A class of problem dealing with narrow-band reflectors under oblique light incidence has been selected, and a group of design procedures leading to satisfactory results is analyzed. Initially performance criteria and design restrictions are set. Various designs are then created and analyzed. Conclusions can then be drawn on the relative merits of the designs. Two different types of target function have been chosen. A global search for 50 or more layers with varied optical thicknesses between 0 and \( \lambda/2 \) and a needle design method have been applied to the synthesis. Gradient and variable metrics have been used for further refinement. Results are compared, and it is shown that all the design methods used yield similar results for this problem. The issue of how to determine the best method is addressed. © 1998 Optical Society of America

1. Introduction

Sophisticated interference coatings have been introduced into many applications in optics and optoelectronics. The demand for such coatings is a new challenge to designers. Often these demands are not purely scientific considerations but also demands restricted by the available technology and time. Designers are often faced with demanding requests from customers. Even worse, there may be no appropriate solutions already available (no procedures in handbooks), so they must look for an original solution, which takes time and increases product price.

For these problems selecting a suitable starting design is not easy. Here thin-film synthesis methods can be quite effective because they do not require a starting design, or if they do, they generate their own starting design automatically. The synthesis is then followed by a refinement method. A good comparison of the different refinement methods and their effectiveness has been presented by Dobrowolski and Kemp.\(^1\) We checked some of the refinement methods and noted that, in the sense of finding the minimum, they work in a similar way. Thus full attention is given to the synthesis strategy.

In this paper we define a case study for analyzing the efficiency of some synthesis methods and for determining the best strategy. Two synthesis methods have been chosen for analysis and comparison: global synthesis, sometimes called also the comprehensive search,\(^2,3\) and Tikhonravov’s needle design method.\(^4,5\) Gradient and variable metrics have been used for refinement. In Refs. 6–8 the effect of the needle design method on some different types of coatings is described, but in this paper we attempt to analyze how the chosen methods and the strategy of their application have led to solutions. Solutions are different, and here we try to select the best or the most typical one for the chosen case study. We performed all calculations using TFCalc software.\(^9\)

2. Case Study and Synthesis Procedures

We have chosen a type of problem dealing with narrow-band reflectors under oblique incidence. Initially goals and restrictions are defined. We then analyzed and compared the various designs. We impose the following requirements at oblique incidence (45 deg):

- \( \text{Reflectance, } <1\% \text{ for wavelengths of } 420–530 \text{ nm.} \)
- \( \text{Reflectance, } >99\% \text{ for wavelengths of } 530–570 \text{ nm.} \)
- \( \text{Reflectance, } <1\% \text{ for wavelengths of } 570–600 \text{ nm.} \)
- \( \text{Reflectance, } >99\% \text{ for wavelengths of } 600–650 \text{ nm.} \)
- \( \text{Reflectance, } <1\% \text{ for wavelengths of } 650–750 \text{ nm.} \)
Reflectance is the average reflectance $R = (R_s + R_p)/2$. The refractive indices used for the design were as follows: $H, n = 2.35, k = 5 \times 10^{-4}$; $L, n = 1.46, k = 0$. The refractive indices were not dispersed. The incident medium was air, and the substrate material was BK7 optical glass. The number of layers should not exceed 50, and the minimum physical thickness of a layer was restricted to 10 nm. The maximum thickness was restricted to 300 nm.

The multilayer synthesis and refinement have been performed by use of two target functions: a discrete target function consisting of 226 points spaced 2 nm apart and a continuous target function in which requirements $(A) \sim (E)$ were entered as five continuous optimization targets. The target functions have been constructed following these requirements. The reflectance bands had half of the tolerance of the transmittance bands.

Optical thicknesses in the global search synthesis have been varied between 0 and $\lambda/2$. For all design strategies $\lambda$ was 550 nm. It actually has no special meaning, because the final results are physical thicknesses, but $\lambda$ has been given only to simplify the descriptions of some of the starting designs. A wider range of starting optical thicknesses was analyzed as well (0–$\lambda$) but, surprisingly, with no improvements! It was expected that this approach might yield additional good designs owing to the half-order phenomena. It seems that this starting range has been restricted too much to reach them and still achieve better results.

Since this research is very basic, full attention is given to determining the most suitable strategy and not to the time it will require. All calculations were performed on a 166-MHz personal computer. Several stopping criteria during minimization were applied, i.e., deviation of the obtained spectra from the target value ($\Delta$), the number of layers, the total thickness. Designing on the best final design was not easy. The greater the number of parameters chosen, the more difficult it is to determine the best design. Tikhonravov recently discussed the dilemma of choosing the “best” design.10

A. Global Search

We started the global search from 50 alternating layers, where thicknesses were set at $\lambda/4$ initially. Random variations of layer thicknesses were then performed. All generated thicknesses were in the range between 0 and $\lambda/2$. Typically 1000 designs have been generated, in which 5–10 of the best were chosen for further refinement. Later we note that the generation of multilayers starting with 70–75 layers yielded better results because of the decrease in the number of layers during the refinement procedure. Thus the final design fell into the given frames. The five best results of that design strategy are presented in Fig. 1.

B. Modified Global Search

This procedure enables some layers to have more freedom than others. Therefore we call the procedure modified. Namely, we gave a kind of direction to the global search, using what could be called a pseudomulticavity bandpass filter. So, for the initial design for the global search, a six-cavity narrow-band filter—$(HLHL2HLHLHL)^6$—centered at 585 nm, allowing 100% variations in thicknesses, was chosen at the start, and then the global search was performed. H and L are high- and low-index materials as mentioned above. More about multiple-cavity filters can be found in Ref. 11. In this way the spacer layers are favored, i.e., they obtained a higher level of freedom. In other words, the shape of the multidimensional space was significantly changed. We hoped that this approach would enable the minima regions to be found more easily and efficiently. In Fig. 2 we present the best results from using the modified global search method followed by a final refinement.

C. Needle Design

The needle design started from a single H layer. The optical thickness of the starting layer was varied between $7\lambda$ and $20\lambda$, where $\lambda$ was again 550 nm. A

![Fig. 1. Reflectance versus wavelength (nanometers) for the filter obtained from the global search.](image1)

![Fig. 2. Reflectance versus wavelength (nanometers) for the filter obtained from the modified global search.](image2)
thicker starting layer yielded more layers in the multilayer. The best result was obtained with a starting thickness of 20\(\lambda\). Larger starting thicknesses yielded too many layers, and their contribution to the performance was insignificant. Layers were added simultaneously and step by step. The step-by-step needle addition of layers enabled better control of the number of layers but yielded slightly greater deviations. The best results were obtained with a design process consisting of three or four simultaneous needle procedures, followed by refinement. The optimization of layers, which temporarily had zero thickness, has not stopped, but they are able to develop again. This approach actually increases the design time but yields better results. In other words, we did not artificially change the dimensions of the hyperspace, keeping the process going to the end as the needle method predicted.

The spectral characteristics of the filters, obtained by the above-mentioned needle design methods, are presented in Figs. 3 and 4. Figure 3 corresponds to the needle design with the simultaneous insertion of layers and Fig. 4 to the step-by-step addition.

D. Needle-Composite Design
Two notch filters (centered at 550 and 625 nm) have been designed with the needle design and some refinement procedures. The starting thicknesses of a high-index layer for the two needle designs are 10\(\lambda\). The first notch filter, having a high reflectance of 530–570 nm, has 39 layers, and the second, having a high reflectance of 600–650 nm, has 41 layers. Two notch filters were assembled through a simple L transition layer, and then the final variable metric refinement was performed. This approach has been found to give very good results in a relatively short time. The results of the described method are in Fig. 5.

E. Composition of Two Narrow-Band Reflectors
In the fifth method we composed the filter of two narrow-band reflectors of the (L3H)\(^n\) type followed by a refinement technique. We tried to overcome the fact discussed above that the restrictions on variations of the thicknesses for global search have been chosen too narrowly. This method did not yield better results than others because the local minima defined by the narrow-band reflectors were obviously too deep. The result, Fig. 6, is an illustration that the solution for the given case study could not be constructed simply by following the described procedure.

3. Discussion and Conclusions
An overview of the results, which illustrates and compares the final values obtained by different design strategies described in Section 2, is given in Table 1. The profiles of the refractive indices for all five solutions are given in Fig. 7. The metric thicknesses are expressed in micrometers. Following the design strategies above, we tried to determine which approach from the chosen synthesis and refinement

Fig. 3. Reflectance versus wavelength (nanometers) for the filter obtained from the needle method and the simultaneous insertion of layers.

Fig. 4. Reflectance versus wavelength (nanometers) for the filter obtained from the needle method and the step-by-step insertion of layers.

Fig. 5. Reflectance versus wavelength (nm) for the filter obtained by use of the composition of two notch filters created by the needle method.
method combinations gave the best results. The best result, in the sense of deviation from the target value, was obtained by a global search followed by variable metrics refinement. Whether this solution should really be considered to be the best is unclear. Comparing the global-based designs with the needle-based designs, we can see that they all have similar deviations and total thicknesses. Which results are the best is really not easy to answer. However, the needle design implemented in Tcalc 3.2 does not give a superior solution. Maybe this is a possible answer to a question of Tikhonravov et al., namely, this is a type of problem in which other methods give better results even with a lower number of layers.

Two different types of target function have been used for the design. The continuous target function, where requirements \(A\)–\(E\) were entered as five continuous optimization targets, yielded generally better results, although the discrete target function has been well defined in 226 points. On the other hand, the use of a discrete target function enabled a faster approach to the solution. Sometimes a combined use of these two target functions decreases the design time. This method has not been applied here, since the intention was to distinguish one from the other in order to come to reasonable conclusions.

<table>
<thead>
<tr>
<th>Synthesis Procedure</th>
<th>Deviation</th>
<th>Number of Layers</th>
<th>Total Thickness (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global search</td>
<td>8.065</td>
<td>48</td>
<td>6044</td>
</tr>
<tr>
<td>Modified global search</td>
<td>8.198</td>
<td>48</td>
<td>5851</td>
</tr>
<tr>
<td>Needle design—simultaneous</td>
<td>8.118</td>
<td>49</td>
<td>5807</td>
</tr>
<tr>
<td>Needle design—one by one</td>
<td>8.160</td>
<td>49</td>
<td>6227</td>
</tr>
<tr>
<td>Composition of two needles</td>
<td>8.344</td>
<td>49</td>
<td>5640</td>
</tr>
<tr>
<td>Composition of two narrow-band reflectors</td>
<td>9.454</td>
<td>47</td>
<td>9292</td>
</tr>
</tbody>
</table>

The strengths and the weaknesses of the software used are reflected partially in the results of this paper. We attempted to avoid erroneous conclusions that might have been generated by the software itself.

4. Summary

In this paper different design procedures, in which a given study is used, have been analyzed. The weaknesses and the strengths of each for the given problem have been discussed. It has been found that...
they all lead to more or less similar solutions, i.e., deviation values, but they are all different in the physical sense. Determining the absolute best answer is not possible, since it depends on many factors and the specific needs and requirements of the application. However, it is possible to conclude that the global search and needle methods contain synthesis methods that safely lead to excellent solutions. It is not possible to conclude generally that they will always work in a similar way, but in this study they both yielded similar results. The success of one method or some other method depends also on the fine structure of the software used, i.e., the algorithms that support the method itself. Therefore the multilayer design, except in its practical use, will present a permanent challenge for applied mathematics and physics.

References