Emergent relativity: Classical drama at the horizon

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Abstract

Motivated by the black hole information paradox, we propose a reformulation of the classical theory of relativity. The main assumption is that classical space is discretized at very small length scales, while time remains continuous. This breaks Lorentz invariance at short distances, implies the existence of a fundamental preferred reference frame, and indicates that the fundamental kinematic notions of time and space are independent of the dynamical metric field. Special and general relativity are then understood as emergent, effective theories, valid only at sufficiently large scales. Observers moving at ultra-relativistic velocities relative to the preferred frame, as well as freely falling observers near a black hole horizon, see a drama, namely a radical deviation from the predictions of relativity. Physics in the black hole interior encounters an even more radical departure from the relativity theory. Such a reformulation of classical physics in black holes and close to their horizons leads to a simple resolution of the black hole information paradox. The quantization of such a theory with a pre-dynamical time and a discrete pre-dynamical space is, in principle, straightforward.

Keywords: emergent relativity; discrete space; black hole; information paradox

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1 Introduction

1.1 Main ideas

The two pillars of modern physics, quantum mechanics (QM) and general relativity (GR), are in clash with each other, at least in their current formulations. The clash is particularly manifest in the black hole information paradox [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], the resolution of which is expected to significantly alter our basic understanding of either QM or GR, or possibly both.

In this paper we argue that it is GR that needs to be modified. More specifically, we argue that the theory of relativity, *both special and general*, should be significantly reformulated already at the classical level, *prior* to quantization. When the theory of relativity is reformulated appropriately, then, we claim, the black hole information paradox goes away, together with some other conceptual problems related to black holes. To give a spoiler, it will turn out that the principles of the theory of relativity, especially the principle of Lorentz invariance, are not fundamental but emergent, very much like an emergent Lorentz invariance appears in condensed matter physics as a property of the propagation of sound.

This work is inspired by many different approaches and ideas existing in the literature. This includes the firewall proposal [11, 12] suggesting that the resolution of the black hole information paradox requires a drama at the horizon seen even by a freely falling observer, the membrane paradigm for black hole physics [13], the view that gravity is just a spin-2 field *a priori* not related to geometry [14, 15, 16, 17, 18], the Horava gravity [19, 20] violating Lorentz invariance at the level of a fundamental action, the "aether" models of gravity [21, 22, 23, 24] with a preferred frame of coordinates which violates Lorentz invariance, the condensed-matter analog models of gravity and black holes [25, 26, 27, 28] the ideas that high energy physics emerges from more fundamental condensed-matter style theories [29, 30, 31], and the idea that diffeomorphism invariance could be emergent [32]. In the present work we attempt to identify the common and most important idea behind all these approaches, the idea that relativity is not fundamental but emergent.

To prepare the reader to our main new idea, let us start with a very brief recapitulation of the historical route that led to special and general relativity [33]. Guided by the Lorentz invariance of Maxwell equations, Einstein formulated the principles of special relativity. Soon after Minkowski reformulated these principles in terms of spacetime geometry given by the line element

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu. \tag{1}$$

Guided by special relativity and the equivalence principle, Einstein then argued that gravity is actually a spacetime geometry described by a more general line element

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu. \tag{2}$$

Finally, after a more elaborated work, it was found that the dynamics of $g_{\mu\nu}(x)$ can be described by the action

$$A = \int d^4x \sqrt{|g|} \left[\frac{R}{16\pi G_N} + \mathcal{L}_{\rm matter} \right], \tag{3}$$

where $\mathcal{L}_{\rm matter}$ describes matter. For instance, for a real massive scalar field field $\phi(x)$ minimally coupled to gravity we have

$$\mathcal{L}_{\rm matter} = \frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2]. \tag{4}$$

Our main conceptual idea is to turn this whole argument around. In principle, once we know the action (3), that is all we need to know. In particular, we do not longer need to know that $g_{\mu\nu}(x)$ is geometry described by the line element (2). Instead, we can *derive* from the minimal coupling in (4) that a localized configuration of a field $\phi(x)$ follows a geodesic associated with the metric (2). In this sense the geometric interpretation (2) is *derived* from the action (3) with (4), so the geometric interpretation (2) is no longer a separate postulate of GR. Instead, the geometric picture is just an auxiliary intuitive picture, derived from the action.

Now, equipped with the idea that the geometric interpretation of gravity is not an independent fundamental principle, but something that results from the action, we can modify our intuition and think of gravity in a way very different from what we are used to. Since continuous field theory has an infinite number of degrees of freedom, which may result in an infinite entropy, we shall modify the action (3) by assuming that the space is fundamentally discrete. The discreteness of space will be controlled by some small distance parameter l, so that in the continuum limit $l \to 0$ one reconstructs the continuum action (3). At first sight, with such a small l, it will look like a minor and benign modification of the action. Nevertheless, after a careful study of its consequences it will turn out that, in some regimes, *the geometric picture (2) is no longer valid even as an approximation*. In particular, the geometric picture will be radically altered close to the horizon of the black hole, and even more radically inside the black hole. Such a radical modification of GR will, among other things, offer a simple solution of the black hole information paradox. In addition, it will turn out that even special relativity, described by (1) in the absence of gravity, is radically modified in some regimes.

We also note that in this paper we will mainly be concerned with the classical theory, not with a quantum one. The discreteness of space will be imposed at a classical level, it will not be associated with quantum uncertainty. The quantization of our theory with a discrete set of degrees of freedom will in principle be straightforward, but the main novelties, including the radical modification of physics in the black hole interior and close to its horizon, will have a classical origin not depending on quantization.

1.2 Underlying philosophy

Our basic logic is the following. In field theory of a real scalar field, each point of space represents one degree of freedom. So if the space is continuous, then an arbitrarily small portion of space contains an infinite number of points, and hence and infinite number of degrees of freedom. Then, in principle, an arbitrarily small portion of space can store an arbitrarily large amount of entropy, so the black hole shrinking due to Hawking radiation does not diminish its ability to store information. This implies that, if one really accepts that space is continuous, then there is no black hole information paradox [34]. In other words, the view that there *is* a black hole information paradox contains an implicit hidden assumption that space is *not* continuous. In this paper we take this logic very seriously and accept its logical conclusions wherever they lead us, as long as they do not contradict existing experiments.

The very notion of degrees of freedom in physics is something that needs to be specified kinematically, before any attempt to specify their dynamics. In a closed system the number of degrees of freedom must be a fixed constant, it cannot change due to dynamics. On the other hand, the metric $g_{\mu\nu}(x)$ in GR is dynamical. This implies that *the number of degrees of freedom cannot depend on the dynamical metric* $g_{\mu\nu}(x)$. If the number of degrees of freedom in a portion of space depends on a volume at all, this volume must be defined by some pre-dynamical metric different from $g_{\mu\nu}(x)$.

As we have seen, to arrive at a finite number of degrees of freedom in field theory, we need to assume that space is not continuous. However, *a priori*, there is no need to assume that for time. Indeed, time in QM is a continuous variable, just as it is continuous in classical mechanics. Being guided by the principle that we should not make new assumptions unless we have a strong physical reason for that, we take a rather conservative approach and assume that time is continuous.

Thus we seek a theory in which time is continuous while space is discrete. But this obviously *violates Lorentz invariance* from the very start, because Lorentz invariance requires that time and space should be treated on an equal footing.

At first sight, this does not necessarily need to lead to dramatic changes of known physics. The space is naturally expected to have a discrete structure at some very small length scale l. (For example it could be of the order of Planck length, but in this paper we shall not assume that. We shall only assume that l is sufficiently small so that there are no contradictions with currently existing experimental data.) Thus, at the phenomenological level, in most cases one may expect only small deviations from standard physics. However, no matter how small the deviations might be, rejecting Lorentz invariance as a fundamental principle is a big *conceptual* leap. Even when a phenomenological theory in the continuous approximation $l \to 0$ looks Lorentz invariant, at the conceptual level one still must think of space and time as fundamentally different notions. Such a way of thinking may come natural to some physicists (e.g. those who are used to think in terms of condensed matter paradigms), but unnatural to others (e.g. those with a strong GR background). To intuitively understand the logic of the theory presented in this work, the relativists must take an extra effort to unlearn a part of their relativistic intuitions and turn back to their pre-relativistic intuitions they had before they learned the relativity theory. With such pre-relativistic intuitions of space and time, the logic of this work should actually be rather easy to follow.

Of course, we are not asking the readers to completely abandon their relativistic intuitions. Indeed, we shall often switch from a relativistic to a pre-relativistic point of view, and back. Through such switches, we shall establish a sort of dictionary between the two points of view and, hopefully, learn how to think of the same things from two different perspectives. However, it is crucial to keep in mind that the two perspectives will not be fully equivalent. Only the pre-relativistic point of view will be fundamental, while the relativistic point of view will be emergent, valid only at the level of an effective theory.

The difference between the two points of view is in fact easy to explain. At the fundamental level, according to the theory presented in this work, the time and space are Newtonian absolute time and space. There is no Lorentz invariance at the fundamental level, the Lorentz invariance only appears at an emergent level. From the emergent relativistic point of view, this Lorentz non-invariance can be described as an existence of a *preferred frame* of spacetime coordinates, which corresponds to the fundamental split of spacetime into separate notions of space and time. The existence of a preferred frame looks somewhat *ad hoc* from the relativistic point of view, but this only reflects the fact that relativity is not fundamental. From the fundamental point of view there is no preferred frame in spacetime, there are only space and time as separate entities.

1.3 Physical consequences

Our view of space and time will turn out to be much more than a different philosophical conceptualization of the theory of relativity. In some regimes it will lead to physical predictions very different from the standard relativity theory. In particular, while the standard continuous theory with $l = 0$ predicts that a freely falling observer does not observe anything out of the ordinary while approaching the black hole horizon, in the theory with $l \neq 0$ it will turn out that a freely falling observer close to the horizon observes a deviation from the standard theory, proportional to terms such as

$$\frac{l^2}{1 - \frac{2M}{r}}. \tag{5}$$

Far from the horizon it is negligible due to small l. And yet, close to the horizon at $r \to 2M$ this becomes arbitrarily large, as long as l is not strictly zero. This is something that could be measured, at least in principle.

The deviations from the standard theory will turn out to be even more radical in the black hole *interior*. There, according to GR, the Schwarzschild radial coordinate r is a timelike coordinate, while the Schwarzschild time coordinate t is a spacelike coordinate. But the very notions of "timelike" and "spacelike" are intrinsically relativistic, they are determined by the dynamical metric $g_{\mu\nu}(x)$ and do not make sense from the fundamental pre-relativistic point of view. From the fundamental point of view t represents time and r space *in both the exterior and the interior*, which, in the interior, is the exact opposite of the relativistic interpretation. We shall see how that implies that physics in the black hole interior is radically different from expectations based on GR.

As we shall see, it will turn out that such radical deviations from relativity theory resolve the black hole information paradox. In fact, the essence of the resolution is

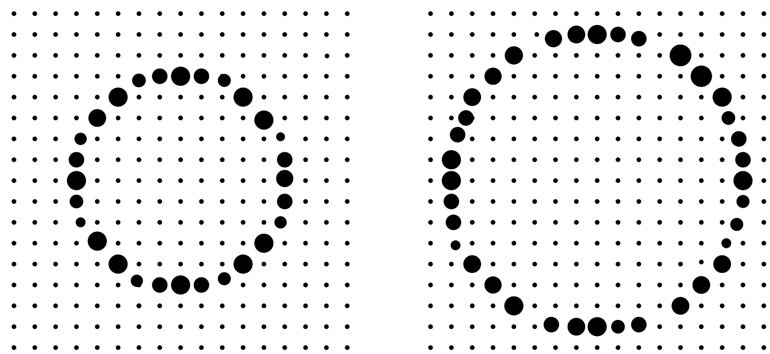

Figure 1: Two snapshots of a black hole, either with an increasing size (from the left to the right) due to income of positive energy, or a decreasing size (from the right to the left) due to income of negative energy. Each dot represents one fundamental degree of freedom on the fixed kinematic space lattice. Each degree of freedom carries a pre-relativistic mass that can change with time. The black hole horizon corresponds to the region where the masses are very large (theoretically infinite), represented by very big dots.

simple to understand intuitively, by visualization at Fig. 1. When the black hole shrinks due to income of negative energy associated with Hawking radiation, some fundamental degrees of freedom that were a part of the interior become a part of the exterior. In this way, information carried by the fundamental degrees of freedom can transfer from the interior to the exterior.

1.4 Organization of the paper

The rest of the paper is organized as follows. The general fundamental principles of the discrete theory are presented in Sec. 2, and then applied to the black hole in Sec. 3. An effective continuous field theory is presented, and its phenomenological consequences are discussed, in Sec. 4. The basic principles of quantization with some conceptual implications are qualitatively outlined in Sec. 5. A discussion of our results and a conclusion are presented in Sec. 6.

2 Fundamental principles

2.1 From background metric to masses and couplings

In this subsection we have two goals. The primary goal is to understand the main conceptual idea how a background metric field $g_{\mu\nu}(x)$ can be interpreted in a solid-state language, as variable masses and couplings on a lattice of harmonic oscillators. The secondary goal is to be very pedantic about the precise values of various proportionality constants that arise in a translation from one language to another.

For simplicity, we will mainly be concerned with a real massless scalar field $\phi(x)$ in a $(1+1)$-dimensional spacetime with a diagonal metric $g_{\mu\nu}(x)$, but as we will see, it will be straightforward to generalize all the results to other kinds of fields and non-diagonal metric in arbitrary number of spacetime dimensions. Since the signs of the metric components will be crucial, it is important to keep in mind that we work with the metric signature

$$(+ - - - \dots). \tag{6}$$

We start from the identification of $(1+1)$-dimensional spacetime coordinates $x = (x^0, x^1) \equiv (ct, \sigma)$, where c is the speed of light. The action is

$$A = \int dt\, L = \int dt \int d\sigma \tilde{\mathcal{L}}, \tag{7}$$

where

$$\tilde{\mathcal{L}} = \sqrt{|g|}\, \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi, \tag{8}$$

g is the determinant of $g_{\mu\nu}$, and the tilde in $\tilde{\mathcal{L}}$ denotes that it transforms as a scalar density. Introducing the notation

$$\tilde{g}^{\mu\nu} = \sqrt{|g|} g^{\mu\nu}, \tag{9}$$

for the diagonal metric the action reduces to

$$A = \int dt \int d\sigma \frac{1}{2} \left[\frac{\tilde{g}^{00}}{c^2} (\partial_t \phi)^2 + \tilde{g}^{11} (\partial_\sigma \phi)^2 \right]. \tag{10}$$

Now the crucial idea is to replace the continuous spatial derivative $\partial_\sigma \phi$ with a *discrete derivative*. Explicitly,

$$\partial_\sigma \phi = \frac{\partial \phi}{\partial \sigma} \to \frac{\phi(\sigma + l) - \phi(\sigma)}{l} = \frac{\phi_{s+1} - \phi_s}{l}. \tag{11}$$

Here l is some very small (but not zero!) constant of the dimension of length, and the variable σ only takes values on a discrete 1-dimensional lattice (i.e. chain) with the spacing l, so that $\sigma = ls$ with s being an integer label. Thus we have $\int d\sigma \to \sum_s l$, and the action becomes

$$A = \int dt \sum_s \frac{1}{2} \left[\frac{l}{c^2} \tilde{g}_s^{00} (\partial_t \phi_s)^2 + \frac{1}{l} \tilde{g}_s^{11} (\phi_{s+1} - \phi_s)^2 \right]. \tag{12}$$

Next we want the factor in front of $(\partial_t \phi_s)^2$ to have a dimension of mass. Hence we write

$$\frac{l}{c^2} = \frac{l}{mc^2} m, \tag{13}$$

where m is an arbitrary constant of the dimension of mass, so the action can be written as

$$A = \frac{l}{mc^2} \int dt \sum_s \frac{1}{2} \left[m_s (\partial_t \phi_s)^2 - k_s (\phi_{s+1} - \phi_s)^2 \right], \tag{14}$$

where we have defined

$$m_s = \tilde{g}_s^{00} m, \quad k_s = -\tilde{g}_s^{11} k, \tag{15}$$

$$k = \frac{mc^2}{l^2}. \tag{16}$$

Furthermore, in Appendix A we have shown that in 2 dimensions the absolute value of the determinant of $\tilde{g}^{\mu\nu}$ is always equal to one. Hence $-\tilde{g}_s^{00}\tilde{g}_s^{11} = 1$, so

$$m_s k_s = mk \tag{17}$$

is constant. Finally, we absorb the factor in front of the action into a redefinition of the field variables ϕ_s, namely, we define the new field variables

$$q_s = \sqrt{\frac{l}{mc^2}}\phi_s. \tag{18}$$

Thus the action takes the final form

$$A = \int dt \sum_s \left[\frac{m_s(\partial_t q_s)^2}{2} - \frac{k_s(q_{s+1} - q_s)^2}{2} \right]. \tag{19}$$

This action has the form of a chain of harmonic oscillators with masses m_s and couplings k_s.

The results above define the dictionary between the relativistic-field-theory language and the solid-state language. The metric $\tilde{g}^{\mu\nu}(x)$ corresponds to the masses m_s and couplings k_s through the formula (15). The dependence of $\tilde{g}^{\mu\nu}(x)$ on $x^1 = \sigma$ corresponds to the dependence of m_s and k_s on the lattice site s. The dependence of $\tilde{g}^{\mu\nu}(x)$ on the time t corresponds to the dependence of m_s and k_s on time. Thus (19), in general, describes an inhomogeneous lattice of harmonic oscillators in a classical time-dependent background, where the time-dependence is encoded in the time-dependence of masses and couplings.

Finally, let us briefly generalize the results above to a non-diagonal metric in D spacetime dimensions. The continuous $(D-1)$-dimensional space is replaced by a fixed $(D-1)$-dimensional lattice, with a fundamental length l that does not depend on the metric $g_{\mu\nu}$. The label s gets generalized to a discrete $(D-1)$-dimensional vector $\mathbf{s} = \sum_{i=1}^{D-1} s_i \mathbf{e}_i$, where s_i are integers and $\mathbf{e}_i$ are unit vectors in $(D-1)$ lattice directions. The action (19) then generalizes to

$$\begin{aligned} A &= \int dt \sum_{\mathbf{s}} \frac{1}{2} \left[k_{\mathbf{s}}^{00}(\partial_t q_{\mathbf{s}})^2 + \sum_{i=1}^{D-1} 2k_{\mathbf{s}}^{0i}(\partial_t q_{\mathbf{s}})(q_{\mathbf{s}+\mathbf{e}_i} - q_{\mathbf{s}}) \right. \\ &\quad \left. + \sum_{i,j=1}^{D-1} k_{\mathbf{s}}^{ij}(q_{\mathbf{s}+\mathbf{e}_i} - q_{\mathbf{s}})(q_{\mathbf{s}+\mathbf{e}_j} - q_{\mathbf{s}}) \right]. \end{aligned} \tag{20}$$

Here $k_{\mathbf{s}}^{00} = m_{\mathbf{s}} \propto \tilde{g}_{\mathbf{s}}^{00}$, $k_{\mathbf{s}}^{0i} \propto \tilde{g}_{\mathbf{s}}^{0i}$ and $k_{\mathbf{s}}^{ij} \propto \tilde{g}_{\mathbf{s}}^{ij}$. For one spatial dimension we have the identification $k_{\mathbf{s}}^{11} = -k_s$.

2.2 From masses and couplings to background metric

The logic of the previous subsection can also be inverted. Suppose that the fundamental dynamical degrees of freedom are some variables $q_s(t)$, where s is a label taking values from some index set. In principle the index set can be either discrete or continuous. However, a continuous index set leads to an infinite number of degrees of freedom, which may lead to infinite entropy in an arbitrarily small portion of space. Hence we assume that the index set is discrete.

Now suppose that the dynamics of these degrees of freedom has the form (19) (subject to the constraint that $m_s k_s$ is a constant, not depending on s and t), describing the action on the 1-dimensional lattice with the spacing l. From this, one wants to derive the action (7) with (8). First, from m_s and k_s one identifies some natural constants m and k. (This can be done in many ways, for example one can take $m = \lim_{s\to\infty} m_s$ and $k = \lim_{s\to\infty} k_s$.) Then one defines the quantities

$$\tilde{g}_s^{00} = \frac{m_s}{m}, \quad \tilde{g}_s^{11} = \frac{k_s}{k}. \tag{21}$$

The corresponding speed c of propagation is given by (16), namely

$$c^2 = l^2 \frac{k}{m}. \tag{22}$$

In the solid-state language, one thinks of c as the speed of "sound" (rather than light) propagating on the lattice. Taking the continuous limit $l \to 0$ (while keeping c^2 fixed), one arrives at the action of the form (7) with

$$\tilde{\mathcal{L}} = \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{23}$$

The $g^{\mu\nu}$ can be determined from $\tilde{g}^{\mu\nu}$ by the formula given in Appendix A, so (23) can finally be written in the form (8). In this way the theory on a continuous space with metric $g^{\mu\nu}$ is derived as an approximation of the fundamental theory (19) on a discrete space and without the metric $g^{\mu\nu}$.

2.3 General framework

So far we discussed only the real scalar field as dynamical, but the same can be done for any dynamical field. Starting with the known continuous action containing at most first derivatives of the fields, one just deforms the theory by putting the space on the discrete lattice, while keeping the time continuous. This means that the spacetime integral is modified as $\int d^D x \to \int dt \sum_{\mathbf{s}} l^{D-1}$, and any first spatial derivative of a field f is replaced with the corresponding discrete derivative

$$\partial_i f \to \frac{f_{\mathbf{s}+\mathbf{e}_i} - f_{\mathbf{s}}}{l}. \tag{24}$$

If the continuous action contains second derivatives, before applying the recipe above one must first perform partial integrations so that it is reduced to an action containing only first derivatives. For example, the Ricci scalar R contains both first and second derivatives of the metric, but the Einstein-Hilbert action can be written as [35]

$$\int d^4x\sqrt{|g|}R = \int d^4x \left[\sqrt{|g|}\mathcal{L}^{(1)} + \partial_\mu V^\mu\right], \tag{25}$$

where $\mathcal{L}^{(1)}$ contains only first derivatives of the metric. The total derivative can then be discarded, leaving an action containing only first derivatives.

Having said that, in this paper we shall not study in much detail the dynamics of gravity. In most cases, we will be dealing with an approximation in which the metric $g^{\mu\nu}$ is given as a background.

Since the space is discrete and time continuous, the theory treats time and space differently, so Lorentz invariance is broken at small distances comparable to the lattice spacing l. From the point of view of relativity theory, this means that there is a preferred frame of reference that splits spacetime into space and time as separate notions.

How to identify this preferred frame in a given spacetime? We do not have a purely theoretical method to do that, so we take a phenomenological approach. For the preferred frame we take the natural *cosmological* frame, with respect to which the cosmic microwave background (CMB) [36] looks homogeneous and isotropic at large scales. Perhaps one could also conceive a different choice of the preferred frame, but since the preferred frame must, in principle, be defined for the whole Universe, the cosmological preferred frame of reference seems to be the most natural one.

3 Schwarzschild black hole

3.1 General properties

Consider a black hole of mass M that reached a stationary equilibrium state, and suppose that its velocity with respect to the preferred cosmological frame is not too large (compared to the speed of light). Thus the black hole can approximately be treated as being at rest with respect to the preferred frame, and due to the equilibrium its properties do not depend on the fundamental time t. The gravitational effect of the black hole vanishes at large distances, so the metric at large distances in the preferred frame takes the form of Minkowski metric. All this means that the preferred frame for the description of the black hole is given by the Schwarzschild coordinates, in which the metric (in units $G_N = c = 1$) takes the usual form

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1-\frac{2M}{r}} - r^2 d\Omega^2. \tag{26}$$

From the GR geometric point of view [37, 38, 39, 40], this black hole could equivalently be described in any other coordinates, like the Kruskal coordinates in which there is no coordinate singularity at $r = 2M$. In other coordinates the metric may become time dependent and not approach Minkowski metric at large distances. However, the crucial point here that cannot be overemphasized, is that this is totally different in the fundamental theory which is not based on the principle of relativity and general covariance. In the fundamental theory, the Schwarzschild coordinates are *the* right coordinates, in which t represents the fundamental time *everywhere*, including the black hole interior. This is consistent with the assumption that the black hole reached the equilibrium, so is t-independent everywhere. Likewise, from the fundamental point of view, the radial coordinate r represents a spatial coordinate *everywhere*, including the interior. All the radical deviations from GR will follow from such an interpretation of the coordinates t and r in (26).

This point is so important that we want to explain it once again, because our whole theory cannot be understood properly if that point is not understood thoroughly. In GR the difference between space and time is defined *dynamically*, in the sense that it depends on the signs of the dynamical field $g_{\mu\nu}(x)$. In particular, the first term in (26) proportional to dt^2 becomes negative at $r < 2M$, meaning that, according to GR, t denotes a spacelike coordinate in the black hole interior. This makes GR different from all other theories in physics. By contrast, we interpret the field $g_{\mu\nu}(x)$ as any other dynamical field in physics, and insist that, at the fundamental level, the notion of time must be defined before the specification of dynamics. Thus the fact that the dynamical field $g_{00}(r)$ becomes negative for $r < 2M$ does not affect the fact that t, at the fundamental *kinematic* level, represents time.

Once this radical reinterpretation of $g_{\mu\nu}(x)$ is accepted, the rest of the analysis is straightforward. For simplicity, in most considerations we ignore the angular part $d\Omega^2$ in (26), so effectively we deal with the 2-dimensional spacetime with the metric

$$g_{00} = 1 - \frac{2M}{r}, \quad g_{rr} = -\frac{1}{1 - \frac{2M}{r}}. \tag{27}$$

Thus $\sqrt{|g|} = 1$, so the masses and couplings (15) in the action (19) are

$$m_s = \frac{m}{1 - \frac{2M}{ls}}, \quad k_s = \left(1 - \frac{2M}{ls}\right) k, \tag{28}$$

where we have used $r = ls$. We see that for $r < 2M$, the masses and couplings are negative. The Hamiltonian associated with the action (19) is

$$H(q,p) = \sum_s \left[\frac{p_s^2}{2m_s} + k_s \frac{(q_{s+1} - q_s)^2}{2} \right], \tag{29}$$

which is positive for $r > 2M$ and negative for $r < 2M$.

Note that, even though a negative Hamiltonian implies negative energies, the strictly negative Hamiltonian does not lead to any instabilities. That is because the strictly negative Hamiltonian is bounded in very much the same way as a strictly positive one, except that one is bounded from above and the other from below. In fact, a negative Hamiltonian H leads to exactly the same equations of motions as the positive one $\bar{H} = -H$. The quickest way to see this is to note that if H is derived from the Lagrangian L, then $\bar{H}$ is derived from the Lagrangian $\bar{L} = -L$, so the extrema of the corresponding actions are the same. Another proof that the two Hamiltonians give the same equations of motion is presented in Appendix B. Also note that m_s and k_s do not depend on time t, so the energy defined by the Hamiltonian (29) is conserved.

Since L and $\bar{L} = -L$, or equivalently H and $\bar{H} = -H$, describe the same physics, in the black hole interior it is more convenient to work with $\bar{L}$ and $\bar{H}$. The corresponding masses $\bar{m}_s = -m_s$ and couplings $\bar{k}_s = -k_s$ are positive in the interior, which corresponds to the picture the physicists are used to.

An interesting issue is what happens at the singularity at $r = 0$. There we have $\bar{m}_0 = -m_0 = 0$, $\bar{k}_0 = -k_0 = \infty$. The harmonic oscillators oscillate wildly, with the infinite local frequency $\omega_0 = \sqrt{\bar{k}_0/\bar{m}_0}$. Physically this is probably wrong, it is expected that a more fundamental theory should modify the theory close to the singularity. Our interpretation of gravity in terms of harmonic oscillators offers a new intuitive picture, and such intuition may guide new ideas how to modify physics close to the singularity. Just for the sake of illustration, one intuitive idea is that $r = 0$ might represent a "wall" from which waves propagating through the lattice reflect, so we need a reflecting boundary condition at $r = 0$. Such a boundary condition can be achieved by postulating $\bar{m}_0 = \infty$, $\bar{k}_0 =$ finite. But at the present level of understanding that is only a speculation, we cannot make any definite claims about the physics at the singularity, so in this paper we shall not consider this issue any further.

A more interesting place to consider, about which we can make claims with a much higher level of confidence, is the horizon at $r = 2M$. Close to the horizon we have

$$m_s \to \pm\infty, \quad k_s \to \pm 0, \tag{30}$$

where the plus (minus) sign refers to the black hole exterior (interior). The vanishing couplings at the horizon imply that lattice vibrations cannot propagate across the horizon. Likewise, the infinite masses imply that the oscillators have infinite resistance to any change of their target-space "velocity" $\partial_t q_s(t)$. This means that any information carried by lattice vibrations cannot propagate from one side to the other. Not only that information carried by lattice vibrations cannot escape from the black hole, but it also cannot fall *into* the black hole.

The result that information cannot fall into the black hole is compatible with the standard view of relativity from the point of view of observer far from the black hole. Indeed, this is not a coincidence, because the fundamental time t coincides with the time t of the observer far from the horizon. But what about a freely falling observer? In the standard theory of relativity, a freely falling observer approaches the horizon after a finite proper time of the observer. Is this picture valid also in our modified theory? Does a freely falling observer observes anything out of the ordinary as she approaches the horizon?

This indeed is where our theory differs dramatically from the standard relativity theory. A short answer is that *even a freely falling observer observes a drama at the horizon.* Namely, even though the fundamental distance l is very small, from the point of view of the freely falling observer the effect of non-zero l enhances dramatically. The effect which does not exist in the strict limit $l \to 0$, becomes a large effect when

l is small but non-zero. We shall see that in more detail in Sec. 4.

Another dramatic difference with respect to the standard relativity theory occurs in the black hole interior. In GR, r represents a timelike coordinate in the black hole interior. More precisely, the future direction for the *black* hole corresponds to a *decreasing* r (for the *white* hole it is the opposite), so any propagation forward in time implies propagation with decreasing r. That is why, according to GR, any propagation forward in time inside the black hole inevitably ends up in the singularity at $r = 0$. But this is not so in the fundamental theory. There r is a space coordinate, while time is t, so propagation forward in time does not prevent propagation with increasing r. From the point of view of GR, a propagation with increasing r corresponds to propagation backwards in time, which is considered unphysical. But in the fundamental theory such a propagation is perfectly physical, implying that propagations do not necessarily end up in the singularity.

3.2 Stability: The Higgs potential

So far we have discussed only the massless field in (8). As we shall see shortly, additional nontrivial issues arise in the black hole interior if we add also a Higgs potential

$$V(\phi) = -\frac{b_2}{2}\phi^2 + \frac{b_4}{4}\phi^4 + \mathcal{O}(\phi^6), \tag{31}$$

where b_2 and b_4 are positive constants, so that the action in D dimensions is

$$A = \int d^D x \sqrt{|g|} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \tag{32}$$

Then, in the 2-dimensional black hole background, the Hamiltonian (29) generalizes to

$$H(q,p) = \sum_s \left[\frac{p_s^2}{2m_s} + k_s \frac{(q_{s+1} - q_s)^2}{2} + U(q_s) \right], \tag{33}$$

where

$$U(q_s) = -\frac{\kappa_2}{2} q_s^2 + \frac{\kappa_4}{4} q_s^4 + \mathcal{O}(q_s^6), \tag{34}$$

with $\kappa_2 = mc^2 b_2$, $\kappa_4 = (m^2 c^4/l) b_4$. In the interior m_s and k_s are negative, so it is more convenient to work with the Hamiltonian $\bar{H} = -H$ (see Appendix B), leading to

$$\bar{H}(q,\bar{p}) = \sum_s \left[\frac{\bar{p}_s^2}{2\bar{m}_s} + \bar{k}_s \frac{(q_{s+1} - q_s)^2}{2} + \bar{U}(q_s) \right]. \tag{35}$$

with $\bar{m}_s = -m_s$, $\bar{k}_s = -k_s$ being positive, and $\bar{U}(q_s) = -U(q_s)$.

The upshot of this calculation is that the relevant potential in the interior is $\bar{U}$, which is the *negative* of the potential U in the exterior. If $U(q_s)$ is not bounded from above (as it is not if the terms $\mathcal{O}(q_s^6)$ in (34) are absent), then $\bar{U}(q_s)$ is not bounded from below, making the dynamics in the interior unstable. But we require

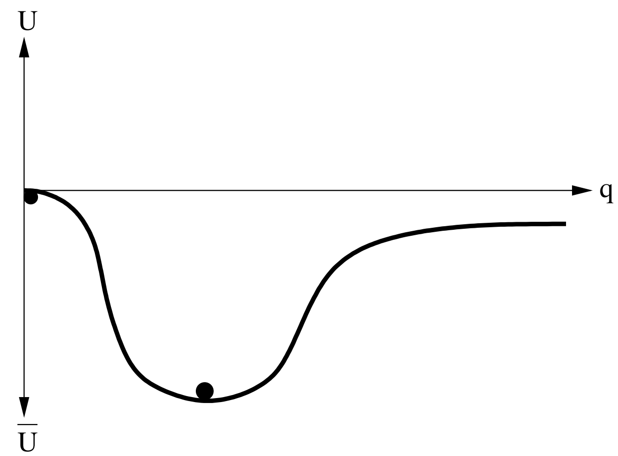

Figure 2: The Higgs potential $U(q)$ and its negative $\bar{U}(q) = -U(q)$. The $U(q)$ has the minimum at $q \neq 0$, while the $\bar{U}(q)$ has the minimum at $q = 0$. Both $U(q)$ and $\bar{U}(q)$ are bounded from below.

that physics should be stable in both exterior and interior, which implies that both U and $\bar{U}$ should be bounded from below. That is only possible if $U(q_s)$ has the property

$$\lim_{q_s \to \infty} U(q_s) = \text{finite}, \tag{36}$$

that is, if the potential is flat for large q_s. An example of such a potential is sketched at Fig. 2. A minimum of $U(q_s)$ is a maximum of $\bar{U}(q_s)$, and *vice versa*. The minimum of U at Fig. 2 corresponds to the vacuum value v of the Higgs field in the Standard Model of particle physics [41], which determines the masses of $W^{\pm}$ and Z^0 bosons by the Higgs mechanism, as well as the masses of quarks and leptons by the Yukawa couplings. In the interior, however, this value v corresponds to the maximum of $\bar{U}$, so it does not correspond to a vacuum value. This means that masses of elementary particles in the black hole interior are very different from those in the exterior, implying that whole of physics is very different. In particular, at Fig. 2 we see that $\bar{U}$ has a minimum corresponding to the vacuum value $v' = 0$, implying that *all elementary particles of the Standard Model have zero mass.* In particular, if the electron has zero mass, then stable atoms are impossible, so there is no chemistry, and no biological observer can survive in the black hole interior. Needless to say, that is radically different from GR which (if the black hole is sufficiently large so that the tidal forces can be neglected) predicts that nothing out of the ordinary should happen to observers after they cross the horizon and enter the black hole interior.

A slightly less dramatic modification of particle physics in the interior would take place if the potential $\bar{U}$ had a minimum at some non-zero value $v' \neq 0$. However, then the mass terms of the $W^{\pm}$ and Z^0 bosons [41] would behave like unstable "tachyons", so the requirement that physics in the interior should be stable suggests that v' should be zero in the interior, as in Fig. 2. Thus it seems very plausible that all elementary particles of the Standard Model acquire zero mass in the interior, as described above.

To conclude, already from purely classical considerations we have seen that physics in the black hole interior is radically different from that in the exterior. In the quantum case, we shall see in Sec. 5 that the difference is even more radical.

3.3 Stability: The angular part

So far we have been ignoring the angular part of the metric (26), now we handle this as well. In the continuum limit, the standard Hamiltonian density for the massless field ϕ can be written as

$$\tilde{\mathcal{H}} = \frac{\tilde{g}^{00}(\partial_0\phi)^2}{2} + \frac{-\tilde{g}^{ij}\partial_i\phi\partial_j\phi}{2}, \tag{37}$$

where $\tilde{g}^{ij}$ is diagonal with components $\tilde{g}^{rr}$, $\tilde{g}^{\theta\theta}$ and $\tilde{g}^{\varphi\varphi}$. In the black hole interior, the dynamics can equivalently be described by its negative

$$\overline{\tilde{\mathcal{H}}} = -\tilde{\mathcal{H}} = \frac{-\tilde{g}^{00}(\partial_0\phi)^2}{2} + \frac{\tilde{g}^{ij}\partial_i\phi\partial_j\phi}{2}, \tag{38}$$

which is convenient because $-\tilde{g}^{00}$ and $\tilde{g}^{rr}$ are positive in the interior. The problem, however, is that the angular parts $\tilde{g}^{\theta\theta}$ and $\tilde{g}^{\varphi\varphi}$ are negative, thus making the Hamiltonian $\overline{\tilde{\mathcal{H}}}$ unbounded from below, which makes the dynamics unstable.

The problem can be removed in a way similar to the removal of the problem with the Higgs potential in the previous subsection. For that purpose we introduce the notation

$$\tilde{\mathcal{S}} \equiv -\frac{\tilde{g}^{ij}\partial_i\phi\partial_j\phi}{2}, \tag{39}$$

and generalize (37) to

$$\tilde{\mathcal{H}} = \frac{\tilde{g}^{00}(\partial_0\phi)^2}{2} + W(\tilde{\mathcal{S}}), \tag{40}$$

where $W(\tilde{\mathcal{S}})$ is a function which is bounded from both above and below, and for small $\tilde{\mathcal{S}}$ behaves as $W(\tilde{\mathcal{S}}) = \tilde{\mathcal{S}} + \mathcal{O}(\tilde{\mathcal{S}}^2)$. The boundedness implies that $\lim_{\tilde{\mathcal{S}}\to\infty} W(\tilde{\mathcal{S}}) =$ finite, similarly to (36) for the Higgs potential.

Note that $W(\tilde{\mathcal{S}})$ in (40) is compatible with local Lorentz invariance only if $W(\tilde{\mathcal{S}}) = \tilde{\mathcal{S}}$, i.e., the generalization (40) is not allowed in Lorentz invariant theories. Thus the generalization (40) violates Lorentz invariance already in the continuum limit. Nevertheless, according to our general prescription in Sec. 2.3, the continuum derivatives in $W(\tilde{\mathcal{S}})$ should be replaced by the discrete ones.

4 Effective field theory

4.1 General prescription

The effects of the discrete space with the fundamental length l can also be described by an effective field theory in the continuum. The idea is to start from the action of any field theory in the continuum, such as

$$A = \int d^D x \sqrt{|g|} \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{41}$$

and replace all the continuous spatial derivatives ∂_i with the corresponding discrete derivatives. For instance, the action (41) gets replaced by the effective action

$$A_{\text{eff}} = \int d^D x \sqrt{|g|} \frac{1}{2} g^{\mu\nu} \hat{d}_\mu \phi \hat{d}_\nu \phi, \tag{42}$$

where $\hat{d}_0 \equiv \partial_0$, $\hat{d}_i \equiv d_i$, for $i = 1, \ldots, D-1$ and d_i is the discrete derivative.

To define the discrete derivative d_i itself, let us consider one spatial coordinate, say x^1, and call it y. The discrete derivative d_y of an arbitrary function $f(y)$ can be defined symmetrically as

$$d_y f(y) = \frac{f(y + l/2) - f(y - l/2)}{l}. \tag{43}$$

From the Taylor expansion

$$f(y + l/2) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{l}{2} \right)^n \partial_y^n f(y), \tag{44}$$

and similarly for $f(y - l/2)$, after a straightforward algebra we find

$$d_y f(y) = \sum_{n=0}^{\infty} \frac{l^{2n}}{2^{2n}(2n+1)!} \partial_y^{2n+1} f(y). \tag{45}$$

Isolating the $n = 0$ term, this can also be written as

$$d_y f(y) = \partial_y f + \mathcal{D}_y f, \tag{46}$$

where

$$\mathcal{D}_y f = \sum_{n=1}^{\infty} \frac{l^{2n}}{2^{2n}(2n+1)!} \partial_y^{2n+1} f(y) \tag{47}$$

is the deviation from the ordinary continuous derivative. For practical purposes, one may only be interested in the lowest nontrivial order in the expansion in l, namely

$$d_y f(y) = \partial_y f + \frac{l^2}{24} \partial_y^3 f + \mathcal{O}(l^4). \tag{48}$$

Note that the recipe of replacing the ordinary derivative with the discrete one is prescribed on the level of action. The equations of motion then need to be derived by the usual methods from the effective action, and, in general, the resulting equations of motion do *not* reduce to ordinary equations of motion with ordinary derivatives merely replaced by the discrete ones. The discrete derivative does not obey the Leibniz rule, i.e. $d_i(fh) \neq (d_i f)h + f(d_i h)$, and the derivation of the equations of motion is non-trivial.

In the rest of the paper we will mainly be interested in a diagonal metric in $D = 2$ dimensions. Hence, applying the results above to the action (42), in the lowest order in l we find

$$A_{\text{eff}} = \int d^2 x \sqrt{|g|} \frac{1}{2} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{l^2}{12} g^{11} (\partial_1 \phi)(\partial_1^3 \phi) \right]. \tag{49}$$

4.2 Why relativity works?

At wave lengths which are large comparad to inter-atomic distances, a sound wave obeys the wave equation

$$\frac{1}{c_s^2}\frac{\partial^2\phi}{\partial t^2} - \boldsymbol{\nabla}^2\phi = 0, \qquad (50)$$

where c_s is the speed of sound. This takes the same form as the wave equation for the propagation of light, except that the latter involves the speed of light c instead of c_s. Just as the light wave equation is Lorentz invariant with respect to Lorentz transformations involving the constant c, the sound wave equation is Lorentz invariant with respect to Lorentz transformations involving the constant c_s.

Nevertheless there is a difference. When an observer moves with the velocity $v < c$ with respect to the source of light, she observes that the light always propagates with the same velocity c. And yet, when an observer moves with the velocity $v < c_s$ with respect to the source of sound, she does *not* observe that the sound always propagates with the same velocity c_s. Somehow, the relativity principle works for the measurement of light, but an analogous relativity principle does not work for the measurement of sound. Why is that? Where does the difference come from?

The difference does not stem from the difference between light and sound. Instead, the difference arises from properties of the *measuring apparatuses*. The measuring apparatuses are themselves physical systems. When we measure light, we do it with an apparatus made of matter obeying the laws which are themselves Lorentz invariant with respect to c. By contrast, when we measure sound, the apparatus is not made of matter obeying the laws which are Lorentz invariant with respect to c_s. Instead, the latter apparatus also obeys the laws which are Lorentz invariant with respect to c.

The moral is that relativity works because the measuring apparatuses, like clocks, rulers, particle detectors, etc., are all made of matter that obeys the laws of physics which are Lorentz invariant in the usual sense, namely, with respect to c.

But we have seen that this Lorentz invariance is only approximately valid. To some extent the measuring apparatuses themselves violate Lorentz invariance and, in principle, it should be possible to see that in experiments. In principle, to understand the effects of this, one should model the workings of various experimental apparatuses as described by the Lorentz violating laws of physics. In practice, however, that would be very complicated. Hence we apply a simpler approach. We use an approximation in which the apparatus obeys Lorentz invariant laws and study how such an idealized apparatus responds to a system obeying Lorentz non-invariant laws. For simplicity and definiteness, we study how such an idealized apparatus sees the effective action (49). We are particularly interested in cases where the deviations from the predictions of standard relativity theory are large. In such cases the lowest order approximation cannot be trusted, so the results obtained from (49) cannot be interpreted as definite predictions. Nevertheless, such large deviations clearly indicate that the predictions of the standard relativity theory are wrong in such regimes. When such a large deviation from the prediction of standard relativity theory occurs, we refer to it as a *drama*.

In the following subsections within this section we study concrete examples of such dramas.

4.3 Drama at high velocity

Consider a flat Minkowski background metric so that the action (49) simplifies to

$$A_{\rm eff} = \int dt dx \frac{1}{2} \left[(\partial_t \phi)^2 - (\partial_x \phi)^2 - \frac{l^2}{12} (\partial_x \phi)(\partial_x^3 \phi) \right], \tag{51}$$

where we work in units $c = 1$. Suppose that the ideal measuring apparatus moves with the constant velocity $-v$ in the x-direction, so that the space and time as seen by the apparatus are described by the coordinates x',t', related to x,t by the Lorentz transformation

$$x' = \gamma(x + vt), \quad t' = \gamma(t + vx), \tag{52}$$

where $\gamma = 1/\sqrt{1 - v^2}$. We have

$$\begin{aligned} \partial_t &= \frac{\partial t'}{\partial t} \partial'_t + \frac{\partial x'}{\partial t} \partial'_x = \gamma(\partial'_t + v\partial'_x), \\ \partial_x &= \frac{\partial x'}{\partial x} \partial'_x + \frac{\partial t'}{\partial x} \partial'_t = \gamma(\partial'_x + v\partial'_t), \end{aligned} \tag{53}$$

where $\partial'_t \equiv \partial/\partial t'$, $\partial'_x \equiv \partial/\partial x'$, so the square bracket in (51) is

$$\begin{aligned} &(\partial_t \phi)^2 - (\partial_x \phi)^2 - \frac{l^2}{12} (\partial_x \phi)(\partial_x^3 \phi) \\ &= \gamma^2 (1 - v^2) [(\partial'_t \phi)^2 - (\partial'_x \phi)^2] - \frac{l^2}{12} \gamma^4 [(\partial'_x + v\partial'_t)\phi][(\partial'_x + v\partial'_t)^3 \phi] \\ &= (\partial'_t \phi)^2 - (\partial'_x \phi)^2 - \frac{\gamma^4 l^2}{12} [(\partial'_x + v\partial'_t)\phi][(\partial'_x + v\partial'_t)^3 \phi]. \end{aligned} \tag{54}$$

The Lorentz invariant combination $(\partial_t \phi)^2 - (\partial_x \phi)^2$, of course, takes in the primed coordinates the same form as in the non-primed coordinates. But the Lorentz non-invariant term proportional to l^2 takes a different form. The most striking feature is that the non-invariant term in primed coordinates gets proportional to

$$\gamma^4 l^2 = \frac{l^2}{(1 - v^2)^2}. \tag{55}$$

Thus, even though l is small, the factor $\gamma^4 l^2$ becomes arbitrarily large when $v \to 1$. In other words, the small Lorentz violation proportional to l^2 gets radically enhanced when the velocity v of the apparatus approaches the speed of light. This indicates a large deviation from the relativity theory at high velocities of the observer, namely a drama in a sense explained in Sec. 4.2.

The drama at high velocities can also be understood qualitatively and intuitively in the following way. From the point of view of the observer at rest, the apparatus at high velocities gets Lorentz contracted. At sufficiently large velocities it gets contracted so much that its size becomes comparable to the fundamental distance l, meaning that the apparatus can probe the discrete nature of space and hence see Lorentz violation as a large effect. In principle, such an effect could be tested experimentally.

Also note that the mere fact that something moves with a velocity close to the speed of light does not, by itself, imply a drama. For instance, light propagates at the speed of light, but from the point of view of a slow observer (relative to the preferred frame), there is no any drama as long as the wave length of light is much larger than l. To see a drama, a probe needs to be so energetic that its wave length becomes comparable to l.

4.4 Drama at Rindler horizon

The drama at high velocities can also be seen by an apparatus with a constant proper acceleration a. The coordinates associated with the accelerated apparatus are the Rindler coordinates [42, 38]

$$x' = \sqrt{x^2 - t^2}, \quad t' = \frac{1}{a}\text{arcth}\frac{t}{x}, \tag{56}$$

describing the accelerating apparatus with the trajectory $x' = 1/a$. The inverse coordinate transformations are

$$x = x'\text{ch}(at'), \quad t = x'\text{sh}(at'). \tag{57}$$

The Minkowski line element

$$ds^2 = dt^2 - dx^2 \tag{58}$$

expressed in the primed coordinates is

$$ds^2 = a^2 x'^2 dt'^2 - dx'^2. \tag{59}$$

Now we want to explore the behavior of the Lorentz violating term in (51). For that purpose we need

$$\partial_x = \frac{\partial x'}{\partial x}\partial'_x + \frac{\partial t'}{\partial x}\partial'_t = \frac{x}{\sqrt{x^2 - t^2}}\partial'_x - \frac{t/(ax)}{\sqrt{x^2 - t^2}}\partial'_t, \tag{60}$$

so we see that the Lorentz violating term is divergent at the horizon $x^2 = t^2$. The lowest divergent term is proportional to

$$\frac{l^2}{(x^2 - t^2)^2}, \tag{61}$$

which is similar to (55), but there are also more divergent terms coming from derivatives of $1/\sqrt{x^2 - t^2}$. This divergence corresponds to a drama at the Rindler horizon.

4.5 Drama at Schwarzschild horizon

Close to the Schwarzschild horizon at $r = 2M$, the Schwarzschild line element (26) (without the angular part) can be approximated with

$$ds^2 = \frac{r-2M}{2M}dt^2 - \frac{2M}{r-2M}dr^2. \tag{62}$$

Introducing the tortoise coordinate

$$dr_* = \sqrt{\frac{2M}{r-2M}}dr \;\Rightarrow\; r_* = 4M\sqrt{\frac{r-2M}{2M}}, \tag{63}$$

(62) can be written as

$$ds^2 = a^2 r_*^2 dt^2 - dr_*^2, \tag{64}$$

where $a = 1/(4M)$. The line element (64) has the same form as the Rindler line element (59). Hence, by analogy with the Rindler case, we can introduce new coordinates r', t' in which the line element, close to the horizon at $r = 2M$, takes the Minkowski form

$$ds^2 = dt'^2 - dr'^2. \tag{65}$$

The corresponding coordinate transformations are

$$r' = r_*(r)\mathrm{ch}(at), \quad t' = r_*(r)\mathrm{sh}(at). \tag{66}$$

In GR, the coordinates t',r' describe what is seen by a freely falling observer close to the horizon.

The analogy above between Rindler and Schwarzschild horizon is well known in standard GR. However, from our fundamental point view, the two horizons are *not* completely analogous. In flat Minkowski background the preferred frame of coordinates x,t is the one in which the metric takes the Minkowski form (58), in which there is *no* coordinate singularity at the Rindler horizon. By contrast, in the Schwarzschild black hole background, the preferred coordinates r,t are the Schwarzschild coordinates in which there *is* a coordinate singularity at the Schwarzschild horizon. This makes the Rindler and the Schwarzschild horizon very different objects in the fundamental Lorentz violating theory.

Remarkably, due to this fundamental *dis*analogy between the two kinds of horizons, in the Schwarzschild case even a *freely falling* observer sees a drama close to the horizon, as we will now show. (A somewhat similar drama close to the horizon has also been anticipated by the firewall proposal [11, 12], but our theoretical explanation of the drama is very different.) The Lorentz violating term in the preferred coordinates is

$$\frac{l^2}{12}g^{rr}(\partial_r\phi)(\partial_r^3\phi) = -\frac{l^2}{12}\left(1-\frac{2M}{r}\right)(\partial_r\phi)(\partial_r^3\phi), \tag{67}$$

so we need the derivative

$$\partial_r = \frac{\partial r'}{\partial r}\partial'_r + \frac{\partial t'}{\partial r}\partial'_t. \tag{68}$$

From the explicit coordinate transformations (66) and (63) we have

$$\frac{\partial r'}{\partial r} = \frac{\partial r_*}{\partial r}\text{ch}(at) = \sqrt{\frac{2M}{r-2M}}\text{ch}(at),$$
$$\frac{\partial t'}{\partial r} = \frac{\partial r_*}{\partial r}\text{sh}(at) = \sqrt{\frac{2M}{r-2M}}\text{sh}(at), \tag{69}$$

so

$$\partial_r = \sqrt{\frac{2M}{r-2M}}[\text{ch}(at)\partial'_r + \text{sh}(at)\partial'_t]. \tag{70}$$

Hence the Lorentz violating term (67) is divergent at $r = 2M$. The lowest divergent term is proportional to

$$l^2 \frac{2M}{r-2M} = \frac{l^2}{1-\frac{2M}{r}}, \tag{71}$$

but there are also more divergent terms coming from derivatives of $1/\sqrt{r-2M}$.

5 Quantization

So far we discussed only the classical theory, now we turn to quantization. Since the theory at the classical level has a form of non-relativistic mechanics, with absolute time and space, and with a discrete set of degrees of freedom, the quantization of the theory is, in principle, straightforward. From the technical point of view quantization reduces to well known principles of non-relativistic QM, of which we do not have much new to say. For that reason, this section will be rather sketchy. The devil, of course, is in the details, but in this work we shall not be concerned with the devil; the details will be left to the future work. Nevertheless, our reformulation of relativity theory leads to some conceptual novelties in quantum theory, and in this section we shall briefly discuss those conceptual novelties.

5.1 QFT in flat spacetime

Consider, for simplicity, a real scalar field in $(1+1)$-spacetime dimensions. The generalization to more dimensions and other kinds of fields will, in principle, be straightforward, at least at the conceptual level. On the discrete space, the quantization is based on the canonical equal-time commutation relation

$$[q_s(t), p_{s'}(t)] = i\hbar\delta_{ss'}. \tag{72}$$

In the continuum limit $l \to 0$ this corresponds to

$$[\phi(t,\sigma), \pi(t,\sigma')] = i\hbar\delta(\sigma-\sigma'), \tag{73}$$

where σ is the continuous spatial coordinate. In the continuum field theory one encounters UV divergences, which must be regularized somehow to renormalize the theory and obtain meaningful results. The quantization in discrete space based on (72) contains such a UV regularization from the very start.

There are also the IR divergences, which are an artifact of making computations in an infinitely large space, and with an infinite time. The infinite time arises from computation of the S-matrix [43], which describes transitions from $t = -\infty$ to $t = \infty$. By considering the system in a more realistic setting, in a finite volume of space during a finite time, the computations become more complicated from the technical point of view, but from the conceptual point of view the IR divergences go away.

The regularization of UV divergences based on the discrete space violates Lorentz invariance. Nevertheless, since l, by assumption, is very small, the effects of such Lorentz violation should be negligible for effects probed with currently existing experiments, such as particle physics experiments at LHC. To probe new physics and distances comparable to l one would need accelerators that accelerate particles to much larger energies than possible with currently existing experimental technologies.

Another possibility to probe new physics at small l would be to have the laboratory that moves with a high velocity, close to the speed of light in the preferred frame of reference, which presumably is the CMB cosmological frame. However, since Earth is not that fast relative to the cosmological CMB frame, it would be very difficult to see such effects in practice.

To conclude, our reformulation of the relativity theory does not have much practical consequences for QFT in flat spacetime and its applications in particle physics, at least with current technologies. From the theoretical point of view, this reformulation of relativity can be thought of as one method of regularization of UV divergences, which perhaps is not a sufficient reason to be particularly excited about.

5.2 QFT in curved spacetime

When the spacetime metric deviates significantly from the flat Minkowski metric, the differences between the standard QFT in curved spacetime [44, 45] and our theory can be significant.

For a warm up, let us start with the Unruh effect [44], dealing with a uniformly accelerating detector in flat spacetime. It is nontrivial because the metric in the accelerated frame deviates significantly from the Minkowski metric, see Sec. 4.4. In the standard approach, the quantum effects of acceleration are described by the so-called Rindler quantization [44], based on taking the time seen by the accelerated observer as the time with respect to which the quantization is performed. The Rindler frame of reference is very different from the preferred frame of reference identified with a Minkowski frame of reference. For instance, the Rindler frame does not cover the space behind the horizon. Moreover, even if we cover the space behind the horizon with another set of coordinates, we have seen in Sec. 4.4 that, close to the Rindler horizon, even classical physics with a Lorentz violating term deviates significantly from the standard relativity theory. In the quantum case, the quantization of the Lorentz violating theory should be performed in the preferred frame, which is a Minkowski frame, and from this point of view Rindler quantization is completely wrong. Nevertheless, it does not mean that the Unruh effect does not exist. The Unruh effect can also be explained by studying a response of an accelerating detector with quantization performed in the Minkowski frame, which leads to predictions [44] very similar to those obtained with Rindler quantization, up to some conceptual differences [46]. The approach to Unruh effect based on Minkowski quantization is essentially correct from the point of view of the fundamental Lorentz violating theory, except that one has to encounter the additional effects due to small l.

Let us now turn to a discussion of Hawking radiation [44] from the black hole. In the standard approach, the particles far from the horizon are based on quantization in the frame that coincides with the preferred frame in the Lorentz violating theory. Thus, we do not expect significant differences between the two theories when applied to the properties of radiation far from the horizon. Indeed, models based on analogue black holes in condensed matter physics with an ultraviolet cutoff show that the thermal nature of radiation far from the horizon is not influenced much by the discreteness of space close to the horizon [47].

Nevertheless, in our theory with the preferred frame, the effects of discreteness are seen by observers close to the horizon. As we have seen in the classical setting in Sec. 4.5, even a freely falling observer observes significant deviations from GR close to the black hole horizon.

The deviations from GR are even more radical in the black hole interior, as we have seen in the classical setting in Sec. 3. In the quantum case, this deviation translates also to the quantum theory, making the difference even more interesting. The quantization in the fundamental theory is again based on the discrete formula (72), which in the continuum limit reduces to (73). But inside the black hole this becomes

$$[\phi(t,r), \pi(t,r')] = i\hbar\delta(r-r'), \tag{74}$$

where t is the Schwarzschild time and r is the Schwarzschild radial coordinate. From the point of view of GR, t is a spacelike coordinate and r a timelike coordinate, so (74) looks like an equal-space commutator, rather than an equal-time commutator. This totally inverts the roles of space and time in the interior and makes the theory radically nonlocal and acausal. And yet, (74) is a perfectly natural consequence of the assumption that the fundamental commutation relations should by pre-dynamical, described by a purely kinematic principle postulated before knowing the solutions of the equations of motion. Since the field $g_{\mu\nu}(x)$ is dynamical in GR, the fundamental equal-time commutation relations should not depend on it. From the point of view of the fundamental theory, there is nothing strange or peculiar with the commutation relation (74). Instead, the peculiar circumstance is that g_{00} becomes negative and g_{rr} positive in the black hole interior, meaning that the masses and couplings of the fundamental degrees of freedom become negative, as explained in Sec. 3.

Now equipped with this understanding of black holes from the fundamental point of view, we can understand how the black hole information paradox [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] resolves in a rather simple and natural way. In this paper we shall not attempt to solve explicitly the equations with the discrete space. From the technical point of view, in Appendix C we present in detail a toy model of Hawking radiation with only two degrees of freedom, giving a gist of some essential properties of Hawking radiation from the point of view of the fundamental theory. But to understand the essence of solution of the information paradox, no detailed calculations are really necessary.

The key of the solution is that, at the fundamental level, the information is stored in the fundamental degrees of freedom. Those degrees form a fixed lattice, and the distance between the neighboring degrees is l, independent of the dynamical metric $g_{\mu\nu}(x)$, as illustrated by Fig. 1 in Introduction. The number of degrees of freedom in the black hole is proportional to its kinematic volume $4R^3\pi/3$, where $R = 2M$ is the Schwarzschild radius. So when the black hole shrinks during the evaporation, the degrees of freedom that were close to the horizon in the interior, after a shrinking become a part of the exterior, as illustrated in Fig. 1. From the point of view of GR, such an escape of degrees of freedom from the inside to the outside looks like an acausal motion backwards in time. Yet, from the fundamental point of view, the degrees of freedom do not move at all. What moves is the horizon, which in the fundamental picture is a place where the pre-relativistic masses of the fundamental degrees of freedom are infinite. So when those fundamental degrees of freedom escape from the black hole by this mechanism, they also carry some information with them. In essence, this is how information escapes from the black hole during the evaporation.

Of course, this picture of information escape lacks some details, even at the purely conceptual level. First, once the information escapes, how can this information be described from the GR point of view? What carries this information? Since the escape mechanism is essentially classical, the field configuration that carries the information is classical too. In the quantum language, this means that the field is in a quasi-classical coherent state, and information is stored in the quantum entanglement between Hawking radiation and those quasi-classical coherent states. This implies that Hawking radiation is accompanied with an additional quasi-classical radiation, similar to the classical radiation created during the classical black hole collapse by which black hole looses classical hair [37, 48]. Interestingly, the result that Hawking radiation is accompanied with an additional quasi-classical radiation has also been obtained by very different arguments in [49].

Second, why does black hole shrinks from the fundamental point of view? Since the Hamiltonian is negative in the interior, this means that the interior Hawking quanta (which are entangled with the outside Hawking particles) have negative energy, which provides total energy conservation. From the GR point of view, this corresponds to the flux of negative energy propagating towards the black hole from the outside, which is a known result in standard QFT in curved spacetime [44]. According to the fundamental theory, standard QFT in curved spacetime should be trusted as an effective theory, as long as one applies it outside of the black hole and not too close to the horizon. The shrinking is then result of back-reaction of the decrease of black hole energy onto the gravitational field $g_{\mu\nu}(x)$, for which $g_{\mu\nu}(x)$ must be treated dynamically.

5.3 Quantum gravity

When one applies the general prescription of canonical quantization to the gravitational degrees of freedom, one obtains a version of canonical quantum gravity [50] on a discretized space. The discretization eliminates the UV divergences of quantum gravity, thus making it UV complete. The UV completeness implies that the theory is, in principle, applicable even without the renormalization, which, at least at the conceptual level, avoids the problem related to the fact that perturbative quantum gravity is technically non-renormalizable [50].

However, the time is continuous, and the action has the property of diffeomorphism invariance with respect to arbitrary time reparametrizations. In other words, variation of the action with respect to the dynamical field g_{00} leads to the Hamiltonian constraint $H = 0$, which in the quantized version leads to the equation

$$H|\psi\rangle = 0, \tag{75}$$

which leads to the problem of time in quantum gravity [51, 52]. Fortunately, this problem resolves in our approach in which the fundamental time is pre-dynamical, i.e. does not depend on dynamical fields such as g_{00}. As shown in [32], any dynamical theory with time-reparametrization invariance can be reinterpreted as a theory *without* time-reparametrization invariance, but with a fixed value of the conserved energy of the whole Universe. The energy is to be fixed only at the classical level, while in quantum theory a general state is in a superposition of different energies. A superposition has a nontrivial time dependence and is described by a time-dependent Schrödinger equation

$$H|\psi(t)\rangle = i\partial_t|\psi(t)\rangle. \tag{76}$$

In this way the time-reparametrization invariance becomes an emergent property, valid only at the classical level. At the fundamental quantum level there is no time-reparametrization invariance, which resolves the problem of time. We refer the reader to [32] for details.

In addition, we note that, with such a reinterpretation of time-reparametrization invariance, the quantum ground state energy does not couple to gravity, which mitigates the cosmological constant problem [32].

5.4 Hidden variables

The Bell theorem [53] implies that any attempt to formulate quantum theory in terms of "hidden" variables, namely variables that have objective properties independent of measurement (and satisfying certain reasonable requirements [54]), must necessarily be non-local, in the sense of involving instantaneous action at a distance. Such non-locality seems incompatible with the theory of relativity, which makes the existence of such variables problematic if the theory of relativity is fundamental. But in our theory relativity is emergent, not fundamental. Hence, from the point of view of our fundamental theory, hidden variables seem like a much more reasonable possibility. The simplest and best known theory of hidden variables in QM is Bohmian mechanics

[55, 56, 57, 58, 59]. In the context of QFT with emergent relativity, the basic principles of such a Bohmian interpretation are outlined in [60, 61].

We have ended this paper with a discussion of hidden variables, but alternatively we could also have started with it. One could start from the insight that the Bell theorem implies non-locality, which implies that the relativity principle and Lorentz invariance cannot be fundamental [62], which can serve as a different motivation to reinterpret the relativity theory as we did in the present paper. This looks to us as an additional circumstantial evidence that the theory presented in this paper could be on the right track.

6 Discussion and conclusion

In this work we have arrived to a rather radical reformulation of relativity theory by starting from rather mild and seemingly innocent assumptions. The requirement of finite entropy leads to a finite number of degrees of freedom, which in the field theory context leads to a discrete space, which leads to Lorentz violation at small distances, which leads to a preferred frame of reference and pre-relativistic notions of space and time, which leads to a radical modification of physics in the black hole interior and close to its horizon. Such a radical modification of black hole physics offers a simple solution of the black hole information paradox. And yet, such a modification does not make the relativity theory as we know it wrong. Instead, the theory of relativity is an emergent effective theory, still valid as a good approximation in circumstances amenable to typical existing experiments, such as measurements far from black hole horizons, by observers not too fast relative to the preferred cosmological CMB frame, who observe photons and other quantum particles with a wave length much larger than the small fundamental length scale l.

Needless to say, such a radical reformulation of relativity theory opens many new questions, and in this paper we could not have answered them all. But we believe that we have answered some of the most critical ones and formulated a framework for the future research. For example, in the GR context we have only studied a Schwarzschild black hole, so the future research may focus on more general black holes, as well as on various cosmological models [36], dS and AdS spacetimes, wormholes [63], etc. All these may acquire fresh new interpretations from the pre-relativistic point of view with pre-dynamical notions of space and time not depending on the dynamical metric field. Such a framework may also lead to new insights regarding black hole entropy, its proportionality to the horizon area rather than its volume, the holographic principle and AdS/CFT correspondence [64, 65, 66], string theory and M-theory [67], as well as many other aspects of quantum gravity. Even someone's views of foundations of quantum mechanics, especially the meaning of non-locality associated with quantum entanglement and the Bell theorem [53], may change significantly from the point of view that relativity is emergent, rather than fundamental.

To conclude, we believe that emergent relativity is an interesting new theoretical framework worthwhile of further research.

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A Metric tensor density

Here we derive some useful properties of the metric tensor density

$$\tilde{g}^{\mu\nu} = \sqrt{|g|} g^{\mu\nu} \tag{77}$$

in D dimensions, where g is the determinant of $g_{\mu\nu}$. Taking the determinant of (77) we obtain $\tilde{g} = \sqrt{|g|}^D g^{-1}$, where $\tilde{g}$ is the determinant of $\tilde{g}^{\mu\nu}$. Hence

$$|\tilde{g}| = |g|^{(D-2)/2}, \tag{78}$$

so for $D = 2$

$$|\tilde{g}|_{D=2} = 1, \tag{79}$$

while for $D \neq 2$

$$\sqrt{|g|} = |\tilde{g}|^{1/(D-2)}. \tag{80}$$

Inserting (80) into (77) we obtain

$$g^{\mu\nu} = |\tilde{g}|^{D-2} \tilde{g}^{\mu\nu}, \tag{81}$$

which expresses $g^{\mu\nu}$ as a function of $\tilde{g}^{\mu\nu}$. In $D = 2$, expressing $g^{\mu\nu}$ as a function of $\tilde{g}^{\mu\nu}$ is ambiguous. The most natural way to resolve the ambiguity is to *choose* that $g^{\mu\nu}$ is given by the analytic extension of (81) to $D = 2$, leading to

$$g^{\mu\nu}|_{D=2} = \tilde{g}^{\mu\nu}. \tag{82}$$

Such a way of resolving the ambiguity corresponds to the choice $|g|_{D=2} = 1$, as can be seen directly from (77).

B Negative Lagrangians and Hamiltonians

Suppose that L is a Lagrangian with a negative kinetic energy term, i.e., that the factor in front of $\dot{q}^2$ is negative. Then $\bar{L} = -L$ has a positive kinetic energy term.

The canonical momentum associated with L is

$$p = \frac{\partial L}{\partial \dot{q}} = -\frac{\partial \bar{L}}{\partial \dot{q}} = -\bar{p}, \tag{83}$$

with the self-explaining notation, so

$$H = p\dot{q} - L = -\bar{p}\dot{q} + \bar{L} = -[\bar{p}\dot{q} - \bar{L}] = -\bar{H}. \tag{84}$$

The Hamiltonian H has a negative kinetic energy term, while $\bar{H} = -H$ has a positive one. The canonical equations of motion are

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{\partial(-H)}{\partial(-p)} = \frac{\partial \bar{H}}{\partial \bar{p}}, \tag{85}$$

$$\dot{p} = -\frac{\partial H}{\partial q} = \frac{\partial \bar{H}}{\partial q} \ \Rightarrow \ \dot{\bar{p}} = -\frac{\partial \bar{H}}{\partial q}. \tag{86}$$

This proves that the Hamiltonian H with the negative kinetic energy term is equivalent to the Hamiltonian $\bar{H} = -H$, in the sense that they lead to the same equations of motion.

C A toy model for Hawking radiation

Here we present a toy model for Hawking radiation containing only two degrees of freedom, with one degree mimicking the black hole exterior and the other the black hole interior. The model is partially inspired by the Jacobson's model [47] with only one degree of freedom.

Consider a system with two degrees of freedom $q_1(t)$, $q_2(t)$, described with a time-dependent Hamiltonian of the form

$$H(t) = \frac{p_1^2}{2m_1(t)} + \frac{p_2^2}{2m_2(t)} + \frac{p_1 p_2}{m_{12}(t)} + V(q_1, q_2, t). \tag{87}$$

The time dependent masses $m_1(t)$, $m_2(t)$ and $m_{12}(t)$, and the time dependent potential $V(q_1, q_2, t)$, mimic the time-dependent gravitational background. Suppose that at the initial time $t = t_{\text{in}}$ the Hamiltonian takes a simple harmonic oscillator form

$$H_{\text{in}} = \frac{p_1^2}{2m_{\text{in}}} + \frac{p_2^2}{2m_{\text{in}}} + \frac{k_{\text{in}}(q_1^2 + q_2^2)}{2}, \tag{88}$$

where m_{in} and k_{in} are positive constants. This mimics a metric background before the formation of a black hole. Also, suppose that at some later time t the Hamiltonian takes the form

$$H = H_+ + H_-, \tag{89}$$

where

$$H_+ = \frac{p_+^2}{2m} + \frac{kq_+^2}{2}, \quad H_- = -\frac{p_-^2}{2m} - \frac{kq_-^2}{2}, \tag{90}$$

and

$$q_\pm = \frac{q_1 \pm q_2}{\sqrt{2}}, \quad p_\pm = \frac{p_1 \pm p_2}{\sqrt{2}} \tag{91}$$

are new canonical positions and momenta. Here m and k are positive constants, so H_+ is positive and H_- is negative. This mimics the metric background after the formation of a black hole, where H_+ corresponds to the black hole exterior and H_- corresponds to the black hole interior. The inverse of (91) is

$$
\begin{aligned}
q_1 &= \frac{q_+ + q_-}{\sqrt{2}}, \quad q_2 = \frac{q_+ - q_-}{\sqrt{2}}, \\
p_1 &= \frac{p_+ + p_-}{\sqrt{2}}, \quad p_2 = \frac{p_+ - p_-}{\sqrt{2}}.
\end{aligned} \tag{92}
$$

Now let us quantize the system. The destruction operators for the initial Hamiltonian $H_{\rm in}$ are

$$
a_s = \sqrt{\frac{m_{\rm in}\omega_{\rm in}}{2\hbar}} q_s + \frac{i}{\sqrt{2m_{\rm in}\hbar\omega_{\rm in}}} p_s, \tag{93}
$$

for $s = 1, 2$. Here $\omega_{\rm in} = \sqrt{k_{\rm in}/m_{\rm in}}$ is the characteristic frequency of $H_{\rm in}$. From the canonical commutation relations $[q_s, p_{s'}] = i\hbar\delta_{ss'}$ it follows that $[a_s, a_{s'}^\dagger] = \delta_{ss'}$. Likewise, for the later Hamiltonian the destruction operators are

$$
a_\pm = \sqrt{\frac{m\omega}{2\hbar}} q_\pm + \frac{i}{\sqrt{2m\hbar\omega}} p_\pm, \tag{94}
$$

where $\omega = \sqrt{k/m}$. From $[q_+, p_+] = [q_-, p_-] = i\hbar$ it follows that $[a_+, a_+^\dagger] = [a_-, a_-^\dagger] = 1$.

Now we want to express one set of destruction operators in terms of the other. First, from (94) we find

$$
q_\pm = \sqrt{\frac{\hbar}{2m\omega}} (a_\pm^\dagger + a_\pm), \quad p_\pm = i\sqrt{\frac{m\hbar\omega}{2}} (a_\pm^\dagger - a_\pm). \tag{95}
$$

Next we insert this into (92), to express q_s and p_s in terms of $a_\pm$ and $a_\pm^\dagger$. Finally, inserting the resulting expressions into (93), after a straightforward algebra we obtain

$$
\begin{aligned}
a_1 &= \alpha_1(a_+ + a_-) + \beta_1(a_+^\dagger + a_-^\dagger), \\
a_2 &= \alpha_2(a_+ - a_-) + \beta_2(a_+^\dagger - a_-^\dagger),
\end{aligned} \tag{96}
$$

where

$$
\begin{aligned}
\alpha_1 = \alpha_2 &= \frac{1}{2\sqrt{2}} \left(\sqrt{\frac{m_{\rm in}m_{\rm in}}{m\omega}} + \sqrt{\frac{m\omega}{m_{\rm in}m_{\rm in}}} \right) \equiv \alpha, \\
\beta_1 = \beta_2 &= \frac{1}{2\sqrt{2}} \left(\sqrt{\frac{m_{\rm in}m_{\rm in}}{m\omega}} - \sqrt{\frac{m\omega}{m_{\rm in}m_{\rm in}}} \right) \equiv \beta,
\end{aligned} \tag{97}
$$

are Bogoliubov coefficients. They satisfy

$$
\alpha_1^2 + \alpha_2^2 - \beta_1^2 - \beta_2^2 = 1. \tag{98}
$$

Now suppose that initially the quantum system is in the vacuum $|0_{\rm in}\rangle$ of $H_{\rm in}$, obeying

$$
a_1|0_{\rm in}\rangle = 0, \quad a_2|0_{\rm in}\rangle = 0. \tag{99}
$$

Next suppose that the Hamiltonian changes from $H_{\rm in}$ to H very rapidly, so that the state does not have time to change significantly. Thus we assume that the system is in the initial vacuum $|0_{\rm in}\rangle$ even at the later time, even though $|0_{\rm in}\rangle$ is not an eigenstate of H. The eigenstates of H are

$$|n_-, n_+\rangle = \frac{(a_-^\dagger)^{n_-}(a_+^\dagger)^{n_+}}{\sqrt{n_-!n_+!}}|0\rangle, \qquad (100)$$

where $|0\rangle$ is the vacuum of H, obeying $a_-|0\rangle = a_+|0\rangle = 0$. We want to express the state of the system $|0_{\rm in}\rangle$ in terms of the eigenstates (100).

Such an expression can be found from scratch (see e.g. [68]), but here we just quote the result:

$$|0_{\rm in}\rangle = \sqrt{1-(\beta/\alpha)^2}\sum_{n=0}^{\infty}\left(-\frac{\beta}{\alpha}\right)^n |n,n\rangle. \qquad (101)$$

Indeed, it is straightforward to check that (101) satisfies both equations in (99), by using (96) and

$$\begin{aligned} a_-|n,n\rangle = \sqrt{n}|n-1,n\rangle, \quad a_+|n,n\rangle = \sqrt{n}|n,n-1\rangle, \\ a_-^\dagger|n,n\rangle = \sqrt{n+1}|n+1,n\rangle, \quad a_+^\dagger|n,n\rangle = \sqrt{n+1}|n,n+1\rangle. \end{aligned} \qquad (102)$$

A state $|n,n\rangle$ in (101) can be thought of as n pairs, with each pair containing one quantum of excitation associated with H_+ and one quantum of excitation associated with H_-. The state (101) mimics Hawking radiation containing pairs of particles, where in each pair one particle is created outside of the black hole and the other inside the black hole [44].

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