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Avoiding Boltzmann Brain domination in holographic dark energy models

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ABSTRACT

In a spatially infinite and eternal universe approaching ultimately a de Sitter (or quasi-de Sitter) regime, structure can form by thermal fluctuations as such a space is thermal. The models of Dark Energy invoking holographic principle fit naturally into such a category, and spontaneous formation of isolated brains in otherwise empty space seems the most perplexing, creating the paradox of Boltzmann Brains (BB). It is thus appropriate to ask if such models can be made free from domination by Boltzmann Brains. Here we consider only the simplest model, but adopt both the local and the global viewpoint in the description of the Universe. In the former case, we find that if a dimensionless model parameter c, which modulates the Dark Energy density, lies outside the exponentially narrow strip around the most natural $c = 1$ line, the theory is rendered BB-safe. In the latter case, the bound on c is exponentially stronger, and seemingly at odds with those bounds on c obtained from various observational tests.

An empty space with positive cosmological constant (attainable after all kinds of matter are emptied out) represents a thermal system, having a non-zero temperature as well as the maximal entropy [1]. At rare occasions such a system (if sufficiently long-lived) would spontaneously form structures as a thermal fluctuation. This assumes a downward shift in entropy, and it is just this drift from the generalized second law of thermodynamics [2–4] that makes virtually any pop-up structure devoid of having a conventional history record [5]. Namely, amongst all observers created spontaneously out of a thermal system the vast majority of them correspond to the smallest fluctuation – isolated brains immersed in thermal equilibrium of the empty space. This constitutes the paradox of Boltzmann Brains (BBs) [6–8] – when ordinary observers (related to the conventional formation and evolution of structures via inflation and subsequent reheating of the early Universe) become vastly outnumbered by those who (having the same impressions and the same frame of mind) form spontaneously out of a sufficiently long-lived vacuum. So it is appealing to see if there is an escape for any otherwise viable cosmological theory from this troublesome situation.

If the vacuum decays fast enough into a different vacuum, the undecayed physical volume then stops growing before the production of BBs is initiated [8]. For our universe the decay time can be calculated to be of order 10^{10} yr [8]. This resolution of the BB paradox, however, poses a serious problem for a description of the multiverse if the global viewpoint in the description of the Universe is adopted [9]. Recently, it was noticed [10] that the BB threat is not specific only for those exotic theories like the string theory multiverse, but such an unpleasant situation may be found even in the vanilla ΛCDM model, in relation with the electroweak vacuum and ordinary physics. It is interesting to note that without new physics, the Page's resolution for the electroweak vacuum works only if the top pole mass lies somewhat beyond the current observational bounds [10]. Also, the role of phantom cosmologies in treating the BB paradox was stressed recently [11].

In the present paper, we consider how holographic dark energy (HDE) models [12–14], as viable set of models for a description of the Dark Energy in the late-time Universe, cope with the intimidation of the BB brains. The form of the vacuum energy in HDE models stems from the holographic principle [15,16], undoubtedly the most amazing ingredient of a modern view of space and time. The fate of the Universe in these models proves notably susceptible to a slight variation in the vacuum energy density [13], allowing behavior not only similar to the cosmological constant, but the phantom case as well. The scenario therefore proves susceptible to the BB domination.

To incorporate the holographic principle in an effective QFT necessarily requires a kind of UV/IR mixing [17]. This is so since in QFTs the entropy $S \sim L^3\Lambda^3$ (where L is the size of the region and Λ is the UV cutoff) scales extensively, and therefore there is always a sufficiently large volume (for any Λ) for which S would

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exceed the absolute Bekenstein–Hawking bound ($\sim M_{Pl}^2 L^2$). After discarding a great deal of states with Schwarzschild radius much larger than the box size (not describable within QFTs), the bound gets more stringent $(M_{Pl}^2 L^2)^{(3/4)}$ [18]. Near saturation, the bound gives the following vacuum energy density,

$$\rho_\Lambda = (3/8\pi) c^2 M_P^2 L^{-2}, \tag{1}$$

where c is a free parameter introduced in [13], with a natural value of order one.

For the sake of demonstration, we shall consider the appearance of BB brains in the simplest (i.e. non-interacting) HDE model [13], where the event horizon of the spatially flat Universe R_h was chosen for L.[1] The model without any additional energy component is easily solvable, yielding for the equation of state [13]

$$\omega = -\frac{1}{3} - \frac{2}{3c}, \tag{2}$$

and R_h scales with the scale factor as

$$R_h = R_{h0}\, a^{1-\frac{1}{c}}, \tag{3}$$

where the subscript "0" indicates the present epoch. The fate of the Universe is strictly dictated by the value of c: for $c \geq 1$ the Universe enters the (quasi-) de Sitter regime ($c = 1$ mimics the cosmological constant), while $c < 1$ corresponds to the phantom regime.

Let us first adopt the local viewpoint. In our case this means that the interior of R_h is everything there is (the interior corresponds to the causally connected region). For $c = 1$, thus R_h being a constant, the number of ordinary observers stays finite for any time in the future, while on the other hand the number of BB observers starts to pile up after a typical timescale (exponentially huge) of order [5]

$$t_{BB} \sim exp(E_{br} R_h) t_{dyn}, \tag{4}$$

where E_{br} is the energy of the BB brain and t_{dyn} is a dynamical timescale typical for the equilibrium system.[2] The system described by (4) is a thermal system revealing the (constant) Gibbons–Hawking temperature given in terms of the inverse radius of the de Sitter space $T_h = 1/(2\pi R_h)$. Besides, it also describes thermal fluctuations obtained from the entropy decrease of the de Sitter space, when the brain of energy E_{br} is formed in the system of size L. Some extremely rare fluctuation would reproduce our whole visible Universe after the Poincaré recurrence time of order $e^{10^{122}} t_{dyn}$ is passed. So for $c = 1$ the Universe is eternally inflating with a constant R_h, and in the absence of the landscape of string theory vacua, or stated simply, in the absence of any other vacuum our vacuum can decay to, the BB problem is unavoidable.

More subtle analysis is required if $c > 1$ or $c < 1$, since in either case R_h (and therefore the Gibbons–Hawking temperature) is time-dependent. First we consider the issue of whether thermodynamic equilibrium is maintained also for a time-dependent temperature T_h. To this end, we adopt a heuristic criterion for maintaining equilibrium in the form

$$\left| \frac{R_h}{\dot{R}_h} \right| \gtrsim \frac{R_h}{c_\gamma}, \tag{5}$$

that is, departures from de Sitter space should be small enough so that the l.h.s. of (5) is always larger than the light-crossing time of the radius R_h. In a two-component flat-space universe ρ_Λ evolution is governed by [13]

$$\Omega'_\Lambda = \Omega_\Lambda^2 (1 - \Omega_\Lambda) \left[\frac{1}{\Omega_\Lambda} + \frac{2}{c\sqrt{\Omega_\Lambda}} \right], \tag{6}$$

where the prime denotes the derivative with respect to lna and $\Omega_\Lambda = \rho_\Lambda / \rho_{crit}$. Combining (6) with (5) for the matter case one arrives at

$$\left| \frac{\sqrt{\Omega_\Lambda}}{c - \sqrt{\Omega_\Lambda}} \right| \gtrsim 1. \tag{7}$$

Employing c close to 1 (see below), one obtains $\Omega_\Lambda > 1/4$. Thus, thermodynamic equilibrium was being established somewhere around the onset of the dark-energy dominated epoch and will stay there for anytime in the future.

Let us consider first the quintessential $c > 1$ case. To this end, we need to find an explicit solution $a(t)$ of the Friedmann equation of the type

$$\dot{a}(t) = H_0\, a^{1/c}(t). \tag{8}$$

With the normalization $a(t_0) = 1$, one finds an explicit solution

$$a(t) = \left[-H_0 t (\frac{1}{c} - 1) + H_0 t_0 (\frac{1}{c} - 1) + 1 \right]^{\frac{c}{c-1}}. \tag{9}$$

Plugging (9) into (3), the central equation to be solved

$$t_{BB} \simeq exp(E_{br} R_h(t_{BB})) t_{dyn}, \tag{10}$$

with $E_{br} \sim 1$ kg and $t_{dyn} \simeq H_0^{-1} \simeq t_0$, can be recast in the form

$$\eta_{BB}(c) \simeq exp\left(10^{68} \left[\eta_{BB}(c)(c-1) + 1\right]\right), \tag{11}$$

where $\eta_{BB} \equiv t_{BB} H_0$. Notice that if Eq. (11) has no solution for any c, then the theory can be considered BB-free. Because of the exponentially huge figures entering (11), it is unfortunately extremely difficult to handle it numerically, and therefore one has to resort to heuristic methods in order to infer some information on the parameter c. For instance, one can easily solve (11) for $c(\eta_{BB})$, i.e.,

$$c(\eta_{BB}) = \frac{\eta_{BB} A - A + ln(\eta_{BB})}{A \eta_{BB}}, \tag{12}$$

where $A = 10^{68}$. By inspecting (12), one sees that for $\eta_{BB} = exp(A)$ and $\eta_{BB} = \infty$ $c = 1$, and therefore somewhere within this interval (12) reveals a maximum. With the maximum expressed in a closed form, we find that we are exposed to the BB threat if

$$1 < c < 1 + \left(\frac{1}{A}\right) e^{-A-1}. \tag{13}$$

Still, since for $c > 1$ the event horizon R_h grows in time, the number of ordinary observers is growing with time as well, making it hard to reckon the real BB threat. All we can say for sure is that if the parameter c lies outside the exponentially narrow strip right to the $c = 1$ line, as given by (13), the theory is safe with respect to the BB invasion.

Let us next consider the phantom regime ($c < 1$). Now the scale factor (9) diverges after finite time is passed – the big rip time [19]. Also, for $c < 1$ R_h decreases in time, falling to zero at the big rip time. The potential BB threat lasts until $R_h^{smallest} \sim 10$ cm is reached, the smallest possible size of the event horizon capable of housing a single BB observer.[3] Thus, if $t_{smallest} \lesssim t_{BB}$, then BB

[1] Since for the purpose of the present paper we are mostly interested in the future evolution of the Universe, even this simplest model can represent virtually all the models having the same choice for L.

[2] The exact expression for t_{dyn} is not of relevance here since we are dealing with exponentially huge figures in front of it. For the sake of rendering our calculation (see below) more compact and not introducing another parameter, we choose H_0^{-1} for t_{dyn}, where H is the Hubble parameter.

[3] Notice that decreasing R_h leads to a much higher BB creation rate, see Eq. (10).

brains are avoided. This means that in addition to (12), an extra constraint

$$1 + \frac{c}{1-c}\left(1 - \frac{B}{c}\right) \lesssim \eta_{BB}(c), \tag{14}$$

is to be considered in the phantom regime. In (14) $1/B = 10^{27}$. Now a straightforward analysis of (12), together with the constraint (14), enables one to obtain in a closed form an interval where one is to be exposed to the BB threat, i.e.,

$$1 > c > 1 + e^{-\frac{A}{B}}(B-1). \tag{15}$$

Our analysis thus shows that only in the exponentially narrow strip around the $c = 1$ line,

$$1 + e^{-\frac{A}{B}}(B-1) < c < 1 + \left(\frac{1}{A}\right) e^{-A-1}, \tag{16}$$

problems with BBs in the simplest HDE model are to be expected. We notice that a restriction on the parameter c under the combined observational tests slightly favor the phantom case [20–23].

Finally, let us adopt the global viewpoint. This means that exponentially huge regions created by the expansion of the Universe, which no one observer can ever probe, are also accepted as a part of reality. As already mentioned, a description of the BB paradox in the global picture leads to strong inconsistencies, if the idea of the string theory landscape is also to solve the cosmological constant problem [9]. Likewise, the popular resolution of the black hole information paradox is to abandon the global viewpoint, and to embrace the local view in the form of the black hole complementarity principle [24]. For these reasons we give much more preference for the local view, but for completeness sake we analyze the global viewpoint as well. To this end, we wholly follow the Page's arguments [7].

Following Page [7], when a four-volume of the Universe exceeds $V_4^{crit} \sim e^{10^{50}} a_{\rm Pl}^4$, one gets more observations by vacuum fluctuation than have occurred during past human history. Specifically,

$$V_4(t) = \int d^4x \sqrt{-g} \sim \int_{t_0}^{t} dt a^3(t), \tag{17}$$

with $a(t)$ from (6), as obtained in the HDE model. Obviously, if $c \geq 1$, the four-volume (17) grows unlimited with time, and without the vacuum decay, the BB problem persists.

Much more interesting is the phantom $c < 1$ case. Now the Universe lasts only until the Big Rip time

$$t_{BR} = t_0 + \frac{c}{(1-c)H_0}, \tag{18}$$

where the scale factor (9) becomes infinite. It can be seen by inspecting Eq. (17) that in this case the four-volume can be made finite for $c < 1/4$. At the same time the requirement $V_4 \lesssim V_4^{crit}$ provides us with a bound on the parameter c

$$c < \frac{1}{4}\left(1 - \frac{1}{4C}\right), \tag{19}$$

where $C = (H_0 l_{Pl})^4 e^{10^{50}}$. This is exponentially more restrictive then the bound obtained by adopting the local view. On the other hand, the bound $c < 1/4$ seems at odds with those bounds obtained from a variety of observational tests [20–23]. To be specific, focusing on the most thorough study [20], we see that the best fit of c for the simplest model [13] gets centered around 0.75, for all data set combinations used in this study. Although there is some fraction of allowed parameter space in which $c > 1$, there is absolutely no room for c as low as 1/4. Also, other studies [21–23] using less precise data gave the best fit value even higher than 0.75, thus moving away even more from the value 1/4. So the observational signatures of the simplest HDE model yet additionally apostrophize the known difficulties of the global view.

Summing up, we have tested how the simplest holographic dark energy model of Li copes with the theoretical conundrum known as the Boltzmann Brain paradox. And the outcome strongly depends on the description of the Universe (whether local or global) one adopts. With the local viewpoint, there is no restriction (up to an exponentially negligible one) on the free parameter of the theory, whilst the global viewpoint sets a restriction on it much stronger than those obtained from observational tests. In absence of any theoretical constraint on the energy density parameter c, our constraints may be considered as a new and useful piece of information corroborating further the genuine quantum-gravity origin of the model.

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