = 0.78(3)$

$\overline{\text{MS}}$ @ $m_b(m_b) = 4.29(12)$ GeV

- $B_1^{(d)} = 0.85(3)(2)$, $B_1^{(s)} = 0.86(3)(1)$
- $B_5^{(d)} = 1.47(8)(9)$, $B_5^{(s)} = 1.57(7)(8)$
- $B_4^{(d)} = 0.95(4)(3)$, $B_4^{(s)} = 0.93(4)(1)$

K, B-meson bag parameters $N_f = 2$ twisted-mass QCD [JHEP 1303 (2013) 089, JHEP 1403 (2014) 0169806471]