DARK MATTER PIONS*

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We study the scenario where dark matter corresponds to a set of pseudo-Goldstone bosons, that we call dark pions, generated by the spontaneous breaking of a symmetry in the dark sector. As a concrete example, we consider an $\mathrm{SU}(N)\times\mathrm{SU}(N)$ broken to the diagonal subgroup that remains an exact symmetry that ensures the stability of the dark pions, and allows a novel-interactions involving neutral gauge bosons and 3 dark pions. We study both experimental and theoretical constraints, and show that the model can accommodate all data in wide regions of parameter space.

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1. Introduction

Dark matter (DM) is the generic name assigned to a hypothesized particle or set of particles whose gravitational effects account for the observed galaxy rotation curves and the velocity dispersion within galaxy clusters [1], the fluctuations in the cosmic background radiation [2], and certain gravitational lensing observations [3]; for this hypothesis to work, DM should comprise 27% of the current mass-energy density of the universe and many (if not most) extensions of the Standard Model (SM) should contain one or more particles that can serve this role [5].

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The interactions of the DM with the SM have also been extensively studied, both in models such as the ones mentioned above, as well as "phenomenological" using effective interactions; this paper will be centered around this last topic.

2. The model

We will assume that aside form the SM sector, there is a dark sector that communicates with the first one through the exchange of heavy mediators whose properties will not be specified except the requirement that they have masses above the available energies. The dark sector, in general, will comprise many particles, all of which are assumed to be invariant under the SM gauge group; the lightest of these is assumed to constitute the main component of the DM. We ensure that these particles are naturally light compared to the rest of the spectrum by assuming they are the pseudo-Goldstone boson generated by the breaking of some continuous symmetry; we do not require this particle to be a massless Goldstone bosons because this type of particle typically generates difficulties when trying to understand the formation of structure in the early universe [6].

Within the effective-interaction paradigm the SM–DM interactions are generated by the Lagrangian of the form of $\mathcal{L}_{\mathrm{DM}\times\mathrm{SM}} \sim \mathcal{O}_{\mathrm{DM}} \times \mathcal{O}_{\mathrm{SM}}$, where the first factor denotes an operator constructed of the DM fields, and is invariant under the exact manifest symmetries that remain in the dark sector after the spontaneous breaking that generates the dark pseudo-Goldstone bosons. $\mathcal{O}_{\mathrm{SM}}$ is a local operator involving the SM fields and invariant under the SM gauge symmetry. Dimensional considerations require that such operator products have a prefactor containing (inverse) powers of some scale, which for the scenario considered here, is fixed by the mediator mass scale. It follows that the higher the dimension of the operators involved, the smaller the effects of the corresponding term: the leading SM–DM effects are determined by the lowest-dimension operators in $\mathcal{L}_{\mathrm{DM}\times\mathrm{SM}}$. The lowest dimension gauge-invariant operators made of SM fields are $|\phi|$ and $B_{\mu\nu}$, where ϕ denotes the scalar isodoublet and B the $U(1)$ gauge field; both have dimension 2. The effective Lagrangian then takes the form

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{DM}} + |\phi|^2 \mathcal{O}_{\mathrm{DM}} + B_{\mu\nu} \mathcal{O}_{\mathrm{DM}}^{\mu\nu} \,, \tag{1}$$

where the first term is the Lagrangian for the Goldstone bosons (henceforth referred to as dark pions) assumed to constitute the DM; we will assume (for simplicity) that this corresponds to $n\ \mathrm{SU}(N)_{\mathrm{L}} \times \mathrm{SU}(N)_{\mathrm{R}}$ non-linear σ model with an added mass term for the dark pions.

We collect the dark-pions fields π^a in a unitary SU(N) field[1] $\Sigma = \exp(i\pi_a T^a/f)$, in terms of which the pure DM Lagrangian becomes

$$\mathcal{L}_{\rm DM} = f^2\,{\rm tr}\Big\{\partial_\mu \Sigma^\dagger\,\partial^\mu \Sigma\Big\} + \tfrac{1}{2}f^2\left(M^2{\rm tr}\Sigma + {\rm H.c.}\right)\,, \tag{2}$$

while the DM–SM interactions are given by

$$\begin{aligned}\mathcal{L}_{\rm int} \;=\; & \tfrac{1}{2}\lambda_h\left(|\phi|^2 - v^2\right){\rm tr}\Big\{\partial_\mu \Sigma^\dagger\,\partial^\mu \Sigma\Big\} + \tfrac{1}{2}f^2\lambda_h'\left(|\phi|^2 - v^2\right)\left({\rm tr}\Sigma + {\rm H.c.}\right)\\ & + B^{\mu\nu}\left(\lambda_V{\rm tr}\Big\{\Sigma^\dagger\partial_\mu\Sigma\,\partial_\nu\Sigma^\dagger\Big\} + {\rm H.c.}\right)\,,\end{aligned} \tag{3}$$

where $v = \langle\phi\rangle \simeq 174$ GeV. The first term is SU(N) $\times$ SU(N) invariant, the remaining terms are only invariant under the diagonal subgroup SU$(N)_V$ under which the π^a transform according to the adjoint representation.

Expanding the total Lagrangian $\mathcal{L} = \mathcal{L}_{\rm DM} + \mathcal{L}_{\rm int}$ in powers of π^a, we find

$$\begin{aligned}\mathcal{L} \;=\; & \tfrac{1}{2}(\partial\boldsymbol{\pi})^2 - \tfrac{1}{2}M^2\boldsymbol{\pi}^2 - \frac{\lambda_V}{f^3}B^{\mu\nu}f_{abc}\partial_\mu\pi_a\,\partial_\nu\pi_b\,\pi_c + \lambda_h'\left(\frac{vh}{\sqrt{2}} + \frac{1}{4}h^2\right)\boldsymbol{\pi}^2\\ & + \frac{\lambda_h}{f^2}\left(\frac{vh}{\sqrt{2}} + \frac{1}{4}h^2\right)(\partial\boldsymbol{\pi})^2 + \frac{N}{16f^2(N^2-2)}\left[\left(\partial\boldsymbol{\pi}^2\right)^2 - \mu^2\left(\boldsymbol{\pi}^2\right)^2\right]\,,\end{aligned} \tag{4}$$

where $\mu^2 = (6 - 4/N^2)M^2/(N^2+1)$. The terms $\propto \lambda_h$ represent $\pi - h$ derivative couplings, while those $\propto \lambda_h'$ correspond to the usual Higgs portal interaction. Terms containing λ_V correspond to interactions between the Z and photon with the dark pions, while the last two terms represent the usual quartic dark-pion self-interactions present in the chiral Lagrangian.

All SM states are singlets under SU$(N)_V$ and all the dark pions are mass degenerate; it follows that the π^a are stable. The reaction $Z \to 3\pi$ is allowed since a state with 3 dark pions can be an SU$(N)_V$ singlet. Note that the dark sector contains N conserved charges that may not vanish: this model allows a variety of ways to explore the asymmetric DM.

The effects of the Higgs portal interaction $\propto \lambda_h'$ have been studied extensively [7] and are well understood. Our interest is to understand the effects of the remaining interactions. For this reason, we will adopt the simplifying assumption $\lambda_h' = 0$. The effects of this interaction can of course be added, but at the cost of complicating the discussion.

[1] Group generators are normalized according to ${\rm tr}\{T_aT_b\} = \delta_{ab}$, the structure constants are defined by $[T_a, T_b] = if_{abc}T_c$; the quantity f (no to be confused with the structure constants) is referred to as the dark-pion decay constant and has units of mass.

3. Thermal history

Given the interactions and properties of the dark pions, it is a straightforward exercise to determine their relic abundance. In doing so, we will assume that the SM remains in equilibrium and that during the epochs of interest the densities are never high enough for quantum statistics to be important. The relevant Boltzmann equations (BE) are then

$$\begin{aligned} \dot{n}_a + 3Hn_a &= -\mathcal{C}_a = -\sum_{b,c,d} \int d\Phi |\mathcal{A}_{a+b\to c+d}|^2 (f_a f_b - f_c f_d)\,, \\ d\Phi &= d\Pi_a\, d\Pi_b\, d\Pi_c\, d\Pi_d (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d)\,, \end{aligned} \tag{5}$$

where n_a denotes the number density of π^a and f_a the corresponding phase-space density; $\mathcal{A}$ denotes the Lorentz-invariant matrix element summed over initial and final states, and including the appropriate symmetry factors; we also used $d\Pi = g d^3\boldsymbol{p}/[(2\pi)^3 E_{\boldsymbol{p}}]$ for the usual Lorentz-invariant phase-space element for a particle with g internal degrees of freedom ($g = 1$ for the π^a).

We re-write the equations in terms of $Y = n/s$ (s is the conserved entropy density) [8] and simplify the discussion by taking $N = 2$, with fields $\pi_{\pm\,0}$, and one conserved charge $q = Y_- - Y_+$. Then, letting $x = M/T$

$$Y_r' = -\sqrt{\pi g(T)/(45G)}\left(M/x^2\right) C_r(Y)\,, \quad C_r(Y) = \mathcal{C}_r/s^2\,, \quad (r = 0, \pm)\,, \tag{6}$$

where $g(T)$ is the relativistic number of degrees of freedom at temperature T and a prime denotes an x derivative

$$\begin{aligned} C_0(Y) &= \left(Y_0^2 - Y_0^{(\mathrm{eq})\,2}\right)\langle\sigma v\rangle_{\pi_0\pi_0\to SM} + \left(Y_0^2 - Y_+Y_-\right)\langle\sigma v\rangle_{\pi_0\pi_0\to\pi_+\pi_-} \\ &\quad + \left[Y_0 Y_0^{(\mathrm{eq})} - Y_+Y_- + (Y_+ + Y_-)\left(Y_0 - Y_0^{(\mathrm{eq})}\right)\right]\langle\sigma v\rangle_{\pi_+\pi_-\to\pi_0 V}\,, \\ C_+(Y) &= \left(Y_+Y_- - Y_0^{(\mathrm{eq})\,2}\right)\langle\sigma v\rangle_{\pi_0\pi_0\to SM} + \left(Y_+Y_- - Y_0^2\right)\langle\sigma v\rangle_{\pi_+\pi_-\to\pi_0\pi_0} \\ &\quad + \left(Y_+Y_- - Y_0 Y_0^{(\mathrm{eq})}\right)\langle\sigma v\rangle_{\pi_+\pi_-\to\pi_0 V}\,, \end{aligned} \tag{7}$$

where $Y^{(\mathrm{eq})}$ are the equilibrium distributions. These equations can be solved numerically with the boundary conditions $Y \to Y^{(\mathrm{eq})}$ as $x \to 1$.

In order to understand the role of $q = Y_- - Y_+$, it is useful to write the equations in terms of q and the two combinations $Y_t = Y_0 + Y_+ + Y_-$ and $Y_d = (Y_+ + Y_-)/2 - Y_0$, noting that the DM relic abundance is proportional to $Y_t(x \to \infty)$. Y_t is useful because its BE is even under $q \leftrightarrow -q$ and, if $u(x) = (\partial T_t/\partial q^2)_{q=0}$, then $u(x_i) > 0$, $u'(x) > 0$. It then follows that $Y_t(q = 0, x) < Y_t(q \neq 0, x)$, so that

$$\Omega_{\mathrm{DM}}(f, M, \lambda_h, \lambda_V; q = 0) < \Omega_{\mathrm{DM}}(f, M, \lambda_h, \lambda_V; q)\,, \tag{8}$$

that is, given the WMAP/PLANCK [2] estimates for the relic abundance $\Omega_{\rm CDM}$ (within the cold DM–CDM-scenario), the corresponding constraint on the model becomes $\Omega_{\rm DM}(f, M, \lambda_h, \lambda_V; q = 0) < \Omega_{\rm CDM}$, since the deficit can always be made up by introducing an appropriate non-zero q.

Numerically we find that q is irrelevant if it is smaller than 10^{-13}. If $q > 10^{-12}$ it dominates the abundance; in this case, $3.4 \times 10^{-10} < |q|M < 4.7 \times 10^{-10}$, when $M < 100$ and is in GeV units. Finally, for $10^{-13} < q < 10^{-12}$, it is of the same order as the Y as illustrated in figure 1.

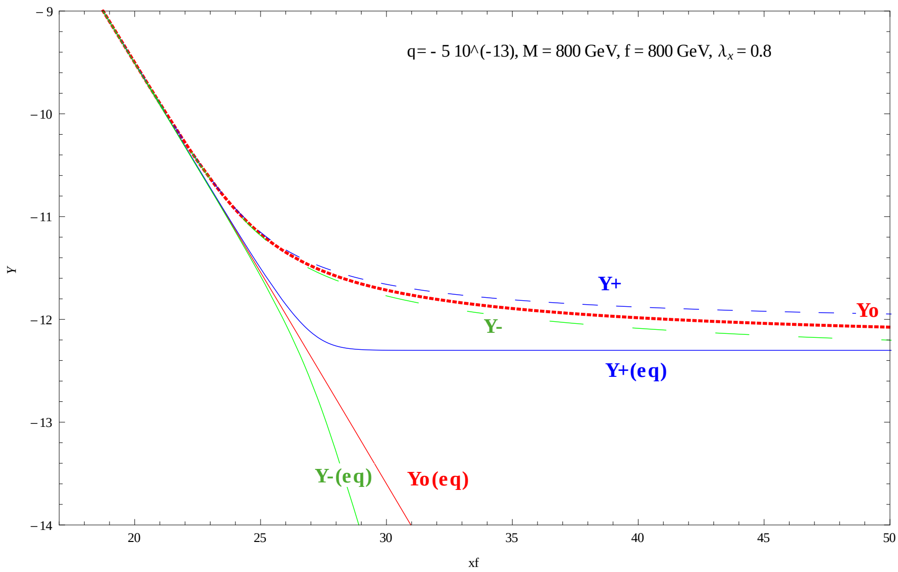

Fig. 1. Example of solution to the BE (6) for $q \neq 0$.

4. Constraints on the model

The model is constrained by the CDM results [2], by Higgs decay data [11] and direct detection limits, as well as by theoretical consistency conditions [10]; as illustrated in figure 2, there are large regions in parameter space allowed by these restrictions.

⊙ *Restrictions derived from the WMAP/PLANCK observations:* using $0.094 \le \Omega_{\rm CDM} \le 0.130$, we find (here and in the following, M and f are in GeV units)

$$4.04 \times 10^{-7} \le \left(\lambda_h\, M/f^2\right)^2 + 0.93 \left(\lambda_V\, M^2/f^3\right)^2 \le 5.59 \times 10^{-7} \delta_{q,0}\,, \qquad (9)$$

where we used the fact that Y_t has a simple scaling behavior dependence on the couplings λ_h, λ_V [9]; the numerical coefficients above were obtained by numerically solving the BE.

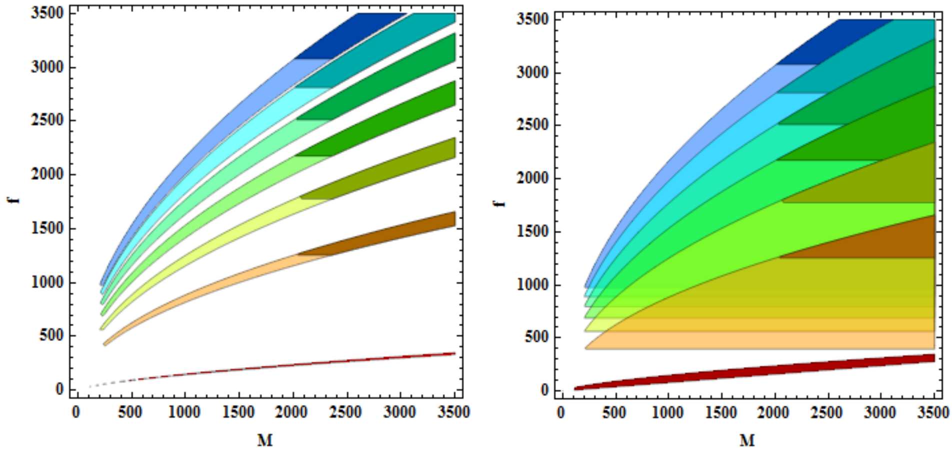

Fig. 2. The colored areas denote parameter regions allowed by the constraints when $\lambda_V = 0.0023$, and for $\lambda_h = 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3$ (from the bottom up); the left (right) panel corresponds to $q = 0$ ($q \neq 0$). The darker colored regions correspond to the limits expected from the proposed XENON1T [12] experiment. For $q \neq 0$, the allowed region do not have a lower bound because of (8).

⊙ *No large deviations in Higgs decays:* when kinematically allowed (*i.e.* $M < 62.5$), $\Gamma(h \to \pi\pi) < 4\text{MeV}$ [11]. This translates into

$$f > 5.9|\lambda_h|^{1/2} \left|7812.5 - M^2\right|^{1/2} \left[1 - (M/62.5)^2\right]^{1/8} . \tag{10}$$

⊙ *Direct detection constraints:* the absence of any events in the XENON100 experiment [12] leads to the simple restriction [9]

$$f > 562.3|\lambda_h|^{1/2} . \tag{11}$$

⊙ *Consistency of the model:* we demand that loop corrections to the various couplings are at most of the same order as their tree-level values [10]. Most stringent is the constraint involving λ_V

$$4\pi f/M \geq \max\left\{\sqrt{4\pi\lambda_V}\, , 1\right\} . \tag{12}$$

5. Conclusions

The model presented explores the possibility that the DM corresponds to a set of pseudo-Goldstone bosons generated by the breaking of some unknown symmetry in the dark sector; for the specific model considered here, the dark sector is assumed to contain a unitary $\mathrm{SU}(N) \times \mathrm{SU}(N)$ chiral symmetry explicitly broken to $\mathrm{SU}(N)$. The resulting model can comfortably accommodate existing restrictions and has the novel feature of a coupling of the neutral gauge bosons to 3 (and more) dark pions, which provides the dominant DM–SM interaction in certain regions of parameter space.

In addition, we found that the relic abundance depends on the dark-pion mass M; it also obeys a simple scaling dependence on the coupling constants λ_h, λ_V that appear only in the combinations $\lambda_h M/f^2$ and $\lambda_V M^2/f^3$. We considered in detail only the $M > 50\,\mathrm{GeV}$ region; smaller dark-pion masses are allowed only when $\lambda_h \ll 1$ in which case the $Z \to 3\pi$ decays opens up and must be included in the calculations.

It is worth noting also that the current model does not provide a resolution for the tension between the XENON100 and the DAMA/LIBRA results [13]. The collider signature of the dark-pions is mainly missing energy, which makes them difficult to detect in such experiments.

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